## Fair votes in practice STV for Westminster

Britain has chosen proportional voting systems for other countries, for example for West Germany after WW2, and for our devolved governments. It's time we introduced it for our own UK Parliament, says Denis Mollison of HeriotWatt University, who devised the scheme recently put forward by the Liberal Democrats as an amendment to the Constitutional Reform bill.

In a recent discussion of voting systems on "Any Questions", Brian Paddick said "If I have to choose between simplicity and fairness, I'll choose fairness every time".
There are plenty of arguments for the Single Transferable Vote on grounds of fairness, as we shall see. But it can also be claimed that STV is simpler than our present "first past the post" (FPTP) system in the aspects that really matter. It may seem simple just to put a cross against your favourite candidate, but around half of UK constituencies have elected the same party since 1945 , so in any of these it is hardly worth voting. Indeed, many political analysts, and party organisers, believe that the result is largely dictated by a very few swing voters, variously estimated in the hundreds, or even merely tens, of thousands. Certainly in most constituencies at most two parties will be in the running, so that the intelligent choice (at least in the short-term) for supporters of other candidates will be to vote tactically for one of these two. Note the unnatural distribution of votes in Figure 1(a), where the case of close 3 -party contests (the centre of the triangle) seems to act as a "repeller".
What is not obvious in Figure 1(a) is the huge bias possible in the FPTP system. Figure 1(b) shows the votes required for various levels of majority; note that the actual outcome of the election in votes $(38,38,24) \%$, shown by the asterisk, falls within Labour majority territory, with the outcome in seats being $(54,37,9) \%$.


Two of the three other voting systems now in use in Scotland - the d'Hondt system for the European Parliament and the Additional Member System for the Scottish Parliament - do achieve a good degree of proportionality, but suffer similarly from wasted votes and incentives to vote tactically. In my own electoral ward in 2007, it made good sense to vote for two different parties, neither of them my first preference, on the Scottish Parliament's constituency and topup ballot papers, if I wanted my votes not to be wasted. And both systems also suffer the democratic deficiency of party lists of candidates. [Though this criticism of such proportional systems can be over-egged; voters have even less say in which individual rather than party is elected in safe FPTP seats.]
In contrast, under STV

- votes are cast for individuals not party lists;
- there are no safe seats, so every vote matters;
- the great majority of voters end up with at least one MP they voted for;
- the overall result is broadly proportional, at least among parties attracting around $15 \%$ support or more;
- minority parties and independents can get elected if they can attract a similar level of support locally;
- but it is more difficult for extremist parties, because they tend not to be anyone's second preference.


## The Single Transferable Vote

The Single Transferable Vote (STV) was devised by British and Danish social reformers of the 19th century: the earliest version seems to have been in 1819, when members of Thomas Hill's Society for Literary and Scientific Improvement of Birmingham chose their Committee by sorting themselves into groups of five each of which chose one of their number as a representative.
The UK Parliament voted several times on STV in the earlier part of last century, and STV was used for the University seats that existed from 1918 to 1950. The UK also introduced STV in 1921 for the Irish Parliaments, North and South, during the troubled period that led up to Irish independence: the Irish have taken to the system to the extent that they have twice voted in referenda to keep it.
The idea of STV, within any one constituency, is to achieve exact proportionality in a way that gives equal weight to each voter, and as far as possible avoids wasted votes (see box for details). Because voting is for individuals not parties, the proportionality is implicit. So for example, if voters were actually influenced far more by the hair colour of candidates than their political party, the outcome would be proportional as between fair and dark haired representatives.
Because proportionality is at the local constituency level, there is a threshold level of support required for election. Candidates can be certain of election if their first preferences exceed the quota (e.g. $20 \%$ in a 4 -seat constituency), and have a reasonable chance on anything above about half the quota provided they can attract transfers from spare votes of other candidates - a requirement that makes election more difficult for extremist candidates. There is a balance between proportionality across the country, and localism, having a small number of seats $s$ per constituency, so that the constituency is not too geographically extended, and the number of candidates for voters to choose between is not too large. The Northern Ireland Assembly uses $s=6$, Scottish Councils $s=3$ to 4, while the Dail, the Irish Parliament, uses $s=3$ to 5 .

## How STV works

Each voter puts the candidates in order of preference.
(1) If a candidate has enough votes, she is elected, and any surplus shared among her voters' next preferences. To be precise, if the candidate has $v$ votes but only needs $q$, she keeps a proportion $k=q / v$ of each vote, and passes the remainder on. (2) At any stage where Rule (1), repeated as necessary, has not filled all places, the candidate with fewest votes is excluded, transferring her votes to those voters' next preferences.
Notes: There are a number of slightly different variants of STV, depending on exactly how transfers are made. For its own elections, the Royal Statistical Society uses Meek's method, which implements the simple rule that surpluses and transfers are passed on to the voter's next choice, regardless of whether that candidate has been elected or not. The only problem with this method is that it requires iteration, and thus the use of a computer. On the other hand it is a stable and transparent algorithm, which has advantages for audit and for describing the result. Indeed, the state of the count at any stage is determined simply by the current "keep values" $k$ of each candidate.
How many votes should you need for election? If the total number of votes is $v$, and there are $s$ seats to fill, then a "quota" of $q=v /(s+1)$ suffices ${ }^{a}$. Meek's method has the further advantage that when some votes become "non-transferable", because of voters not specifying further preferences, the quota can be adjusted correspondingly.
${ }^{a}$ Except in the highly improbable case of an $(s+1)$-way tie.

## An STV scheme for the UK Parliament

The STV scheme put forward recently as an amendment to the Constitutional Reform Bill aims to divide the UK into mainly 4 or 5 seat constituencies, but with a fair degree of flexibility so as to follow natural boundaries as far as possible. This flexibility, in numbers both of seats per constituency and of voters per seat, is partly inspired by looking at the work involved in past Boundary Commissions, both in the UK and in Ireland. Their aim of equalizing voters per seat leads not only to some distinctly unnatural boundaries, but also to a long and slow process: for example, the new boundaries of this year's general election are based on electoral data for 2000.

The present scheme uses the following rules:

1. Constituencies preferably elect either 4 or 5 MPs ; where the other rules make this difficult, either 3 or 6 is allowed.
2. Constituencies are assigned one MP for every 100,000 electors or part thereof; thus a constituency with between 400,001 and 500,000 electors is assigned 5 MPs.
3. Constituencies are based on Local Authority areas.
4. Exceptions to Rules 1 and 2: in line with current practice, some allowance is made for sparsely-populated and isolated areas. Smaller numbers of MPs (generally 2 or 3, but in three exceptional cases just 1) are allowed in sparsely populated areas; and the ceiling on the number of electors per MP is reduced to 90,000 for areas where the population density is less than $150 / \mathrm{sq} \mathrm{km}$, and 80,000 where it is less than $30 / \mathrm{sq} \mathrm{km}$.

United Kingdom: Local Authority Districts, Counties and Unitary Authorities, ${ }^{1} 2009$


2: The UK divided into 118 STV constituencies using Local Authority boundaries.
For the detailed schedule of constituencies, see http://www.ma.hw.ac.uk/~denis/stv4uk/

The map illustrates the result of applying these rules, using data on the electoral registers current in 2009. It gives a Parliament of 512 MPs, just over $20 \%$ fewer than at present, in 118 constituencies.
There are great advantages to the flexibility allowed in rules 1 and 2 . Constituency boundaries can be chosen as fit best with natural communities, here equated with Local Authority areas. The number of seats for each can be updated with each year's electoral register, with boundaries only needing to be reconsidered if that takes them outwith the 3 to 6 range. The price of this naturalness and convenience is an inevitable variability in electors per MP: for example, as the number of electors passes 400,000 , this ratio jumps from 100,000 to 80,000 . However, this variability compares favorably with what is achieved by the micro-management of our present boundary commissions, despite their overt aim of equalizing electorates (Figure 3).


3: A comparison of the variability of electorate per MP for the present UK constituencies (FPTP, average 70,000 ) and the proposed scheme (STV, average 88,300 ). On a variety of standard statistical measures, the STV scheme has significantly less variability.

For England, roughly half the traditional (ceremonial) counties can be used as single constituencies. The larger counties and metropolitan areas are split into a number of constituencies, ranging up to 11 for Greater London, with each being a grouping of one or more district or unitary Local Authorities.
For Wales, the eight ceremonial counties are each used as a single constituency, with number of MPs varying from 2 for the sparsely populated counties of Gwynedd and Powys up to 5 for Gwent. For Scotland, the 32 Councils are grouped into 16 constituencies. For Northern Ireland, the 26 Councils are grouped into 3 constituencies.
The distribution of the number of MPs per constituency is

| Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3 | 5 | 13 | 38 | 46 | 13 |

## The 2005 General Election re-run

As an example, let's look at the most recent UK General Election, and estimate how it might have turned out using STV rather than the present "first past the post" (FPTP) system.
Assume for the moment that the votes cast in 2005 represented genuine first preferences, and that we can estimate second preferences using data from the Scottish local elections held under STV in 2007.
The following table shows how the percentages of seats for the main parties under STV would have been much closer to the voters' first preferences ${ }^{1}$ :

| Party | Con | Lab | LD | Nat | Other |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Votes | 33.2 | 36.2 | 22.6 | 2.2 | 5.7 |
| MPs under STV | 34.9 | 39.5 | 23.2 | 2.2 | 0.2 |
| MPs (actual) | 31.5 | 56.8 | 9.9 | 1.3 | 0.5 |

STV gives a result very close to proportionality, certainly far closer than the present system, which gave one party a large overall majority on only $36.2 \%$ of the votes (indeed, Labour won $25 \%$ more seats than the Conservatives, despite having only $3 \%$ more votes). The main difference from proportionality under STV is that minorities and independents with less than $10 \%$ of the vote get very few MPs (as they do of course under the present system). The Nationalist parties get their fair share of seats $(2.2 \%)$, because their vote is concentrated locally. In practice, minority parties and independents can be expected to win more seats, because voters can express their real preferences under STV without fear of their vote being wasted. It is relatively easier for moderate parties to win seats than for extremists, because the latter are unlikely to attract the transfers of second preferences they need to build the vote required to win a seat ( $16.7 \%$ in a 5 -member constituency).
If we look at the breakdown of MPs in each constituency, we find that there are no safe seats: there are no single-party seats (except for the three special single MP cases). Indeed, the great majority of constituencies have representatives of all 3 main parties; and $91 \%$ of voters end up with an MP of their first preference party, compared with only $48 \%$ under the present system.

## STV in practice and detail: the example of Scottish councils

The transfer estimates used in the preceding estimates were taken from data for the 2007 council elections in Scotland.
Statisticians should be particularly grateful to the councils of Glasgow and East Ayrshire, the only two to make their full data sets available. These make possible a host of interesting analyses, ranging from those of practical interest to the parties and electoral officers, to abstract questions about how electors view parties in "political space" and whether preferences ever in practice show cyclic tendencies as in the notorious "Condorcet paradox".
The introduction of STV in 2007 changed the local government landscape of Scotland. Many councils that had been run by the same single party - in almost all cases on a minority of the vote - since they were formed, are now run by coalitions. This outcome has been broadly welcomed, even by some of those who have lost power as a result. As a recent report (Steven

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4: How votes translate into seats under STV, estimated for 2005 for the 41 proposed 5 -seat constituencies in England; here votes of $(36,39,25)$ \% give $(36,40,24) \%$ of seats. [Hexagons show zones of attraction for the case of neutral transfers.]


5: Seats for the majority party as a function of votes under STV (compare Figure 1(b)).
2010) puts it, parties "have to try to work together for a greater good, as opposed to pursuing overtly partisan objectives on the false premise that they alone have the right to drive policy on the basis of $20-30 \%$ of the electorate that actually voted for them".
The election results demonstrate that it is still possible for a party to retain an overall majority of seats on less than $50 \%$ of the vote, particularly if the opposition is fragmented. This was the case in Glasgow, where Labour had $43 \%$, the SNP $25 \%$, followed by three parties (Lib Dem, Conservative and Green) with around $7 \%$ each. Given the fairly high quota required for election in the 3 or 4 member wards ( 25 and $20 \%$ respectively), it is perhaps surprising that the minority parties got any seats at all. Figure 6 shows that in fact transfers were sufficient to give the Liberal Democrats and Greens their fair share of seats once their first preferences reached roughly half the quota; the Conservatives did less well here because they attracted fewer second preferences.
Of immediate practical interest for the major parties is to know whether they put up two few or too many candidates in any contests. In Glasgow, the SNP almost certainly put up too few; re-running the election in some wards with half the vote for their single candidate given to a fictitious twin suggests that they might have won up to 5 more seats ( 27 instead of 22 on a council of 79). In contrast, in one ward in East Ayrshire, Labour put up 3 candidates, but only won one seat despite having first preferences alone nearly sufficient for two. Re-running the election here shows that the problem was that the first of their candidates to be excluded was the most popular with voters of other parties; so they could have won 2 seats, but only if the candidate they dropped was one of the two more popular among Labour voters.
The version of STV used for these elections was "Weighted Inclusive Gregory" (WIG), in which transfers to already elected candidates are not allowed. This introduces an unnecessary discontinuity to the pattern of transfers, but the effect is small in these data sets. However, if we re-run the election with Meek's method, quite large differences appear due to the number of non-transferable votes. As already mentioned, Meek's method, unlike WIG, is able to adjust the quota when votes drop out.


6: Seats as a function of votes under STV, Glasgow council election 2005. Parties (Con, Lab, LD, SNP, Green, Sol) are represented by (blue, pink, orange, yellow, green, red) respectively.


Figure 7 shows the difference this would have made in Craigton; here the final place was filled by Solidarity, with less than $15 \%$ of the vote, where Labour had two candidates elected each with $20 \%$. Under Meek, the quota would at this stage have been reduced to about $17 \%$, freeing up enough transfers from the elected Labour candidates to have given the final place to their colleague. [In terms of overall representation, of course, one might argue that Labour were getting rather more than their fair share of seats, but a more consistent solution to that problem would be to have more seats per ward.]
A significant problem for parties putting up several candidates was the effect of their position, determined by alphabetical order, on the ballot paper. It is not surprising that there is significant preference for the first candidate when a party has two on the list. But how do voters treat the six possible orderings of three candidates of the same party? Labour had 3 candidates in 12 wards in Glasgow: 60 degrees of freedom gives plenty of scope for generalised linear modelling. The main conclusion from this on ballot order is that the "donkey vote" of 123 is very significantly favoured, attracting around $27 \%$; the other five orders are roughly equally popular ( 14 or $15 \%$ each), with some dependence of their precise relative preference on their spacing among other candidates.
The model also yields estimates of the "true" popularity of the candidates. If this is done using 3 separate parameters, comparing candidates pairwise, it reveals highly significant cyclic patterns in some wards. For example, in Greater Pollok, preferences between candidates ABC showed a cyclic paradox with $C>B, B>A, A>C$, in each case by 8 or $9 \%$, where random
effects predict at most 1 or $2 \%$. Such cyclic effects are of great interest to social choice theorists, as they undermine the otherwise ideal pairwise comparison method proposed by the enlightenment philosopher Condorcet (1785) for the general problem of making a single choice from the preferences of voters (see Hill 1988). The explanation in the present case seems to be disappointingly prosaic: Labour campaigning divided some wards in three, with respective leaflets asking supporters to vote CBA, BAC and ACB.
In Australia, the ballot-order problem is dealt with by rotating names on the ballot paper (the "Robson rotation"), which seems very sensible; but also (in Senate elections) by allowing voters to opt for the party's preferred order, which seems contrary to the idea that in STV the voter chooses among individuals. A solution more in line with that spirit would be to allow voters to express equal preference between some (or all!) of the candidates. This can be done straightforwardly with Meek's method: effectively an equal preference is treated as though the vote were divided into equal fractions, one for each possible order. The John Muir Trust has used this sophisticated variant of STV for its trustee elections since 1998.

## Conclusions

Returning to Brian Paddick's comment on the relative merits of fairness and simplicity, it is important to remember that the simplicity we require, as with a car or computer, is at the user interface: the voter should be able to put down their real preferences and expect the system to come up with a fair and proportionate result.
The problem with our present system for electing Parliament is not just that it can deliver an absolute majority of seats on $35 \%$ of the vote. It's far worse, with parties' policies often distorted to compete for the small floating voter populations of marginal constituencies.
Most of Europe has proportional representation for its parliaments, but mainly of the unsatisfactory party-list type - try googling "Berlusconi party candidates"! We invented the better system of STV nearly 200 years ago, a system of fair representation in which all voters matter equally, and where parties can flourish but votes are cast for individuals - it's time we used it for our own Parliament.

## References

More details can be found at http://www.ma.hw.ac.uk/ denis/stv4uk/
de Condorcet, M (1785) "Essai sur l'Application de l'Analyse a la probabilité des Décisions Rendues a la Pluralité des Voix"

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http://www.electoral-reform.org.uk/downloads/Working_with_STV.


[^0]:    ${ }^{1}$ Northern Ireland is excluded; the estimated result there is also very close to proportional, among the main Northern Irish parties.

