Spatio-temporal and Network Modelling of Diseases
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# On age-dependent branching models for surveillance of infectious diseases controlled by additional vaccination

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# Outline

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- An Example with real data of Avian influenza in Vietnam

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- Different types of stochastic models have been used to model the evolution of an infectious disease into a population: branching processes.

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- Infectious period consists of two parts: an incubation or latency part and comparatively very short contact part. Different levels of severity depend on the length of incubation period

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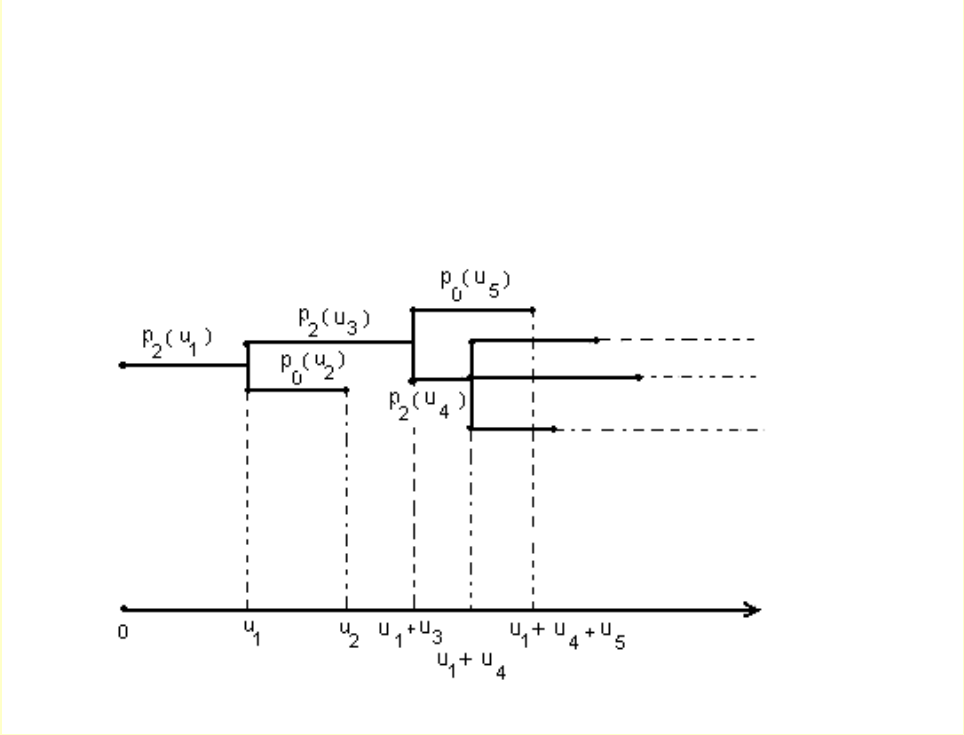
- Let us assume that three types of individuals may exist in the population: infected; healthy but susceptible to catch the infection (susceptible individuals); healthy and immune to the disease
- The disease is spreading when an infected individual is in contact with susceptible individuals.
- We denote by $p_{\alpha,k}(u)$, $u > 0$ the probability that one infected individual at age $u$ contacts $k$ healthy individuals, $k \geq 0$, where $\alpha$ $(0 \leq \alpha \leq 1)$ is the proportion of immunized individuals into the population.

- Following this spreading scheme along time, infected individuals pass on the disease to other susceptible individuals and so on. We model the number of infected individuals in the population by Sevast'yanov's age-dependent branching process: $\{Z_\alpha(t)\}_{t\geq 0}$

  A possible path



- Life-length: $G(t)$

- Family of reproduction laws: $\{p_{\alpha,k}(u)\}_{k\geq 0}$, $u > 0$,
  $m_\alpha(u)$ is the mean of contacts of infected individual at age $u$ when the level of immunized individuals is $\alpha$

- $m_\alpha = \int_0^\infty m_\alpha(u) dG(u)$.

  Intuitively: By life-length we mean the infectious period (measured in real time) till one infected individual infects susceptible individuals or the disease disappears in this individual

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Extinction Time: $T_\alpha = \inf_{t \geq 0}\{Z_\alpha(t) = 0\}$ $\qquad$ $v_\alpha(t) = P(T_\alpha \leq t)$

Intuitively: $T_\alpha$ is the maximal time that the infection survives into the population when the proportion of immune individuals is $\alpha$

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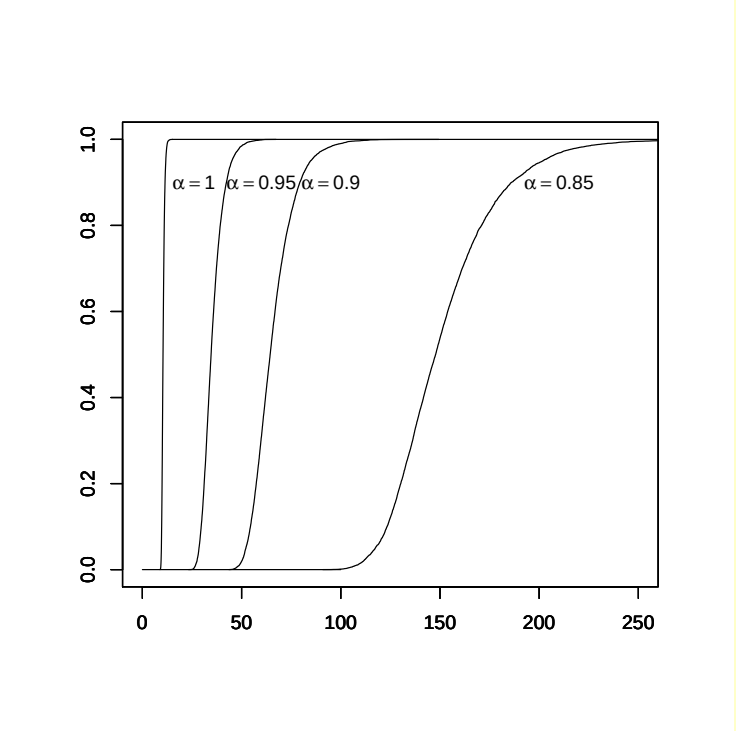


Intuitively, it is clear that the greater is the proportion of the immune individuals, the more probable is that the infectious disease disappears faster

- Mean of the infection extinction time $T_\alpha$

$$\mu_\alpha = E[T_\alpha] = \int_0^\infty (1 - v_\alpha(t))dt$$

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- Continuity property: For each $\varepsilon > 0$ there exist $\eta = \eta(\varepsilon, \alpha) > 0$ such that for all $\alpha^*$ with $m_{\alpha^*} \leq 1$ and $|\alpha - \alpha^*| \leq \eta$,

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- If $\alpha_1 < \alpha_2 \leq 1$, then $\mu_{\alpha_2} \leq \mu_{\alpha_1}$

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- Assume that at an arbitrary time $t_0$ after the infection occurred, we want to vaccinate a proportion $\alpha$ of susceptible individuals. We suppose that the vaccination process finishes at time $t_1$. Therefore, $t_1 - t_0$ is the period of time that is taken for immunization, called the vaccination period. We suppose that vaccination period is fixed a priori and does not depend on $\alpha$.

- Optimal proportion of vaccinated individuals:

  *For fixed $\tau$, we are looking for vaccination policies, which guarantee that the average time to extinction of an infection after vaccination period $t_1$, is less than or equal to $t_1 + \tau$.*

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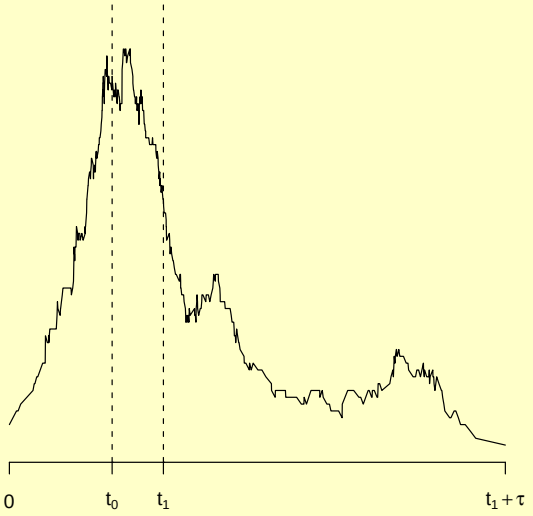


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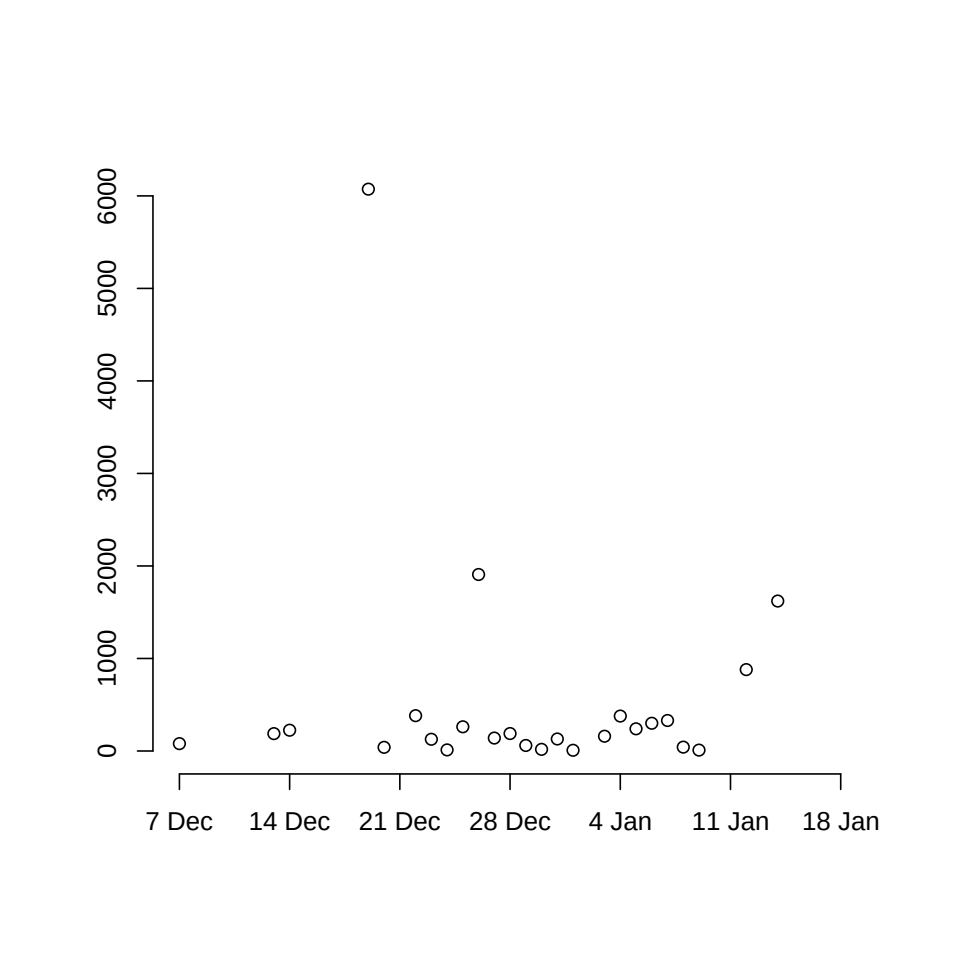
3) incubation period for avian influenza virus is estimated between 3 and 7 days (IDSA, 2007) $\Longrightarrow$ mean of gamma is $5$ and the shape is 16, which guarantee that the survival period in $90\%$ of individuals is between 3 and 7 days

# Epidemic Modelling: Avian influenza in Vietnam

Data for infected domestic birds detected between $7^{th}$ December 2006 and $14^{th}$ January 2007 in South Vietnam



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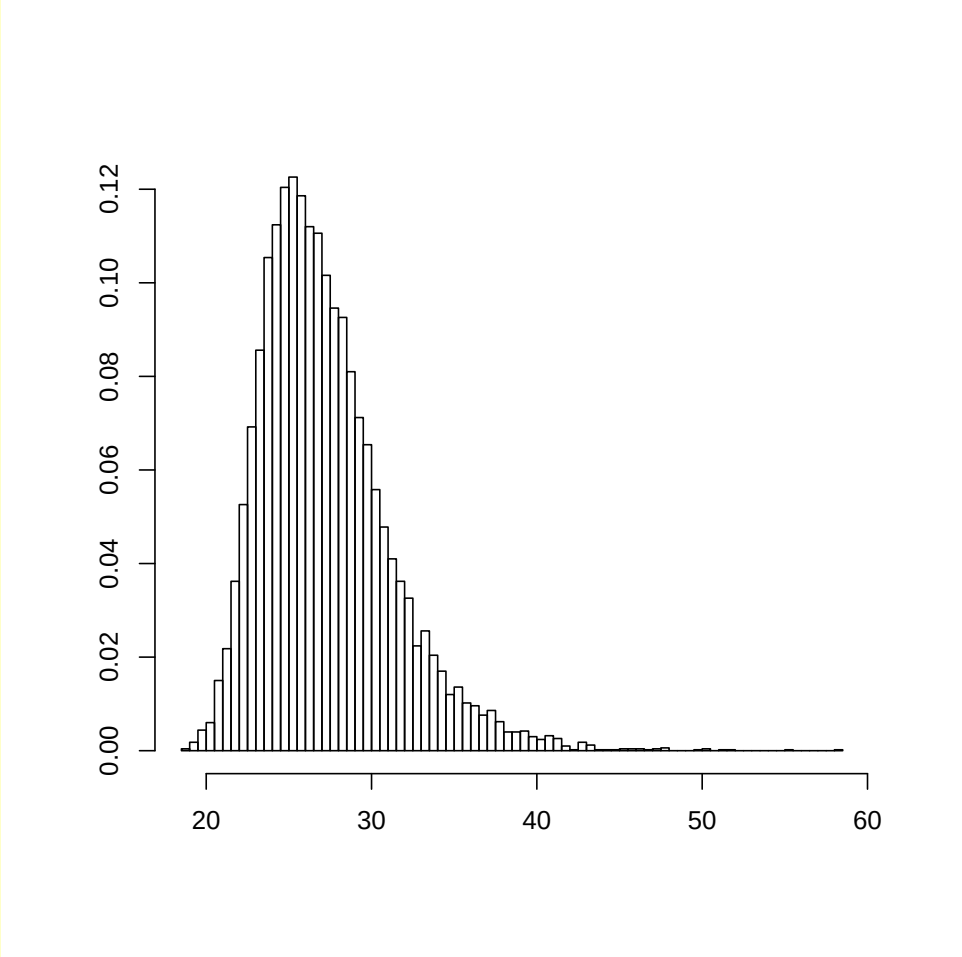
(v) The number of individuals incubating the virus at $19^{th}$ December we estimate by $2132 = 413 * (413/80)$

(vi) $\alpha_{opt}(30, 2132) = 0.97.$

Histogram of simulated extinction times for $\alpha = 0.97$.



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(ii) The optimal vaccination level does not depend on the initial number of individuals incubating the virus (sensitivity analysis)

# Epidemic Modelling: Open questions for further research

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2) How to model the vaccination period?

   By age-dependent branching processes in varying environments.

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Thank you for your attention!