Contact Tracing

Johannes Müller

Technical University Munich
Center for Mathematics

Contact Tracing - Aim

Concept:

If we find an infected individual (so-called "index case"), we ask for (potentially) infectious contacts.

Contact Partners are called in to a doctor. In this way, it is possible to find more infectious persons.

Related Methods:

Clustering ("who has a similar behaviour like you" instead of "with whom did you have contact"), Screening

Diseases:

HIV, SARS etc., Tuberculosis, sexually transmitted diseases (Gonorrhea, Chlamydia)...

Contact Tracing - Questions

- What's the effect of a contact tracing program?

- How is an efficient contact tracing procedure designed?
  - Do we trace as much as we can?
  - Do we trace only the contacts of the primary index case?
  - How hard do we search for contacts?

- How does contact tracing differs in the onset of an epidemic and an endemic state?

- Estimate Parameters?

- Can we find monitoring tools for contact tracing programs?

Contact Tracing - Some Models

(a) Hethcote/York, 1984: Gonorrhea
Deterministic model. Nonlinear correction of the incidence(!) function. No connection to stochastic models.

(b) Lourdes/Azoza, 1995: HIV
Deterministic model. Linear (!) incidence function. No connection to nonlinear stochastic models.

(c) Kretzschmar/Duynhjoven/Severijnen, 1996: Gonorrhea, Chlamydia
Detailed simulation model. No attempt for analysis.

(d) Huerta, Tsimring 2002
Stochastic network ("small world network") and mean field

(e) Fraser, Riley, Anderson, Ferguson 2004
Deterministic PDE-Model; Tracing process is handled in a linear way.

(f) Eames 2006
Pair approximation in stochastic network.

Basic Epidemic Model

Simple stochastic SIS model with random mixing.

Consider a population of N individuals.

Define the standard epidemic model on this population:

- contact rate: β (random mixing)
- probability q for the transmission of the infection per S/I-contact
- recovery rate μ

Contact Tracing - Different Situations

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT

Contact Tracing - Onset

- Onset of the diseases / initial phase of CT
- Onset of the diseases / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT

Graph of Infecteds:

Nodes: Infected and Infectious individuals.

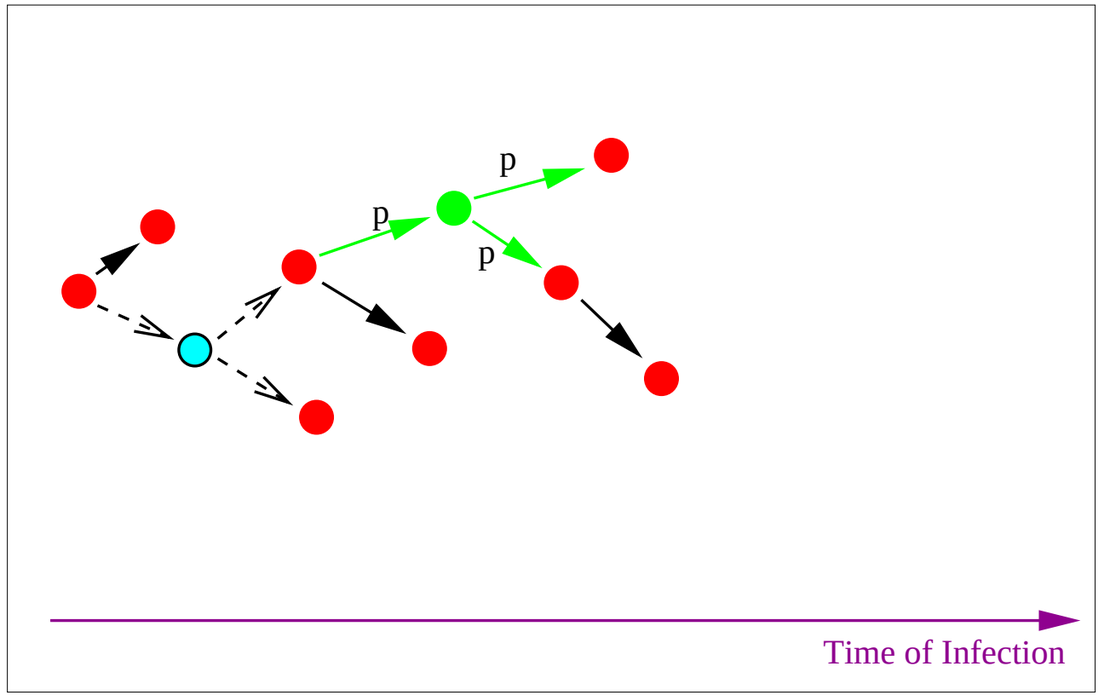
Edges: Directed edge from "Infector" to "Infectee".

Tracing Process:

- Define a screening rate: with rate σ an individual is detected (the primary index case) and removed.
- Edges have the probability p to become detected and to be removed.

Neglected: (many things, especially)

- contact tracing is no instantaneous process.
- no test is perfect.
- diseases are not Markovian processes.
- diseases are not homogeneous.



Contact Tracing - Onset

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT

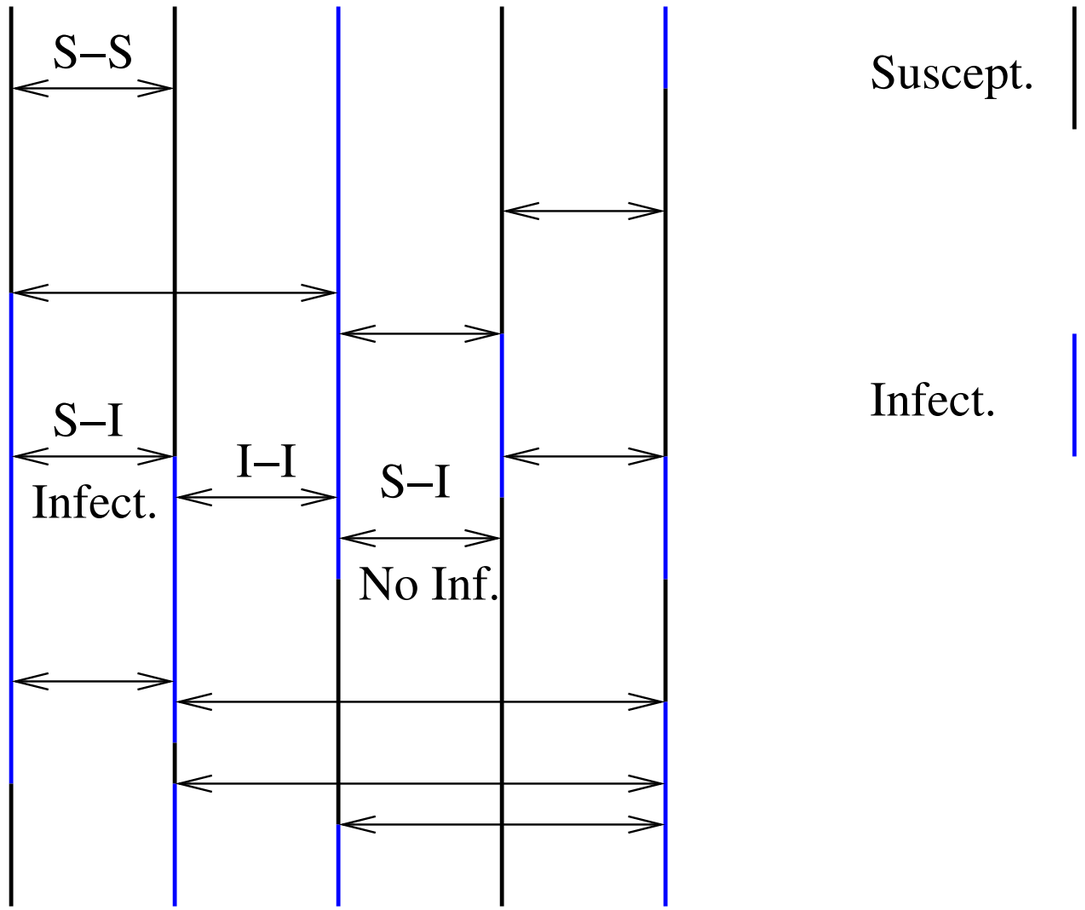
Results in the Onset of the diseases / equilibrium phase of CT:

- The process tends to a stable exponentially growing phase,
  where distributions (e.g. size of connected components) stabilizes
- Formula for the effective reproduction number R_e (though not nice in the general case)
- E(Number of detected cases per index case) $= p\frac{R_0}{1+R_0} + \mathcal{O}(p^2)$
  where p is the tracing probability
- E(size of a connected component in the infection graph) $\leq 1 + R_0$
- Structure of a connected component in the infection graph (without contact tracing)

(M',Kretzschmar, Dietz 2000; M', Möhle 2003; M', Hösel 2007)

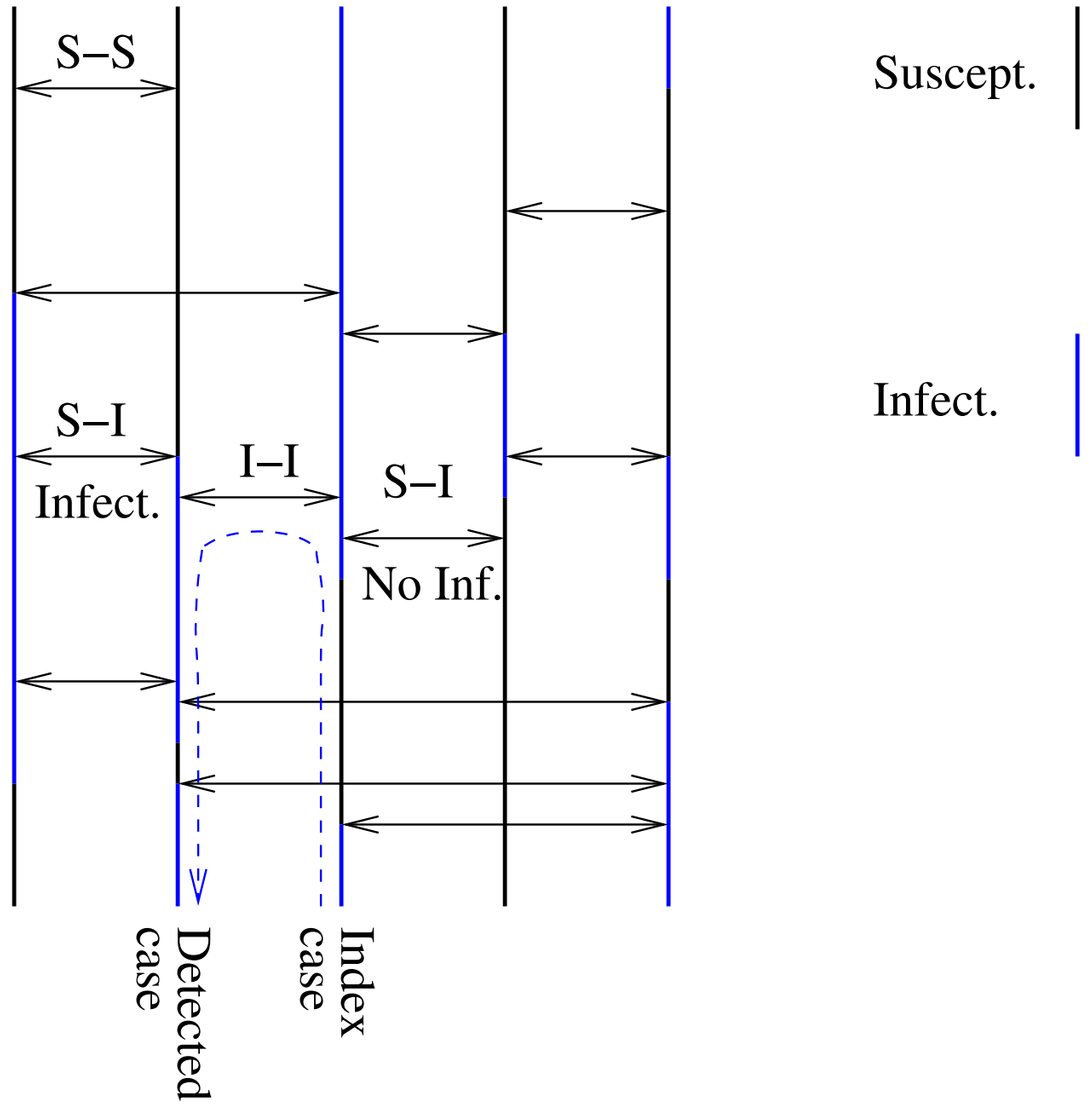
Contact Tracing - Endemic Case

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT



Contact Tracing - Endemic Case

- Onset of the epidemic / initial phase of CT
- Onset of the epidemic / equilibrium phase of CT
- Epidemic equilibrium / initial phase of CT
- Epidemic equilibrium / equilibrium phase of CT
- Disease is subcritical / equilibrium phase of CT



Contact Tracing - Tracing Model in the Endemic State

Two different type of traced contacts:

- Infection Tracing
  Tracing of infectee or infector

- Random Tracing
  Tracing of contacts via that no infection has been transmitted.

Tracing Model:

- All contacts have to be considered.
- In a finite, randomly mixing populations, any person has had contact to all other persons (perhaps long ago).
- Declining of tracing probability with the time since contact is necessary to incorporate.

$$P(\text{tracing of a contact a time units ago}) = p_1 e^{-p_2 a}$$

- Detection/screening rate σ.

Contact Tracing in the Endemic state - Analysis

Central tool:

$$\kappa(a) = P(\text{ individual has been infected } a \text{ time units ago and is infectious since then})$$

$$\frac{d}{da}\kappa(a) = -\kappa(a)\{\mu + \sigma + \text{Removal rate via a traced contact}\}, \qquad \kappa(0) = 1.$$

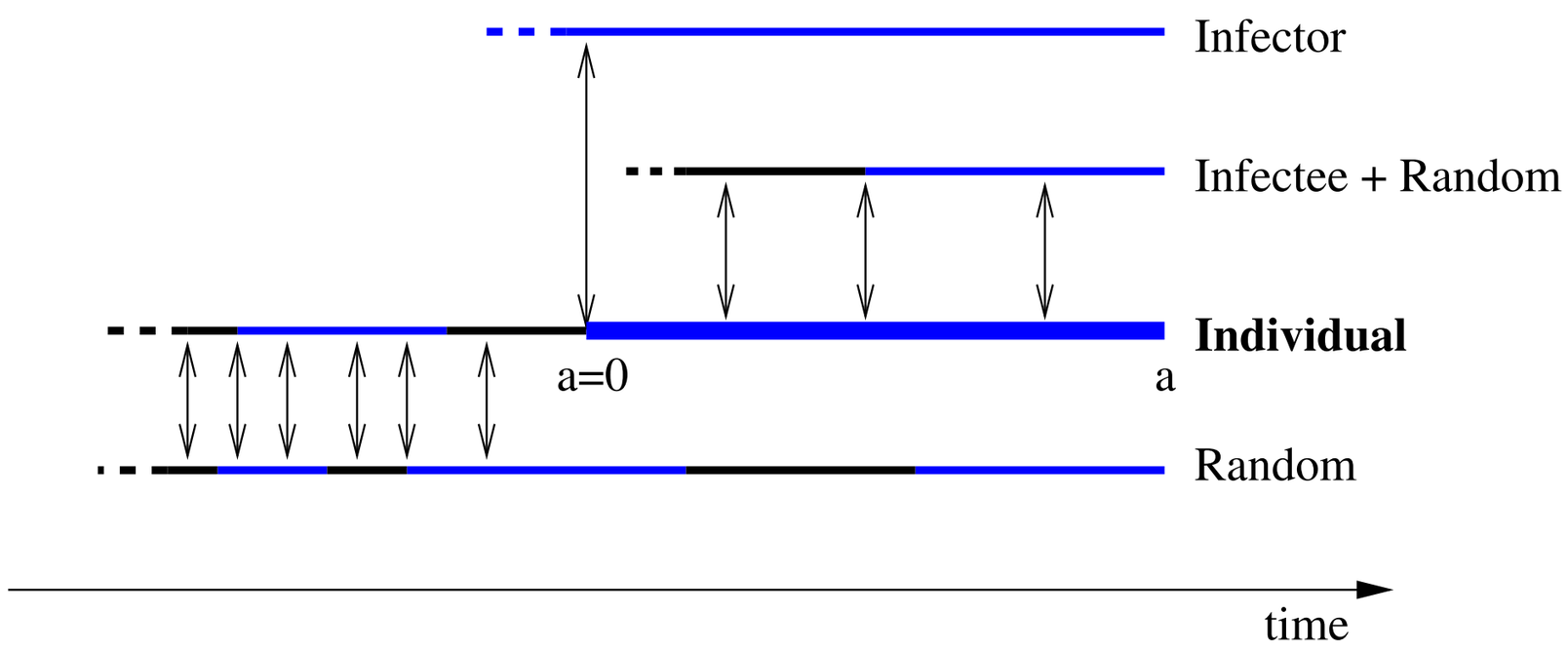
Contact Tracing in the Endemic state - Analysis

Central tool:

$$\kappa(a) = P(\text{ individual has been infected } a \text{ time units ago and is infectious since then})$$

$$\frac{d}{da}\kappa(a) = -\kappa(a)\{\mu + \sigma + \text{Removal via a traced contact}\}, \qquad \kappa(0) = 1.$$

- Assumption: approximately constant prevalence.
- Ten cases (blue: infectious, black: susceptible):



Contact Tracing in the Endemic state - Analysis

Good news: Up to the first order in p_1, the differential equation for κ is possible to work out.

Bad news: The result is terribly lengthy.

Attempt: Approximation.
(a) neglect all contacts that took place before the last infectious period of an individual started
(b) Approximate terms in exponent by quadratic terms in time since infection a

Let

$$\tau_0 = q\beta/(\mu+\sigma) \qquad \text{(Fraction of susceptibles without contact tracing)}$$

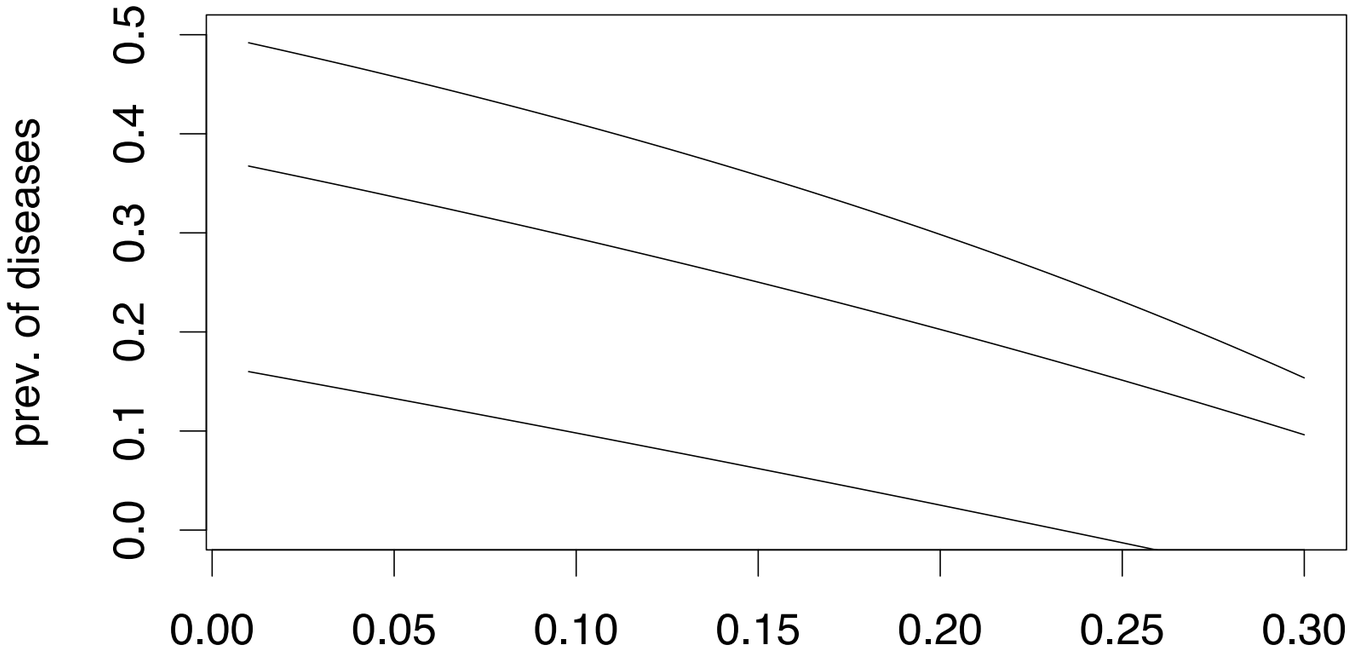
then

$$\kappa(a) \approx \underbrace{e^{-(\mu+\sigma)a}}_{\text{No Screening}} \; \underbrace{e^{-\sigma p_1 a}}_{\text{Infector}} \; \underbrace{e^{-\beta q \tau_0 \; p_1\sigma \; a^2/2}}_{\text{Infectees}} \; \underbrace{e^{-\beta(1-\tau_0) \; p_1\sigma \; a^2/2}}_{\text{Random tracing: I-I}} + \mathcal{O}(p_1^2)$$

Contact Tracing in the Endemic state - Results

Endemic Equilibrium:

$$1 = R_e = q\beta S \int_0^\infty \kappa(a)\, da$$



$(\beta/(\mu+\sigma) \in \{3,4,5\},\, q = 0.4,\, \sigma/(\mu+\sigma) = 0.2)$

Effect even for small values like $p_1 \leq 0.1$

Contact Tracing in the Endemic state - Results

Contact Tracing:

Rough Estimation of scanned persons per index case (do not through away these data!):

$$E(\text{No of traced/scanned contacts per index case}) \approx p_1 R_0/q$$

Rough Estimation of scanned persons per index case:

$$E(\text{Detected Persons per Index Case}) \leq p_1(\underbrace{1}_{\text{Infector}} + \underbrace{1}_{\text{Infectee}} + \underbrace{(1-\tau_0)R_0/q}_{\text{Random Tracing}})$$

Relative number of hits:

$$\frac{\text{detected cases}}{\text{scanned cases}} \leq 2q/R_0 + (1-\tau_0)$$

Screening:

Compare with the case of pure screening:

$$\frac{\text{detected cases}}{\text{screened cases}} = (1-\tau_0)$$

For $R_0 \gg 1$, the relative success (number of hits versus number of tests) is in the same range for contact tracing and screening; the latter is cheaper!

Contact Tracing in the Endemic state - Questions

Are there (endemic) situations where contact tracing my be of use?

• Core groups

- Contact tracing may reveal a core group structure
  (a lot of detailed data is necessary - unrealistically?)
- Contact tracing may reveal core group members
  (clustering may be more efficient in this)

• Asymptomatic cases

- Contact tracing may reveal asymptomatic cases

Summary: Contact Tracing

• Different situations (onset/endemic state) require different mathematical methods and may lead to different conclusions!

Contact tracing in the Endemic State and a Random Mixing Population

• For large populations, we can analyse models for contact tracing in the endemic case.

• Contact tracing is able to reduce the prevalence of diseases even for small tracing probabilities.

• In a random mixing population, screening may be preferable.

• In a core group situation, clustering may be preferable

• If you have many asymptomatic cases, contact tracing may be of use