Scale-Free Network of Dengue in Singapore

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Model's Equations

$$
\begin{aligned}
\frac{\mathrm{d}S_{\mathrm{H}}}{\mathrm{d}t} &= -ab_{\mathrm{H}} I_{\mathrm{M}} \frac{S_{\mathrm{H}}}{N_{\mathrm{H}}} - \mu_{\mathrm{H}} S_{\mathrm{H}} + r_{\mathrm{H}} N_{\mathrm{H}} \left(1 - \frac{N_{\mathrm{H}}}{k_{\mathrm{H}}}\right) \\
\frac{\mathrm{d}I_{\mathrm{H}}}{\mathrm{d}t} &= ab_{\mathrm{H}} I_{\mathrm{M}} \frac{S_{\mathrm{H}}}{N_{\mathrm{H}}} - (\mu_{\mathrm{H}} + \alpha_{\mathrm{H}} + \gamma_{\mathrm{H}}) I_{\mathrm{H}} \\
\frac{\mathrm{d}R_{\mathrm{H}}}{\mathrm{d}t} &= \gamma_{\mathrm{H}} I_{\mathrm{H}} - \mu_{\mathrm{H}} R_{\mathrm{H}} \\
\frac{\mathrm{d}S_{\mathrm{M}}}{\mathrm{d}t} &= p_{\mathrm{S}} (c - d \sin(2\pi f t + \phi)) \\
&\quad \times S_{\mathrm{E}} \theta (c - d \sin(2\pi f t + \phi)) - \mu_{\mathrm{M}} S_{\mathrm{M}} - ab_{\mathrm{M}} S_{\mathrm{M}} \frac{I_{\mathrm{H}}}{N_{\mathrm{H}}} \\
\frac{\mathrm{d}L_{\mathrm{M}}}{\mathrm{d}t} &= ab_{\mathrm{M}} S_{\mathrm{M}} \frac{I_{\mathrm{H}}}{N_{\mathrm{H}}} - \mathrm{e}^{(-\mu_{\mathrm{M}} \tau_{\mathrm{I}})} a S_{\mathrm{M}}(t - \tau_{\mathrm{I}}) \frac{I_{\mathrm{H}}(t - \tau_{\mathrm{I}})}{N_{\mathrm{H}}(t - \tau_{\mathrm{I}})} - \mu_{\mathrm{M}} L_{\mathrm{M}} \\
\frac{\mathrm{d}I_{\mathrm{M}}}{\mathrm{d}t} &= \mathrm{e}^{(-\mu_{\mathrm{M}} \tau_{\mathrm{I}})} ab_{\mathrm{M}} S_{\mathrm{M}}(t - \tau_{\mathrm{I}}) \frac{I_{\mathrm{H}}(t - \tau_{\mathrm{I}})}{N_{\mathrm{H}}(t - \tau_{\mathrm{I}})} - \mu_{\mathrm{M}} I_{\mathrm{M}} \\
&\quad + p_{\mathrm{I}}(c - d \sin(2\pi f t + \phi)) I_{\mathrm{E}} \theta (c - d \sin(2\pi f t + \phi)) \\
\frac{\mathrm{d}S_{\mathrm{E}}}{\mathrm{d}t} &= [r_{\mathrm{M}} S_{\mathrm{M}} + (1 - g) r_{\mathrm{M}} I_{\mathrm{M}}] \left(1 - \frac{S_{\mathrm{E}} + I_{\mathrm{E}}}{k_{\mathrm{E}}}\right) \\
&\quad - \mu_{\mathrm{E}} S_{\mathrm{E}} - p_{\mathrm{S}} (c - d \sin(2\pi f t + \phi)) S_{\mathrm{E}} \theta (c - d \sin(2\pi f t + \phi)) \\
\frac{\mathrm{d}I_{\mathrm{E}}}{\mathrm{d}t} &= g r_{\mathrm{M}} I_{\mathrm{M}} \left(1 - \frac{S_{\mathrm{E}} + I_{\mathrm{E}}}{k_{\mathrm{E}}}\right) - \mu_{\mathrm{E}} I_{\mathrm{E}} \\
&\quad - p_{\mathrm{I}} (c - d \sin(2\pi f t + \phi)) I_{\mathrm{E}} \theta (c - d \sin(2\pi f t + \phi)).
\end{aligned}
$$

$$\frac{\mathrm{d}i_{\mathrm{H}}}{\mathrm{d}t} = ab_{\mathrm{H}}\frac{S_{\mathrm{H}}}{N_{\mathrm{H}}}i_{\mathrm{M}} - (\mu_{\mathrm{H}} + \alpha_{\mathrm{H}} + \gamma_{\mathrm{H}})i_{\mathrm{H}}$$

$$\frac{\mathrm{d}l_{\mathrm{M}}}{\mathrm{d}t} = ab_{\mathrm{M}}\frac{S_{\mathrm{M}}}{N_{\mathrm{H}}}i_{\mathrm{H}} - \mu_{\mathrm{M}}l_{\mathrm{M}} - \mathrm{e}^{(-\mu_{\mathrm{M}}\tau_{\mathrm{I}})}ab_{\mathrm{M}}\frac{S_{\mathrm{M}}(t-\tau_{\mathrm{I}})}{N_{\mathrm{H}}(t-\tau_{\mathrm{I}})}i_{\mathrm{H}}(t-\tau_{\mathrm{I}})$$

$$\begin{aligned}\frac{\mathrm{d}i_{\mathrm{M}}}{\mathrm{d}t} &= \mathrm{e}^{(-\mu_{\mathrm{M}}\tau_{\mathrm{I}})}ab_{\mathrm{M}}\frac{S_{\mathrm{M}}(t-\tau_{\mathrm{I}})}{N_{\mathrm{H}}(t-\tau_{\mathrm{I}})}i_{\mathrm{H}}(t-\tau_{\mathrm{I}}) - \mu_{\mathrm{M}}i_{\mathrm{M}}\\ &\quad + p_{\mathrm{I}}(c - d\sin(\Phi))i_{\mathrm{E}}\theta(c - d\sin(\Phi))\end{aligned}$$

$$\frac{\mathrm{d}i_{\mathrm{E}}}{\mathrm{d}t} = gr_{\mathrm{M}}\left(1 - \frac{S_{\mathrm{E}}}{k_{\mathrm{E}}}\right)i_{\mathrm{M}} - \mu_{\mathrm{E}}i_{\mathrm{E}} - p_{\mathrm{I}}(c - d\sin(\Phi))i_{\mathrm{E}}\theta(c - d\sin(\Phi)),$$

$$
\begin{aligned}
i_{\mathrm{H}} &= c_1 \exp(\lambda t) \\
l_{\mathrm{M}} &= c_2 \exp(\lambda t) \\
i_{\mathrm{M}} &= c_3 \exp(\lambda t) \\
i_{\mathrm{E}} &= c_4 \exp(\lambda t) .
\end{aligned}
$$

$$
\begin{vmatrix}
-(\lambda+\gamma_{\mathrm{H}}+\alpha_{\mathrm{H}}+\mu_{\mathrm{H}}) & 0 & ab_{\mathrm{H}}\frac{S_{\mathrm{H}}(t)}{N_{\mathrm{H}}(t)} & 0 \\
ab_{\mathrm{M}}\frac{S_{\mathrm{M}}}{N_{\mathrm{H}}}-ab_{\mathrm{M}}\mathrm{e}^{(-\mu_{\mathrm{M}}\tau_{\mathrm{I}})}\frac{S_{\mathrm{M}}(t-\tau_{\mathrm{I}})}{N_{\mathrm{H}}(t-\tau_{\mathrm{I}})}\mathrm{e}^{-\lambda\tau_{\mathrm{I}}} & -(\lambda+\mu_{\mathrm{M}}) & 0 & 0 \\
ab_{\mathrm{M}}\mathrm{e}^{(-\mu_{\mathrm{M}}\tau_{\mathrm{I}})}\frac{S_{\mathrm{M}}(t-\tau_{\mathrm{I}})}{N_{\mathrm{H}}(t-\tau_{\mathrm{I}})}\mathrm{e}^{-\lambda\tau} & 0 & -(\lambda+\mu_{\mathrm{M}}) & \begin{array}{l} p_{\mathrm{I}}(c-d\sin\Phi) \\ \theta(c-d\sin(\Phi)) \end{array} \\
0 & 0 & gr_{\mathrm{M}}\left(1-\frac{S_{\mathrm{E}}}{k_{\mathrm{E}}}\right) & \begin{array}{l} -\lambda-\mu_{\mathrm{E}}- \\ p_{\mathrm{I}}(c-d\sin(\Phi)) \\ \theta(c-d\sin(\Phi)) \end{array}
\end{vmatrix} = 0.
$$

An approximate threshold

$$R(t) = \frac{ab_{\mathrm{M}}}{(\gamma_{\mathrm{H}} + \alpha_{\mathrm{H}} + \mu_{\mathrm{H}})} \frac{S_{\mathrm{M}}(t-\tau_{\mathrm{I}})}{N_{\mathrm{H}}(t-\tau_{\mathrm{I}})} \frac{a \exp(-\mu_{\mathrm{M}}\tau_{\mathrm{I}})\, b_{\mathrm{H}} c}{\mu_{\mathrm{M}}} \frac{S_{\mathrm{H}}(t)}{N_{\mathrm{H}}(t)}$$

$$+ \frac{p_{\mathrm{I}}\,(c - d\sin\Phi)\, g r_{\mathrm{M}} \left(1 - \frac{S_{\mathrm{E}}(t)}{k_{\mathrm{E}}}\right) \theta(c - d\sin(\Phi))}{\mu_{\mathrm{M}}\,(\mu_{\mathrm{E}} + p_{\mathrm{I}}\,(c - d\sin\Phi)\,\theta(c - d\sin(\Phi)))} > 1.$$

$$\frac{\partial R}{\partial a} = 2\frac{abc}{\left(\mu_H + \alpha_H + \gamma_H\right)}\frac{N_M\left(t-\tau_I\right)}{N_H\left(t-\tau_I\right)}\frac{e^{-\mu_M \tau_I}}{\mu_M}$$

$$\frac{\partial R}{\partial \mu_E} = -\frac{p_I c_I g r_M \left(1-\frac{S_E}{\kappa_E}\right)}{\mu_M \left(\mu_E + p_I c_I\right)^2}$$

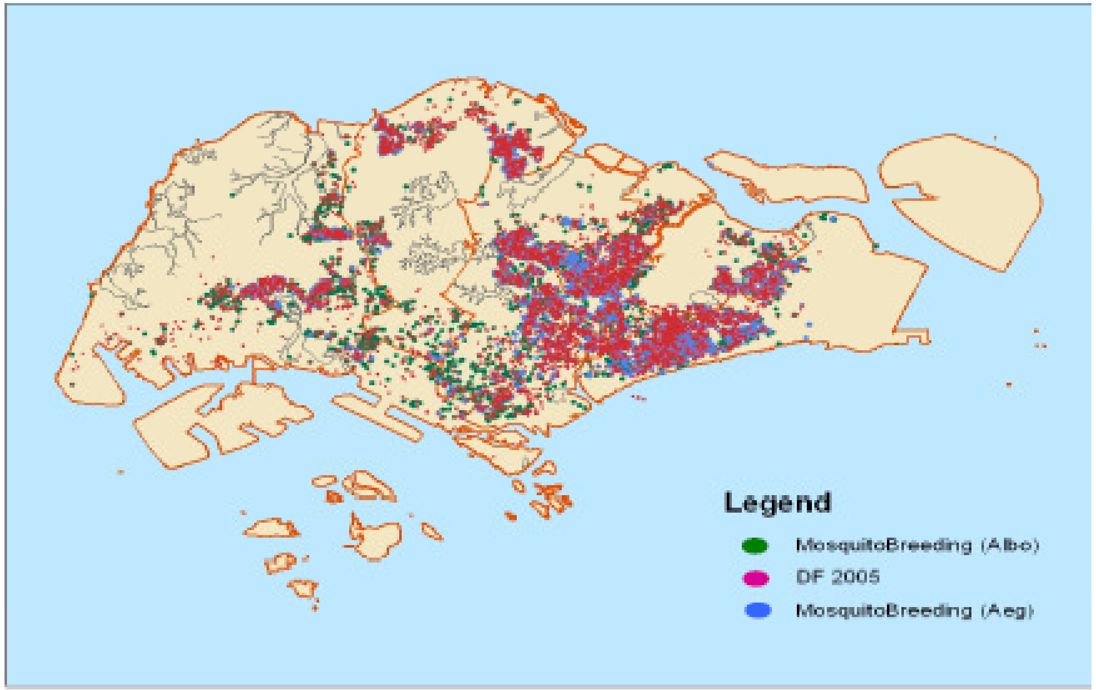
$$\frac{\partial R}{\partial \kappa_E} = \frac{p_I c_I g r_M S_E}{\mu_M \left(\mu_E + p_I c_I\right)\kappa_E{}^2}$$

$$\frac{\partial R}{\partial ac} = \frac{ab}{\left(\mu_H + \alpha_H + \gamma_H\right)}\frac{N_M\left(t-\tau_I\right)}{N_H\left(t-\tau_I\right)}\frac{e^{-\mu_M \tau_I}}{\mu_M}$$

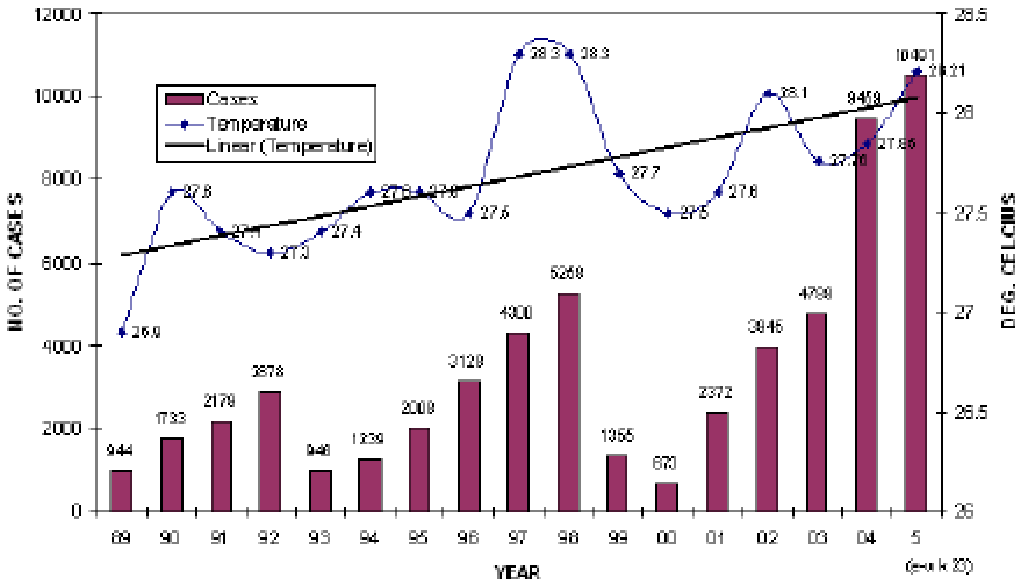
$$\frac{\partial R}{\partial \mu_M} = -\frac{\tau_I a}{\left(\mu_H + \alpha_H + \gamma_H\right)}\frac{N_M\left(t-\tau_I\right)}{N_H\left(t-\tau_I\right)}\frac{a e^{-\mu_M \tau_I} bc}{\mu_M}$$

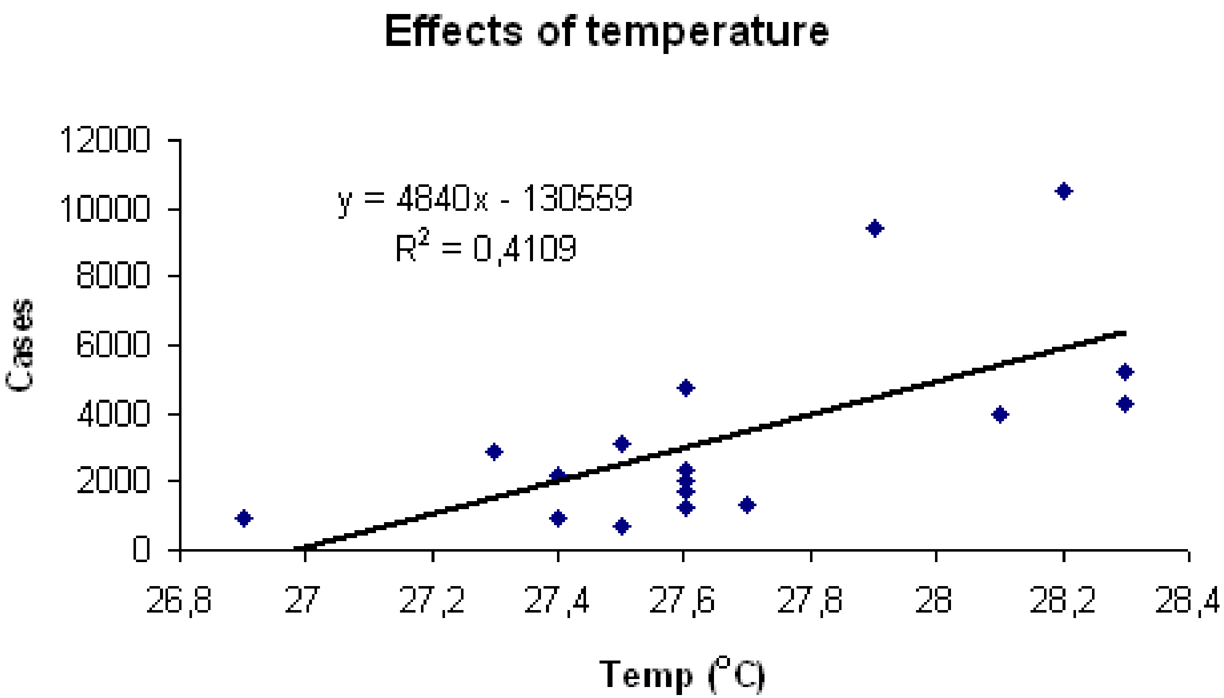
$$-\frac{a}{\left(\mu_H + \alpha_H + \gamma_H\right)\mu_M{}^2}\frac{N_M\left(t-\tau_I\right)}{N_H\left(t-\tau_I\right)}\frac{a e^{-\mu_M \tau_I} bc}{\mu_M} - \frac{\tau_I e^{-\mu_M \tau_I} p_I c_I g r_M \left(1-\frac{S_E}{\kappa_E}\right)}{\mu_M{}^2\left(\mu_E + p_I c_I\right)}$$

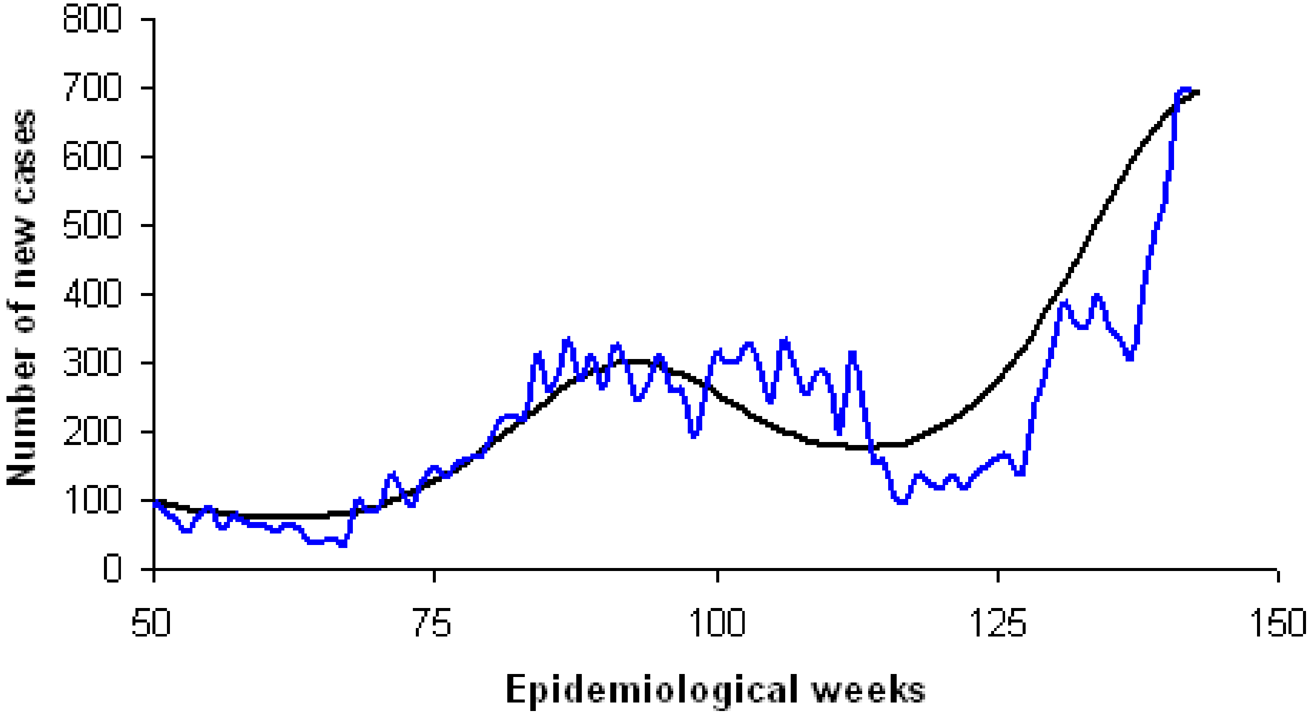
DISTRIBUTION OF DF/DHF CASES & *AEDES* MOSQUITO BREEDING, 2005



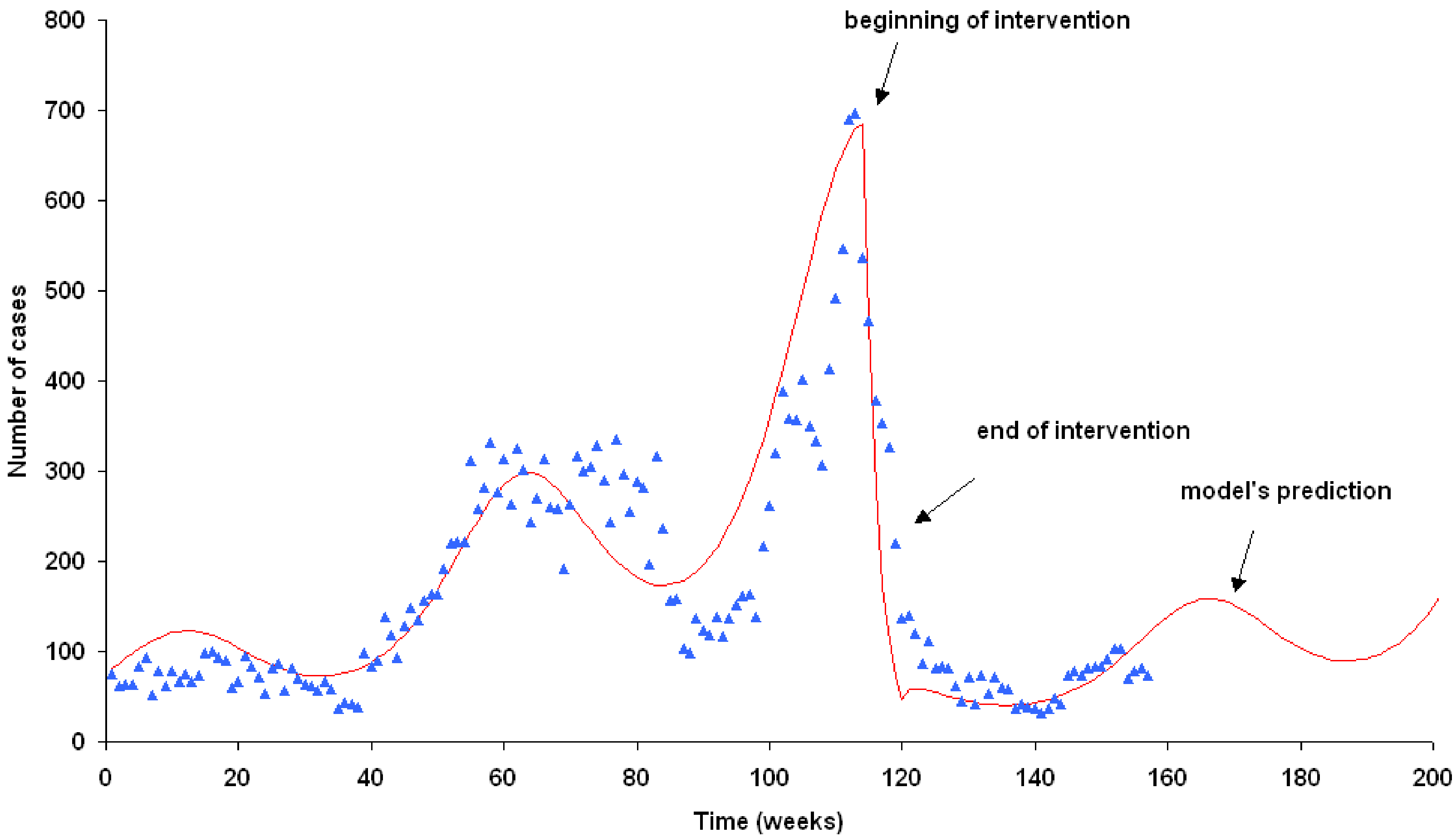
AMBIENT TEMPERATURE AND REPORTED DF/DHF CASES, 1989 - 2005



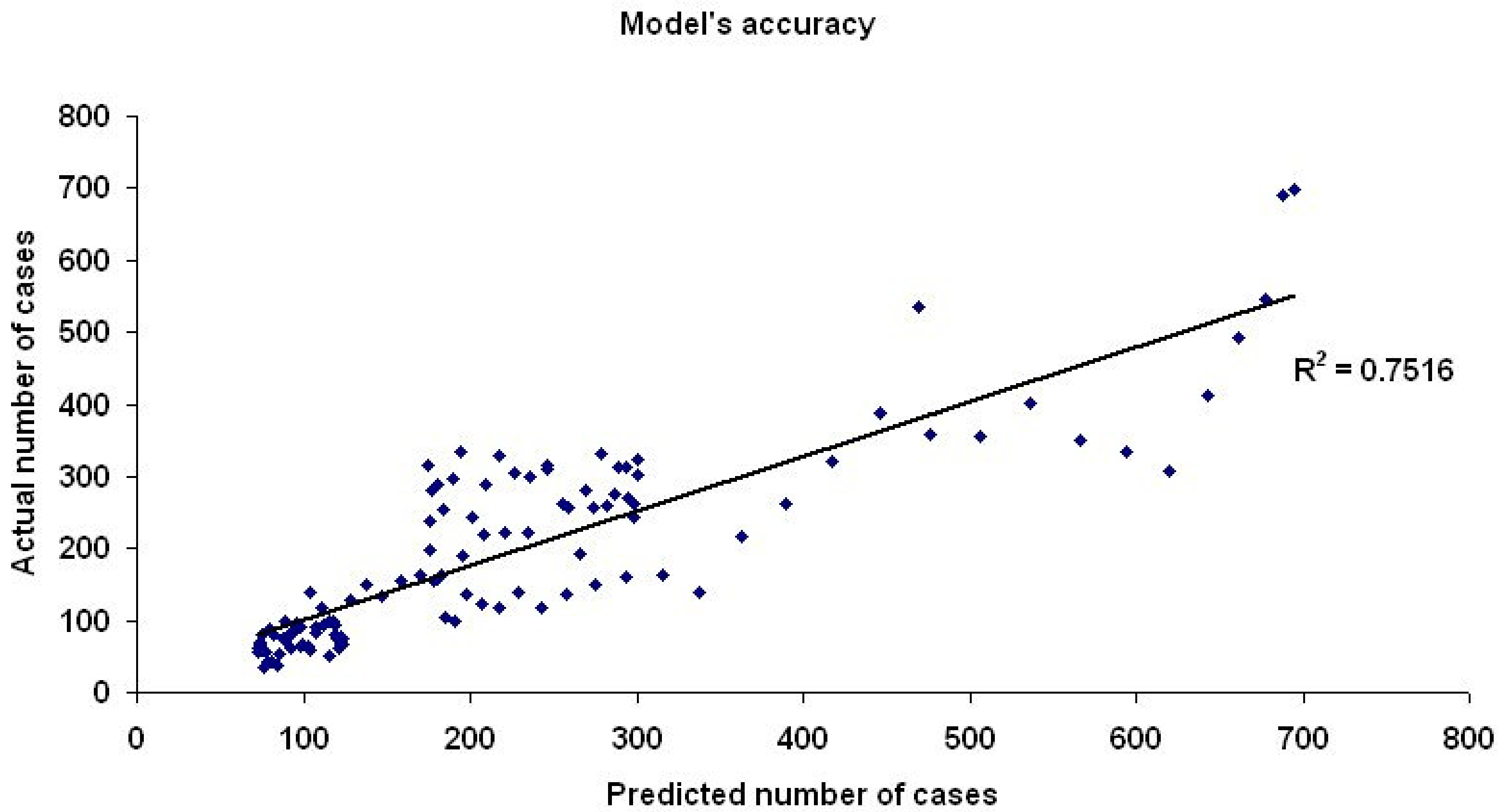

Effects of temperature
y = 4840x - 130559
R2 = 0,4109
Cases
12000
10000
8000
6000
4000
2000
0
26,8
27
27,2
27,4
27,6
27,8
28
28,2
28,4
Temp (oC)



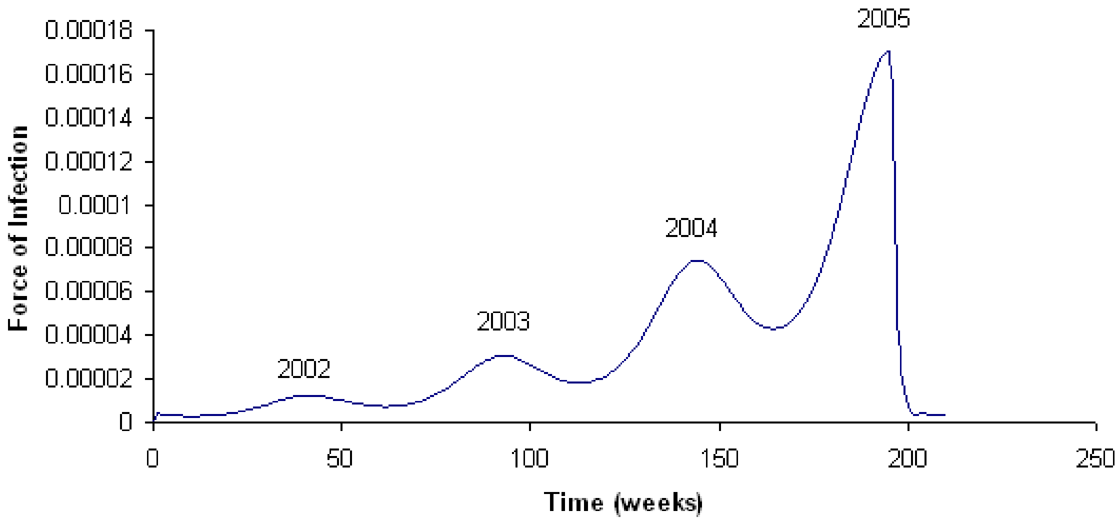
800
700
600
500
400
300
200
100
0
Number of new cases
50
75
100
125
150
Epidemiological weeks



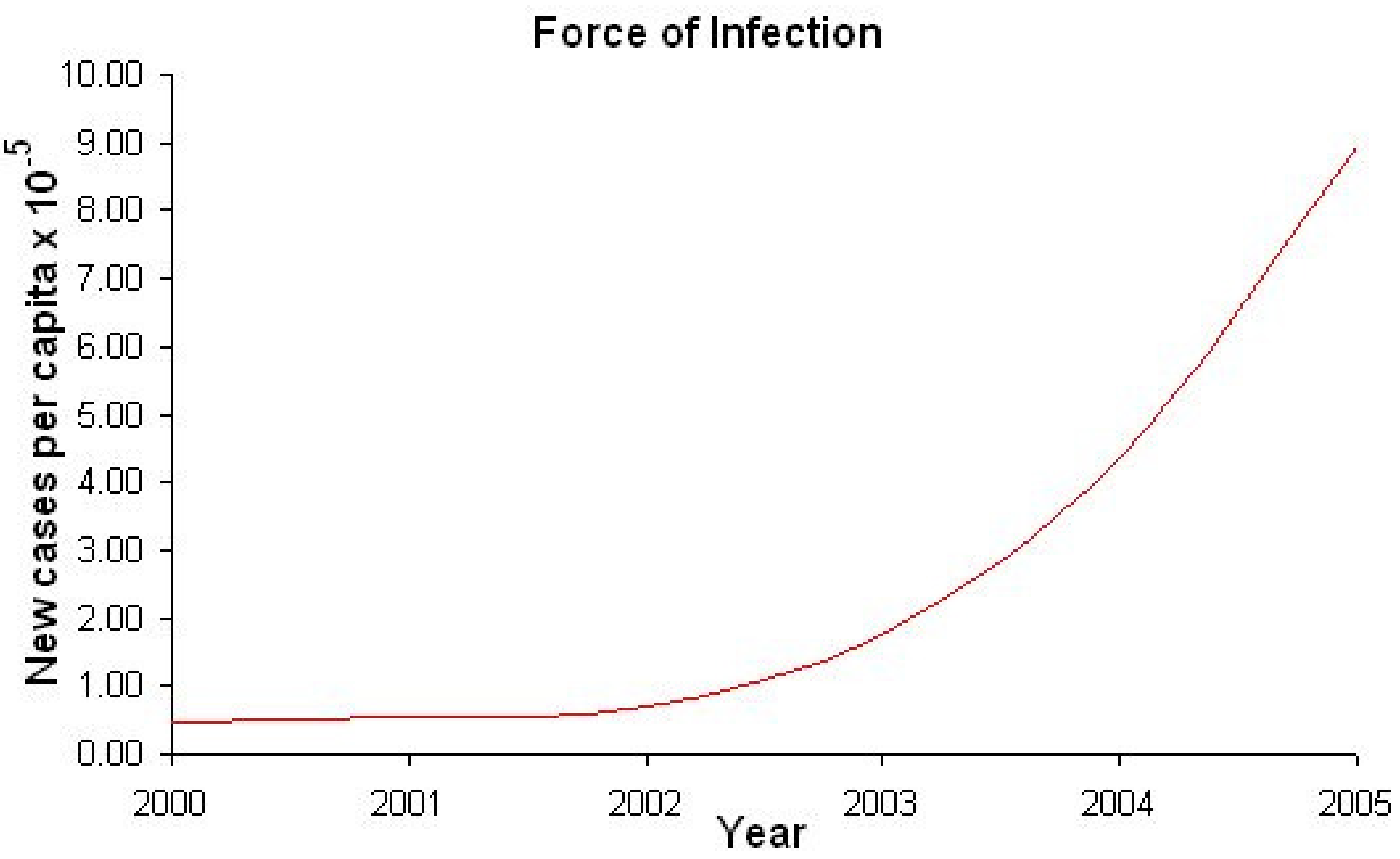
beginning of intervention
end of intervention
model's prediction
Number of cases
Time (weeks)
0
100
200
300
400
500
600
700
800
0
20
40
60
80
100
120
140
160
180
200

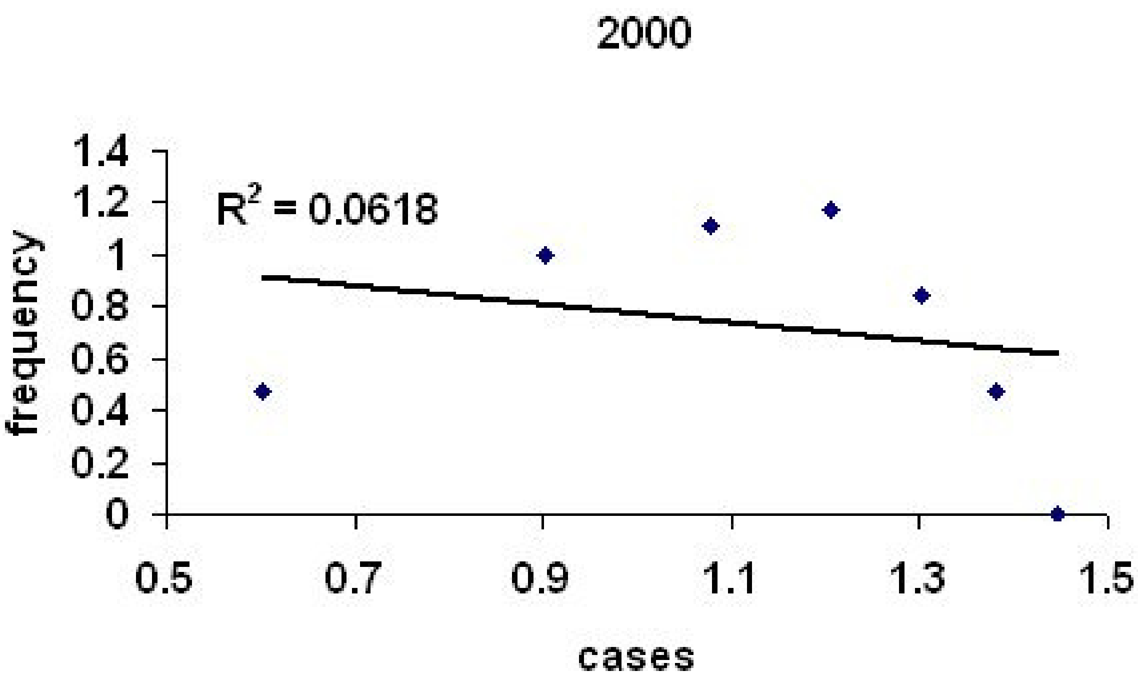


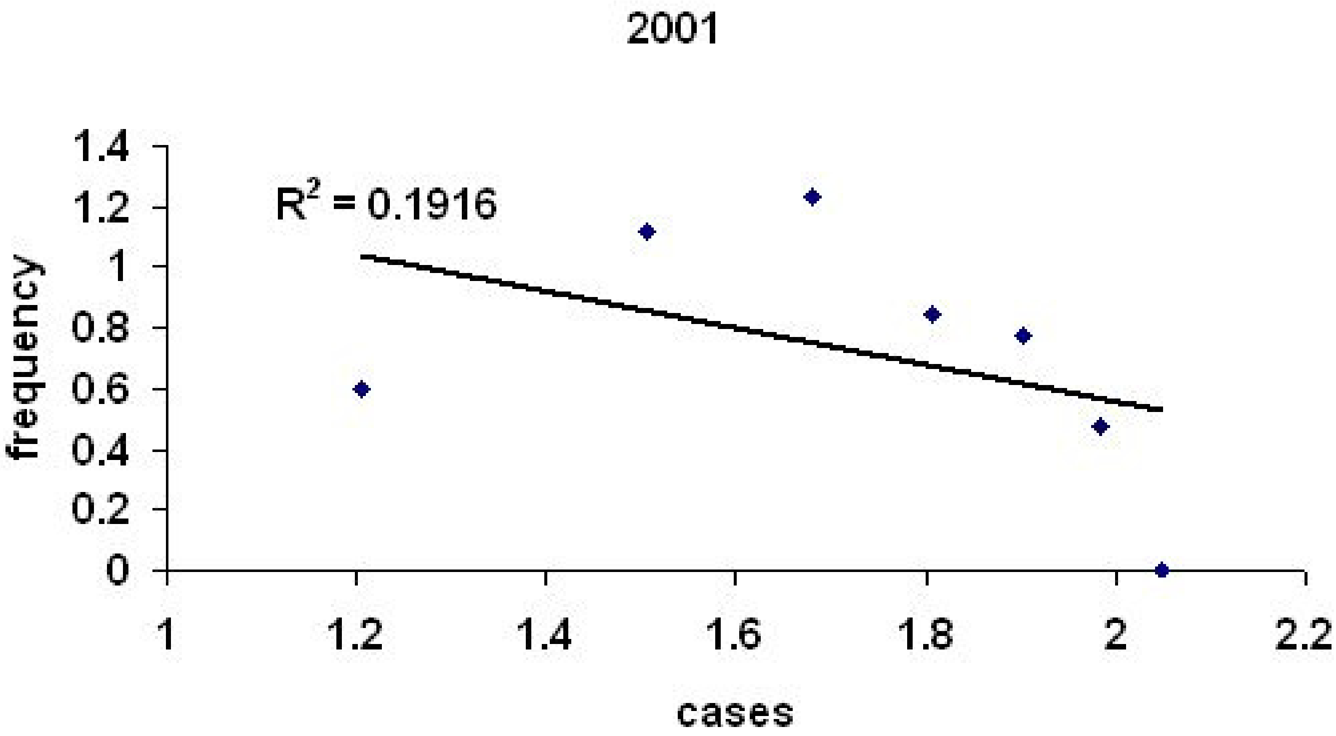
Model's accuracy
800
700
600
500
400
300
200
100
0
Actual number of cases
0
100
200
300
400
500
600
700
800
Predicted number of cases
R² = 0.7516



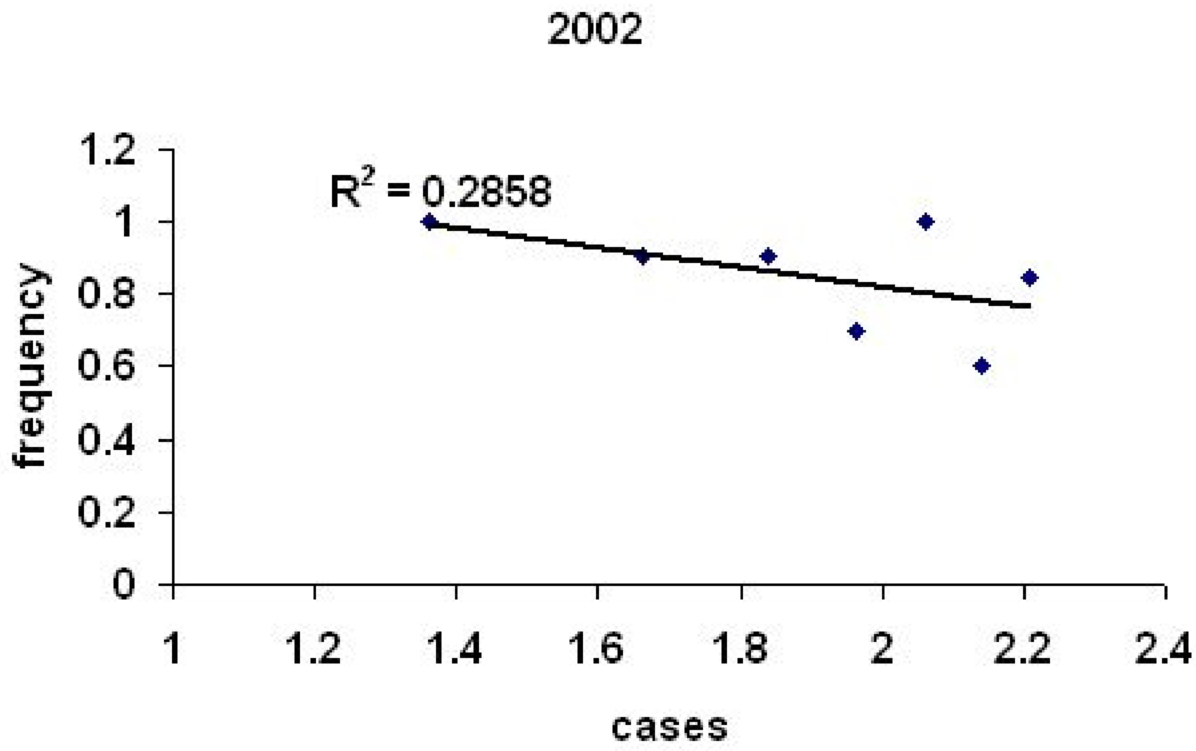
2005
2004
2003
2002
0.00018
0.00016
0.00014
0.00012
0.0001
0.00008
0.00006
0.00004
0.00002
0
Force of Infection
0
50
100
150
200
250
Time (weeks)


Force of Infection
New cases per capita x 10^-5
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
2000
2001
2002
2003
2004
2005
Year

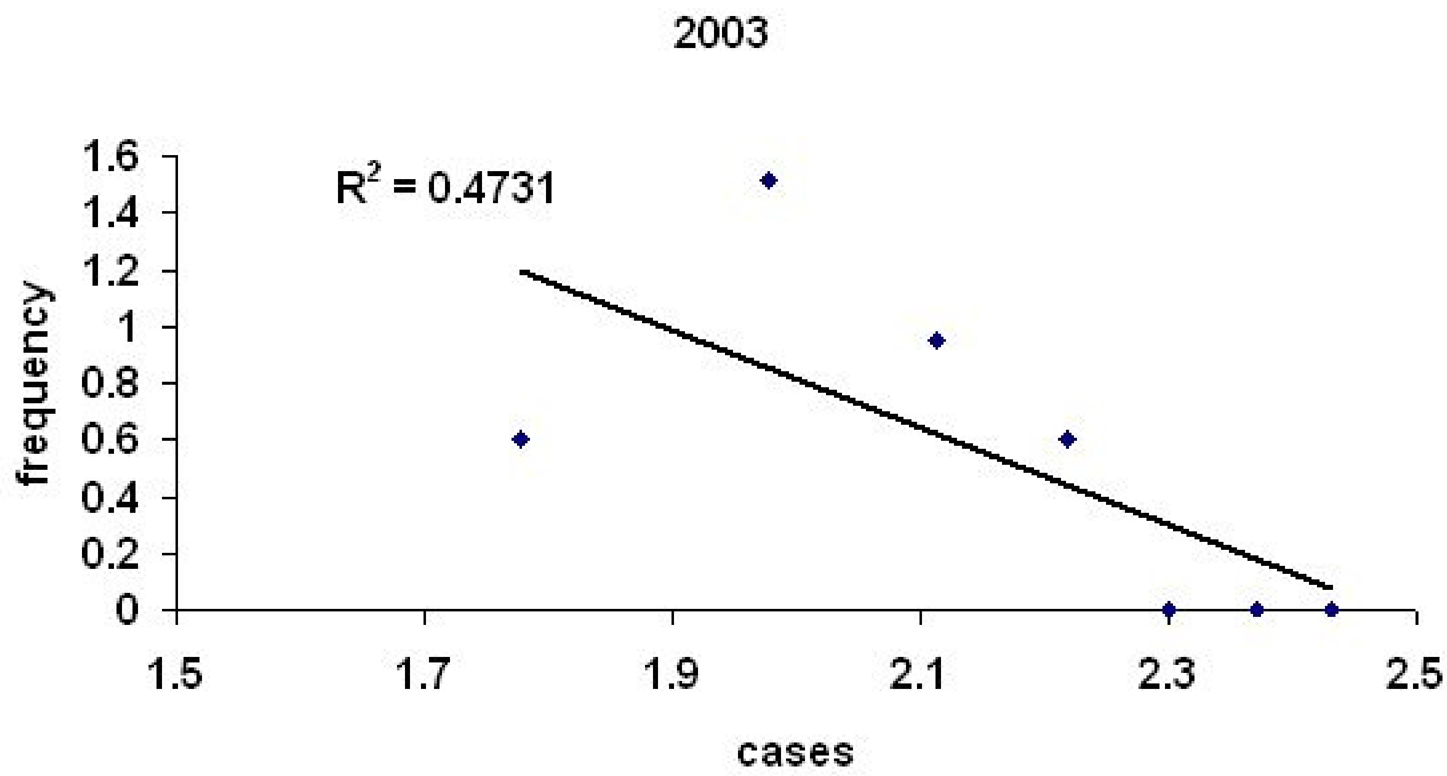

2000
1.4
1.2
1
0.8
0.6
0.4
0.2
0
frequency
R² = 0.0618
0.5
0.7
0.9
1.1
1.3
1.5
cases

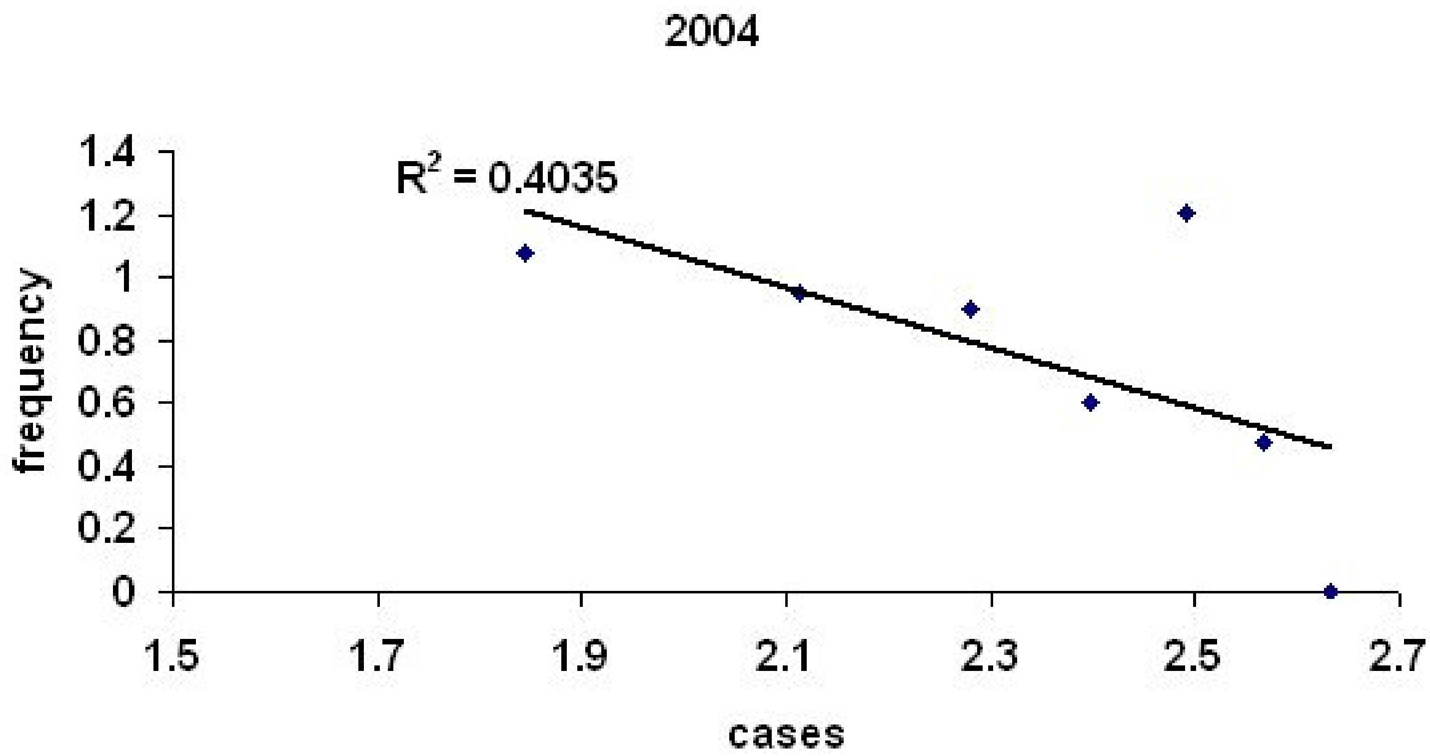


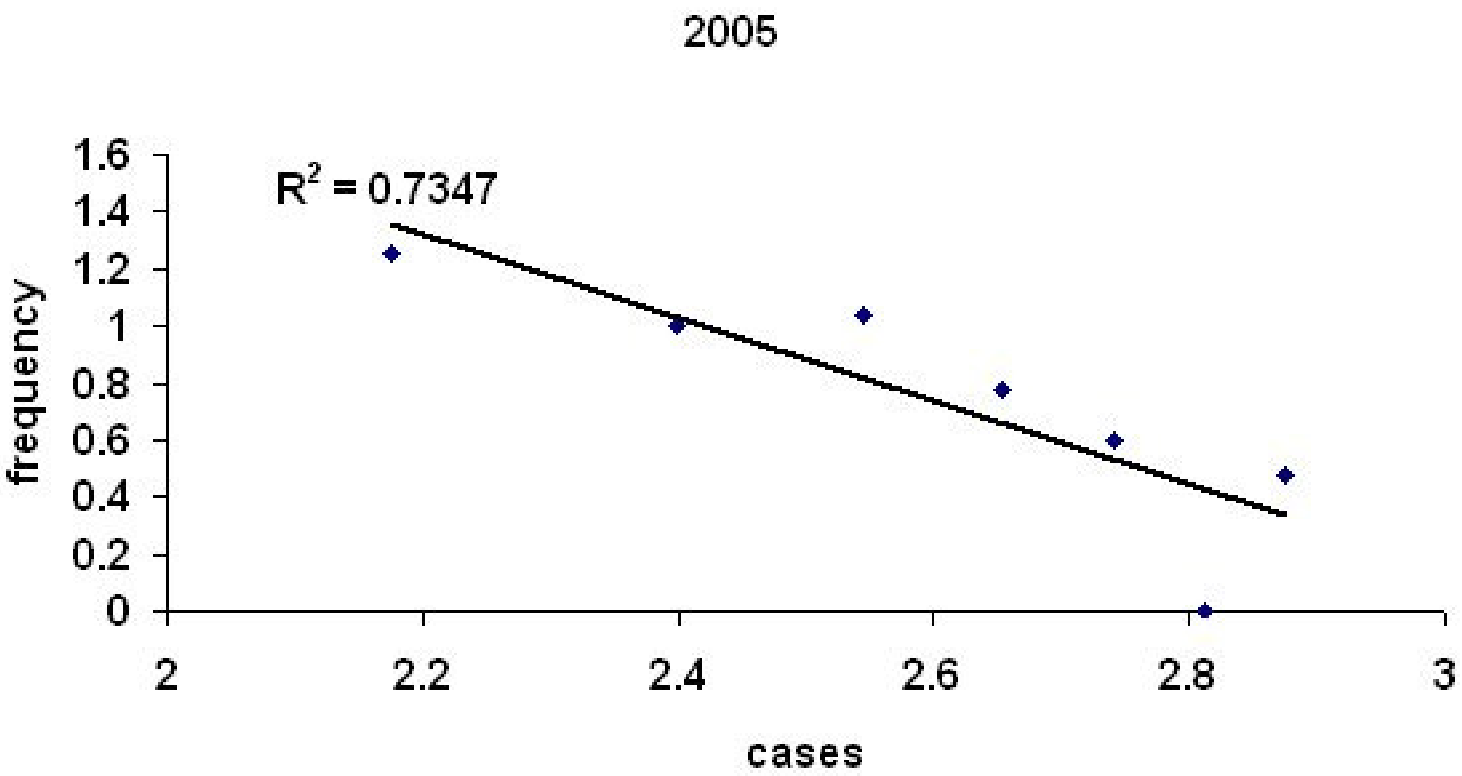
2001
$R^2 = 0.1916$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
frequency
1
1.2
1.4
1.6
1.8
2
2.2
cases

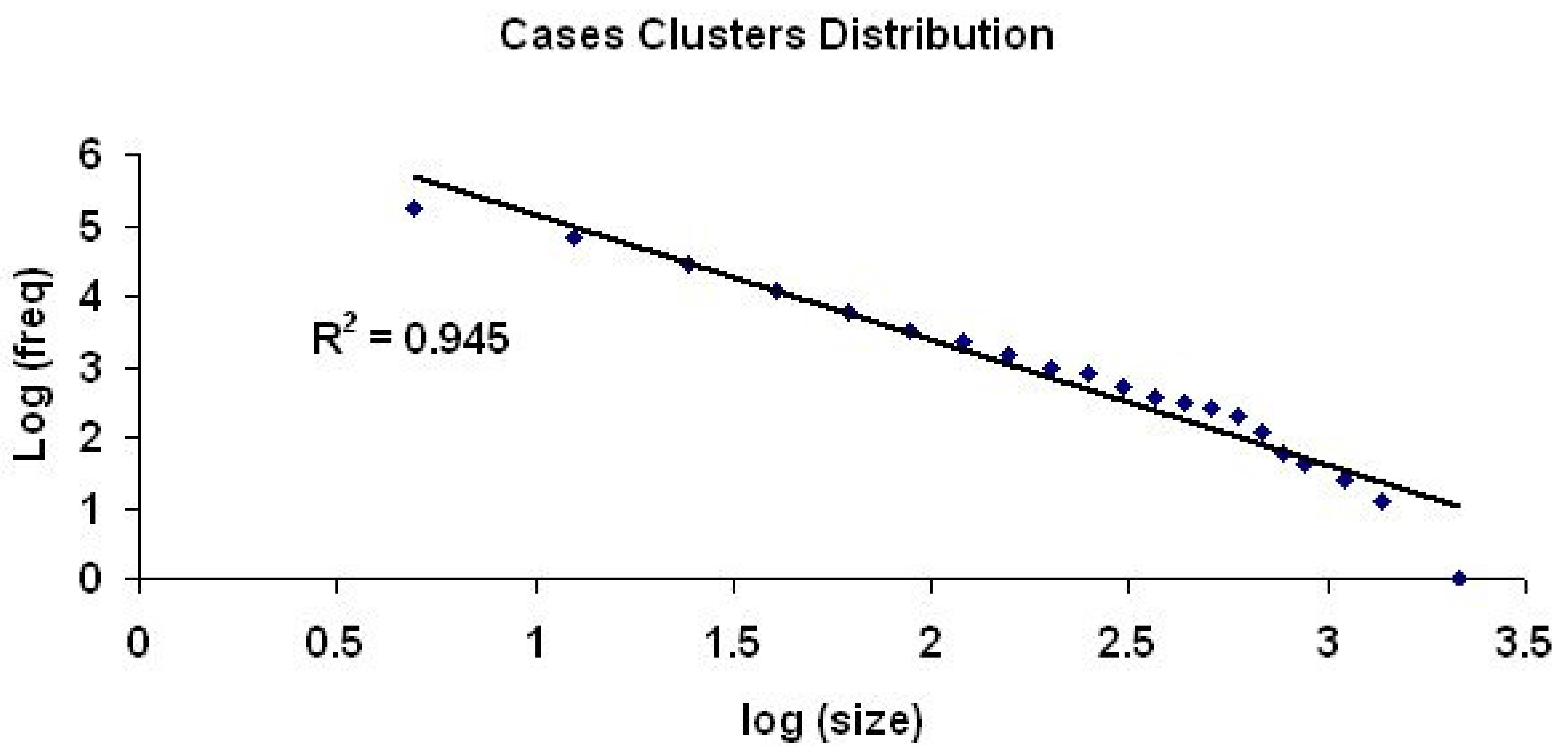


2002
$R^2 = 0.2858$
frequency
1.2
1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
cases


2003
R² = 0.4731
frequency
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1.5
1.7
1.9
2.1
2.3
2.5
cases


2004
$R^2 = 0.4035$
frequency
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1.5
1.7
1.9
2.1
2.3
2.5
2.7
cases


2005
R² = 0.7347
frequency
cases
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
2
2.2
2.4
2.6
2.8
3


Cases Clusters Distribution
Log (freq)
log (size)
R2 = 0.945
6
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
3.5

$$\lambda(t) = ab\int_0^t \frac{S_M(s)}{N_H(s)}\,ds\int_0^t \frac{I_M(s)}{N_M(s)}\,ds$$

$$\frac{\partial\rho_\kappa(t)}{\partial t} = -\rho_\kappa(t) + \lambda\kappa[1-\rho_\kappa(t)]\Theta(\lambda)$$

$$\rho \cong 2\exp\left[-\frac{1}{m\lambda}\right]$$

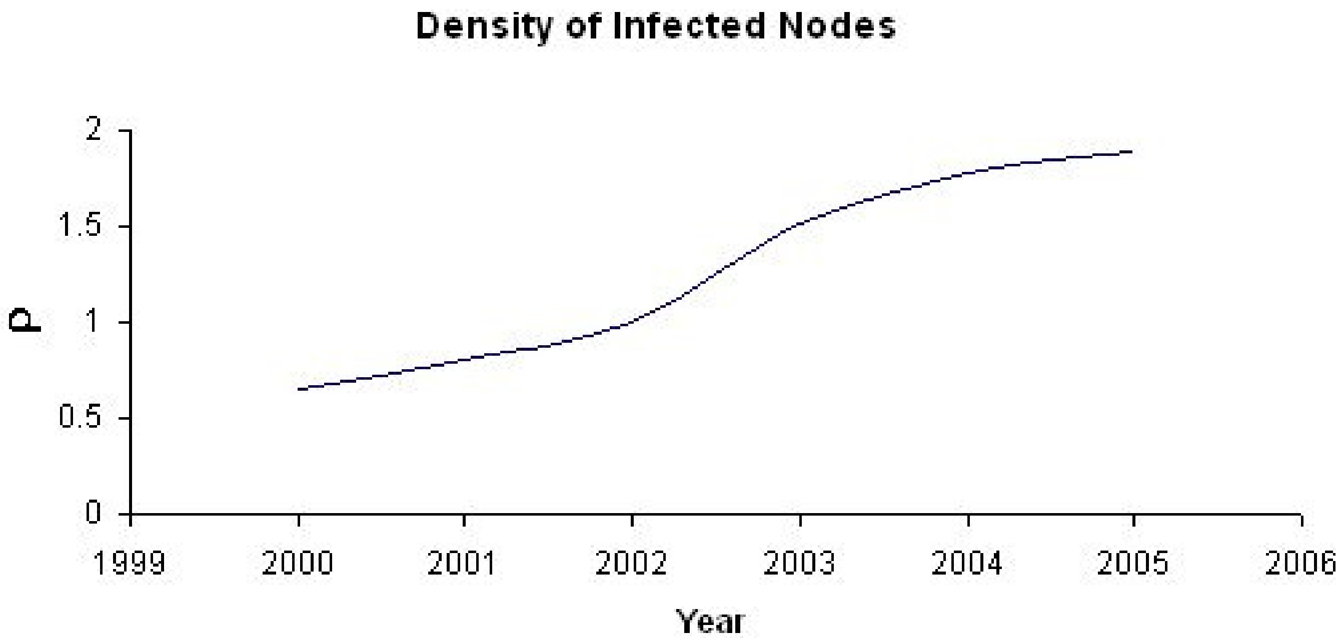

Density of Infected Nodes
P
2
1.5
1
0.5
0
1999
2000
2001
2002
2003
2004
2005
2006
Year

DIAGRAMA Nº 1

FEMEA DE STEGOMYIA VISTA LATERAL