Theory, statistics and politics of preferential voting

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Contents

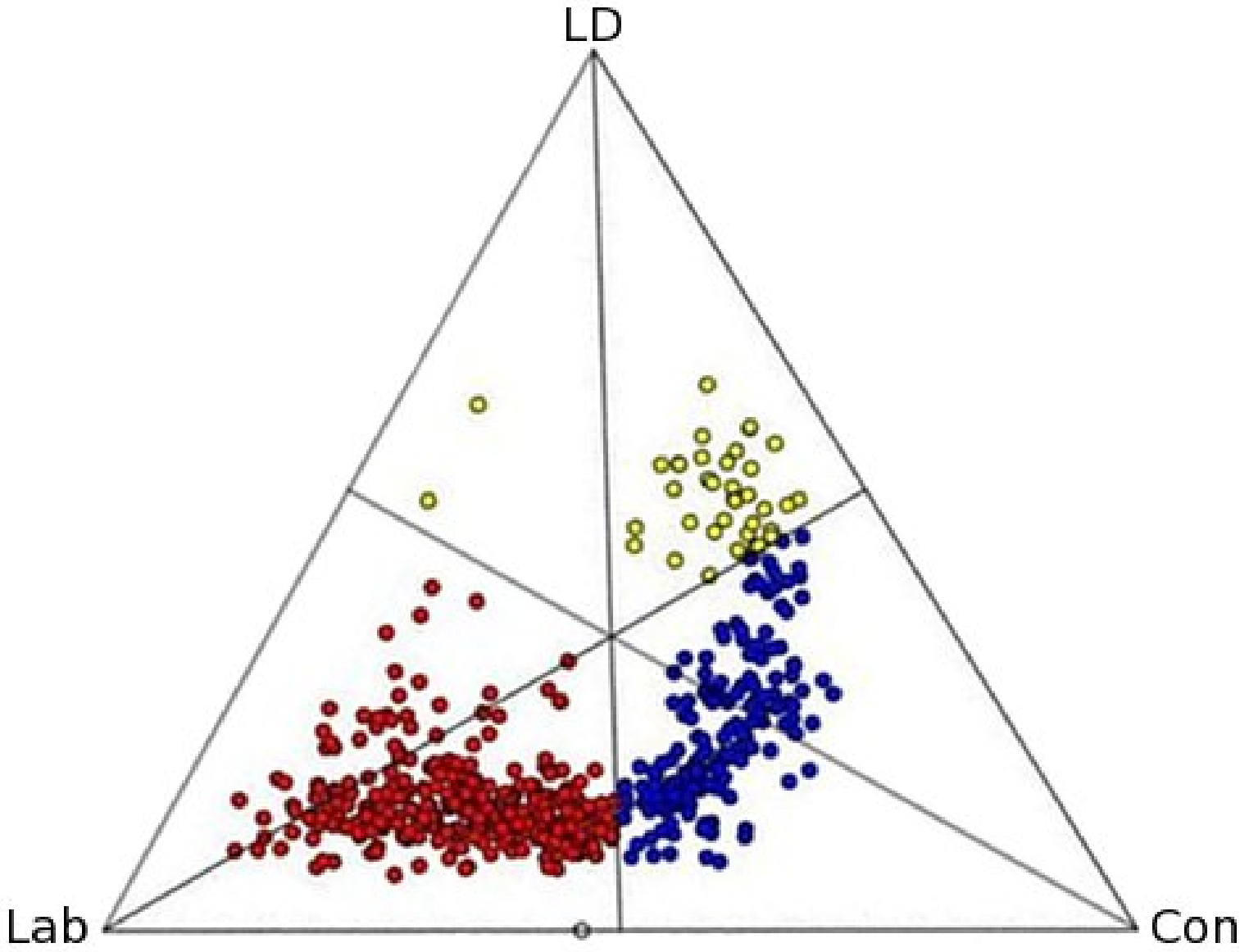
1. Voting systems in Scotland

- UK Parliament – FPTP (plurality)
- Europe – proportional (party lists)
- Scottish Parliament – proportional (FPTP + lists)
- Councils – proportional (STV) (since 2007)

UK Parliament 2005

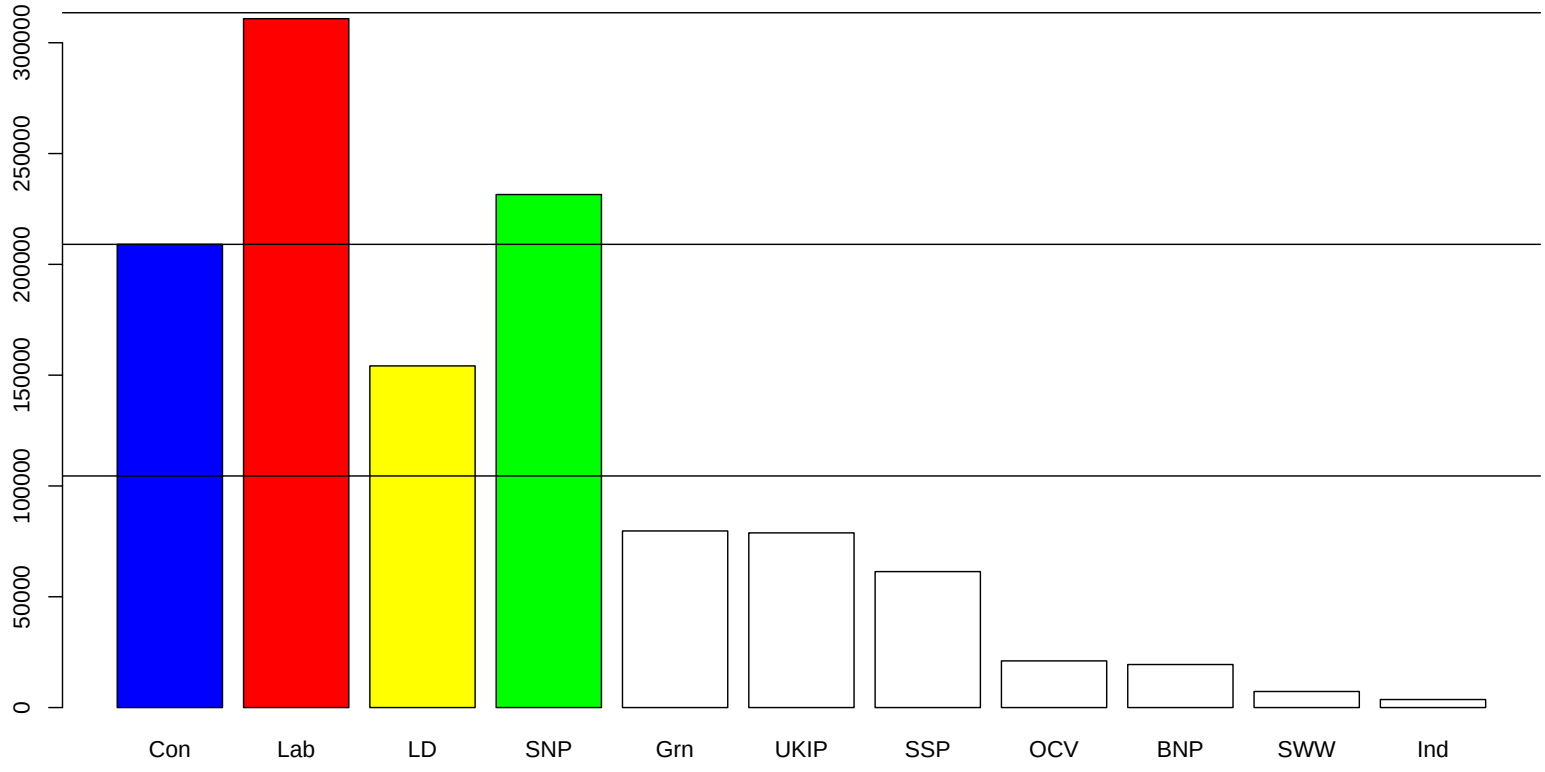
(England data only)

	Con	Lab	LD	other
Vote %	36	35	23	6
Seats %	37	54	9	0


LD
Lab
Con

Problems with FPTP

- Disproportional
- Huge variations in influence between voters ("safe" and "marginal" seats)
- Tactical voting
- 1-member constituencies seldom match communities



European election, Scotland 2004

quota 8.88 % (excesses 15 %, unrepresented 23 %)

Holyrood 2003

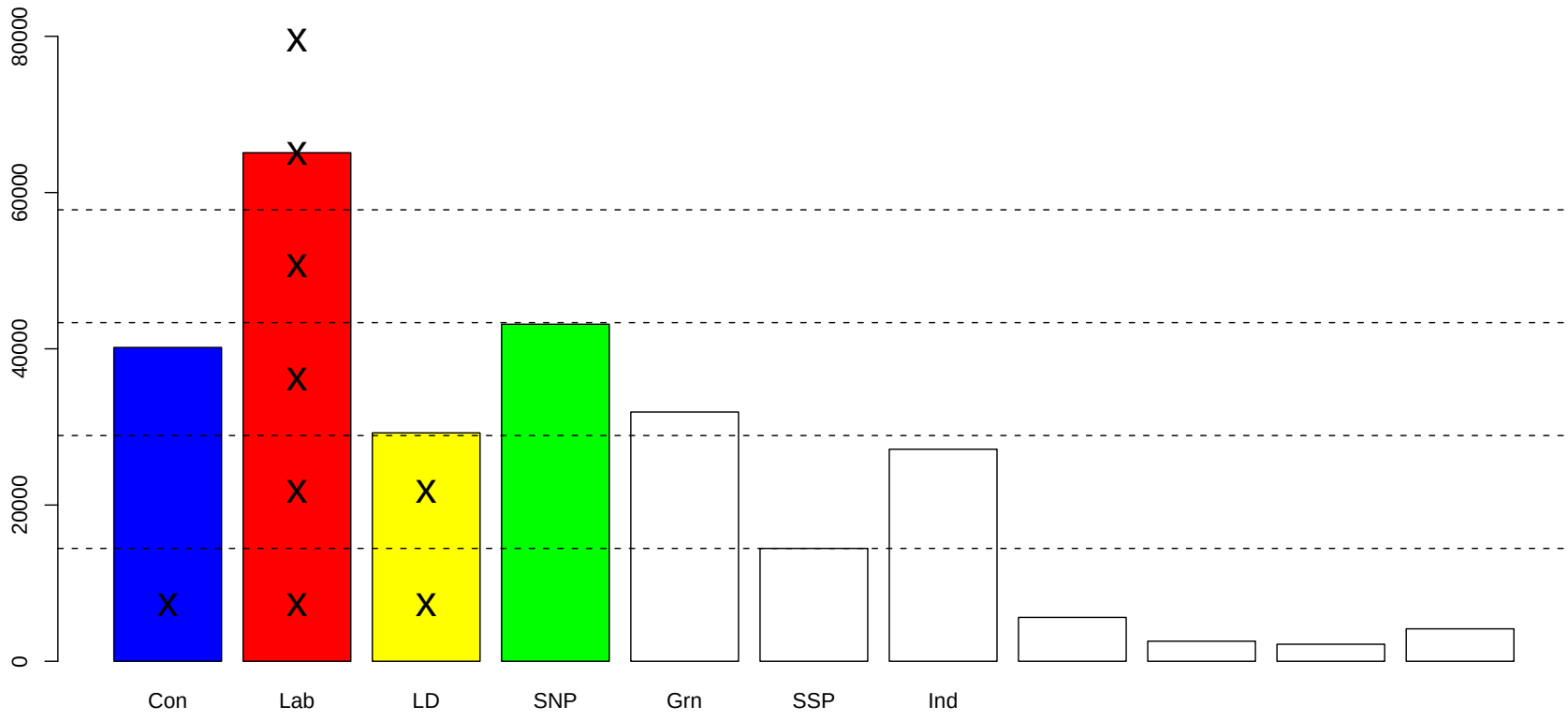
Additional members system *i.e.*

73 seats FPTP

+

56 seats from regional lists

In Lothians, 9 + 7



Holyrood 2003 – Lothians

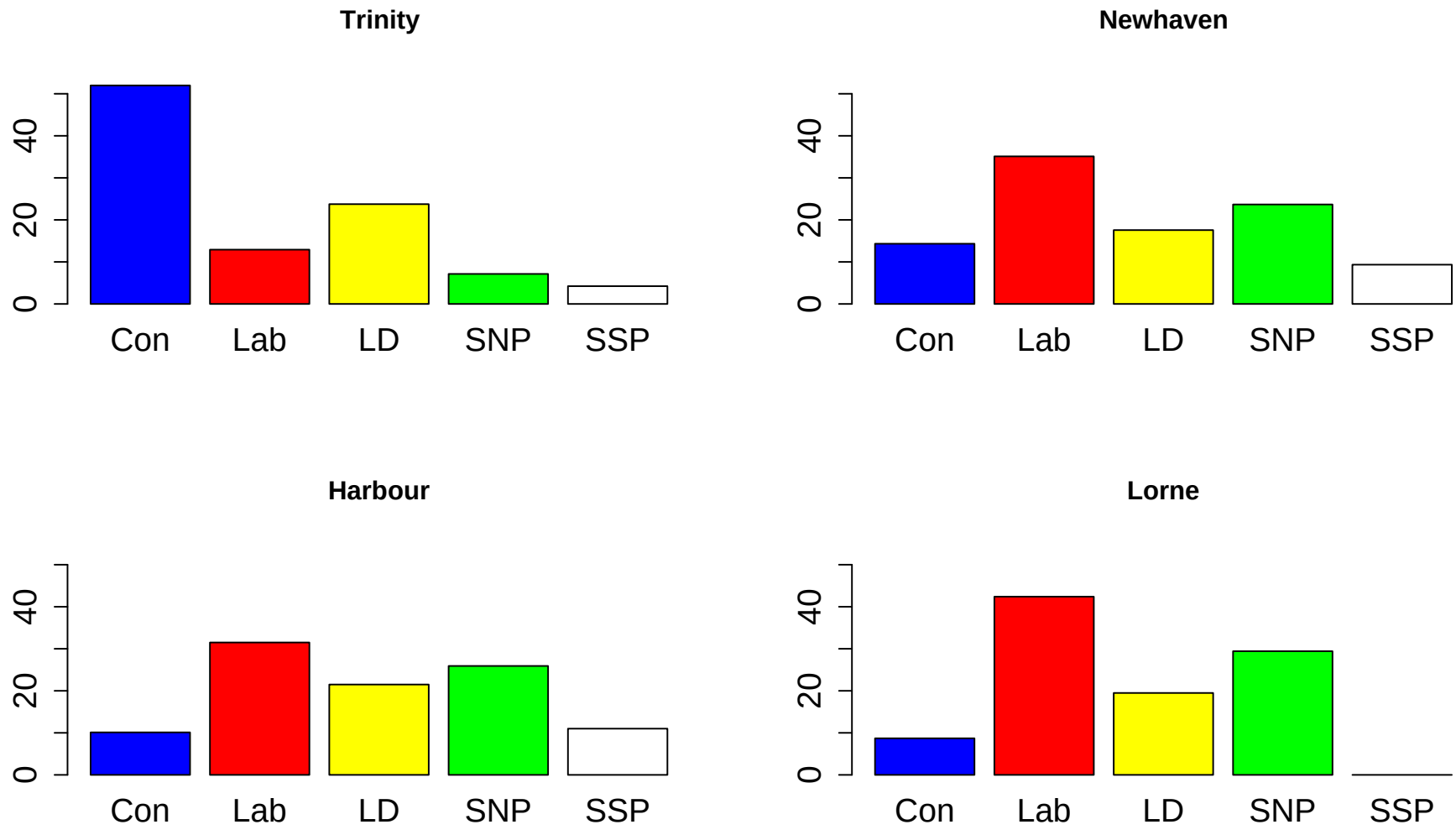
quota 5.54 % (duplicate 41 %, excesses 16 %, unrepresented 5 %)

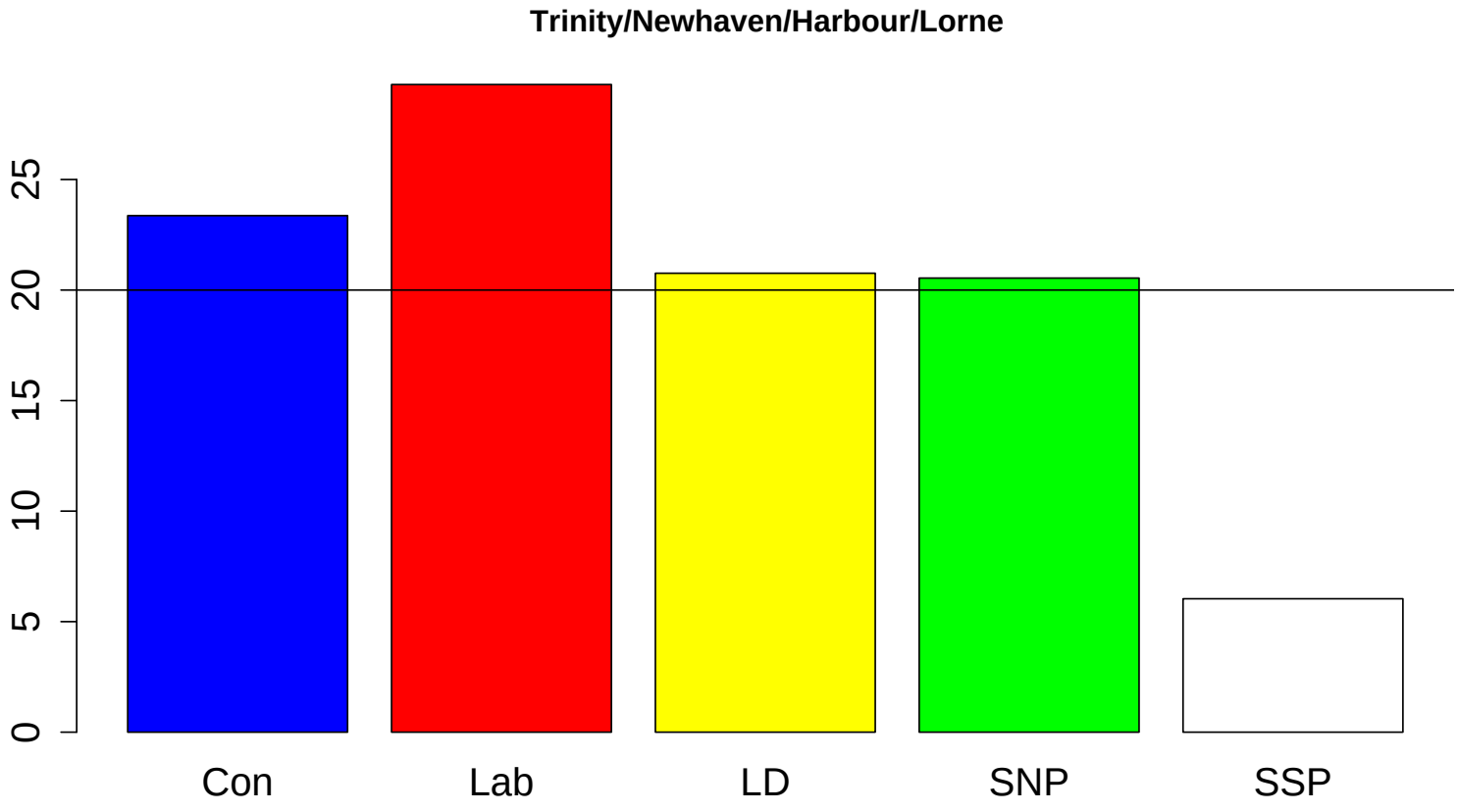
Council elections, Edinburgh 2003 (FPTP)

	Con	Lab	LD	SNP	other
Vote %	24.6	27.4	27.0	15.6	5.4
Seats					

Council elections, Edinburgh 2003

	Con	Lab	LD	SNP	other
Vote %	24.6	27.4	27.0	15.6	5.4
Seats	13	30	15	0	0


Trinity
0
20
40
Con
Lab
LD
SNP
SSP
Newhaven
0
20
40
Con
Lab
LD
SNP
SSP
Harbour
0
20
40
Con
Lab
LD
SNP
SSP
Lorne
0
20
40
Con
Lab
LD
SNP
SSP


Trinity/Newhaven/Harbour/Lorne
0
5
10
15
20
25
Con
Lab
LD
SNP
SSP

2. Idea of STV

Proportional: "one representative for every q votes".

Each voter puts the candidates in order of preference

Vote is used as effectively as possible, minimising wasted votes

Therefore . . .

1. elected if have sufficient share of votes
2. if too many, pass on spare votes
3. if too few, excluded; pass on all votes

From the voter's point of view

no vote is wasted

Details

1. elected if have sufficient share of votes':

declare i elected if

$$v_i > q$$

where $q = N/(s+1)$

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if 4 seats, need more than $\frac{1}{5}$, i.e. *20 %*

2. if too many, pass on spare votes

If $v_i > q$, keep the same proportion k_i of each vote; transfer the remainder of each vote to that voter's next choice (*)

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Note: we call k_i the 'keep value'.

$$k_i v_i = q$$

3. if too few, excluded; pass on all votes

If rules 1 and 2 don't complete the election, exclude the candidate with least votes; transfer each of their votes to the voter's next choice (*)

(*) Notes

If no further preference, remainder of vote is of no value ('non-transferable'); reduce quota q accordingly

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Always transfer to next choice, even if they're already elected.

Why? : fair, conceptually simple, avoids discontinuities

The only problem with this exact expression of the idea of STV is that calculations require a computer.

Historical note

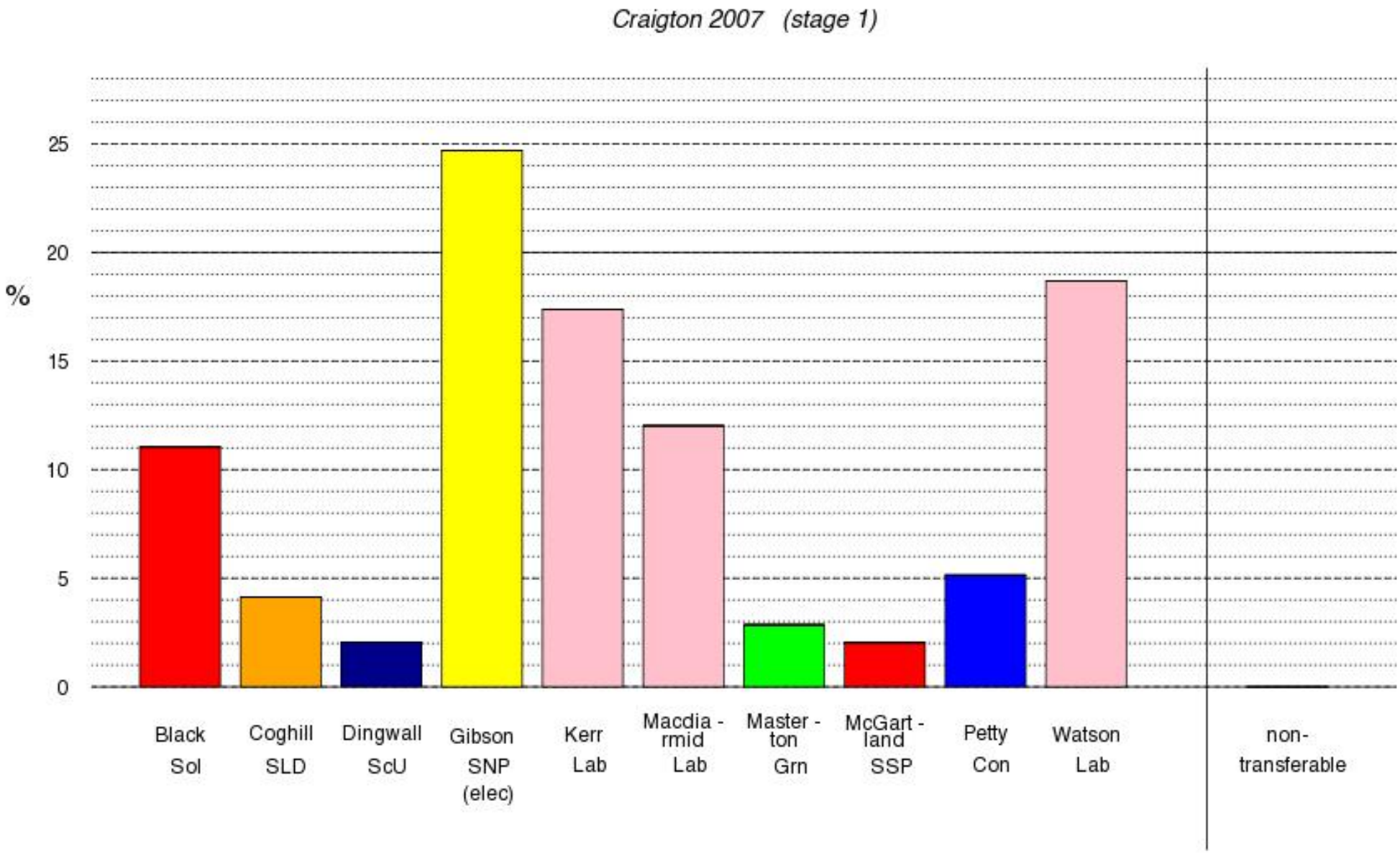
'Exact STV' = Meek's method (1969, 1970)

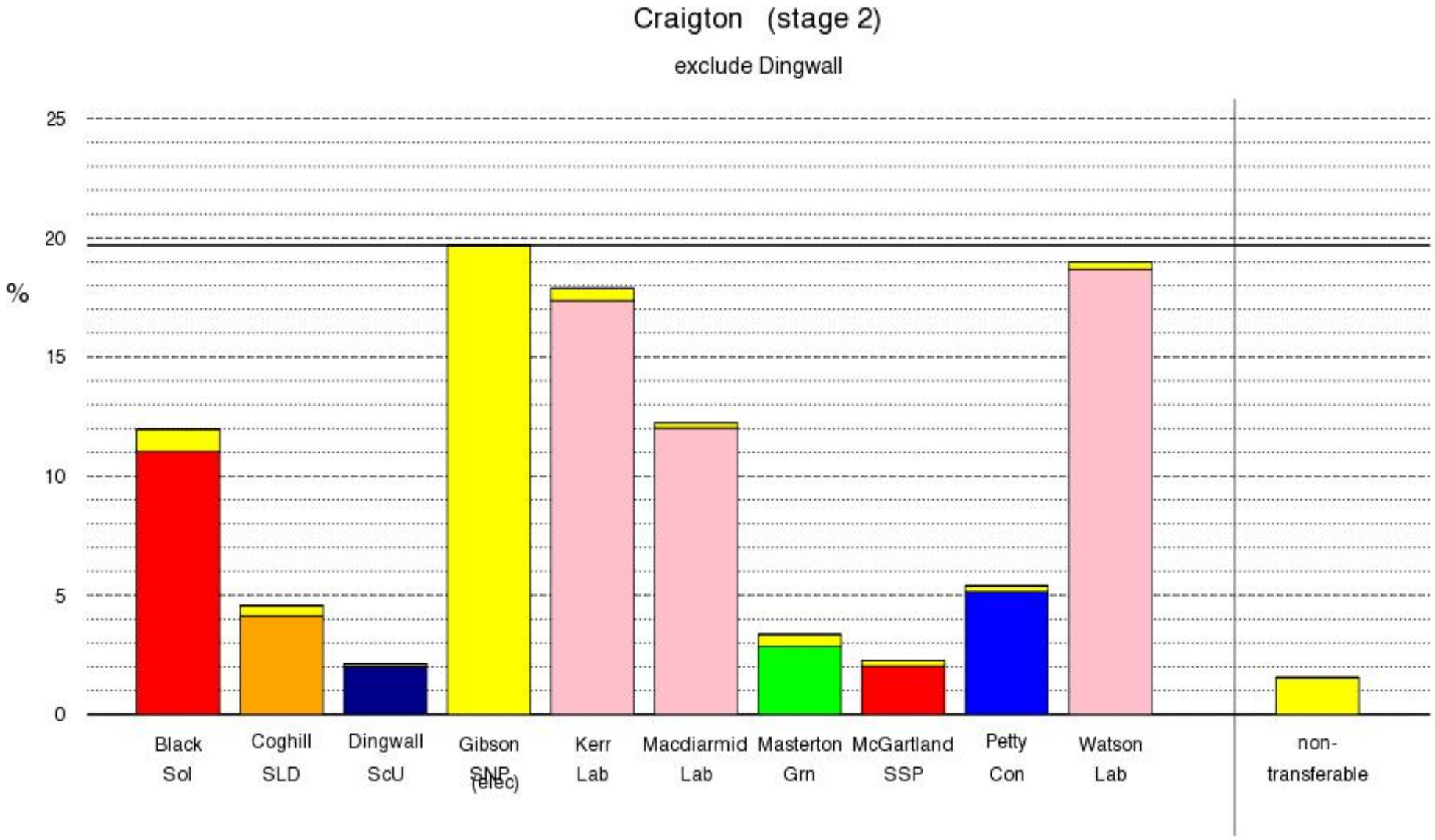
First used by RSS 1980s

Older approximations are widely used (Irish Parliament since 1921)

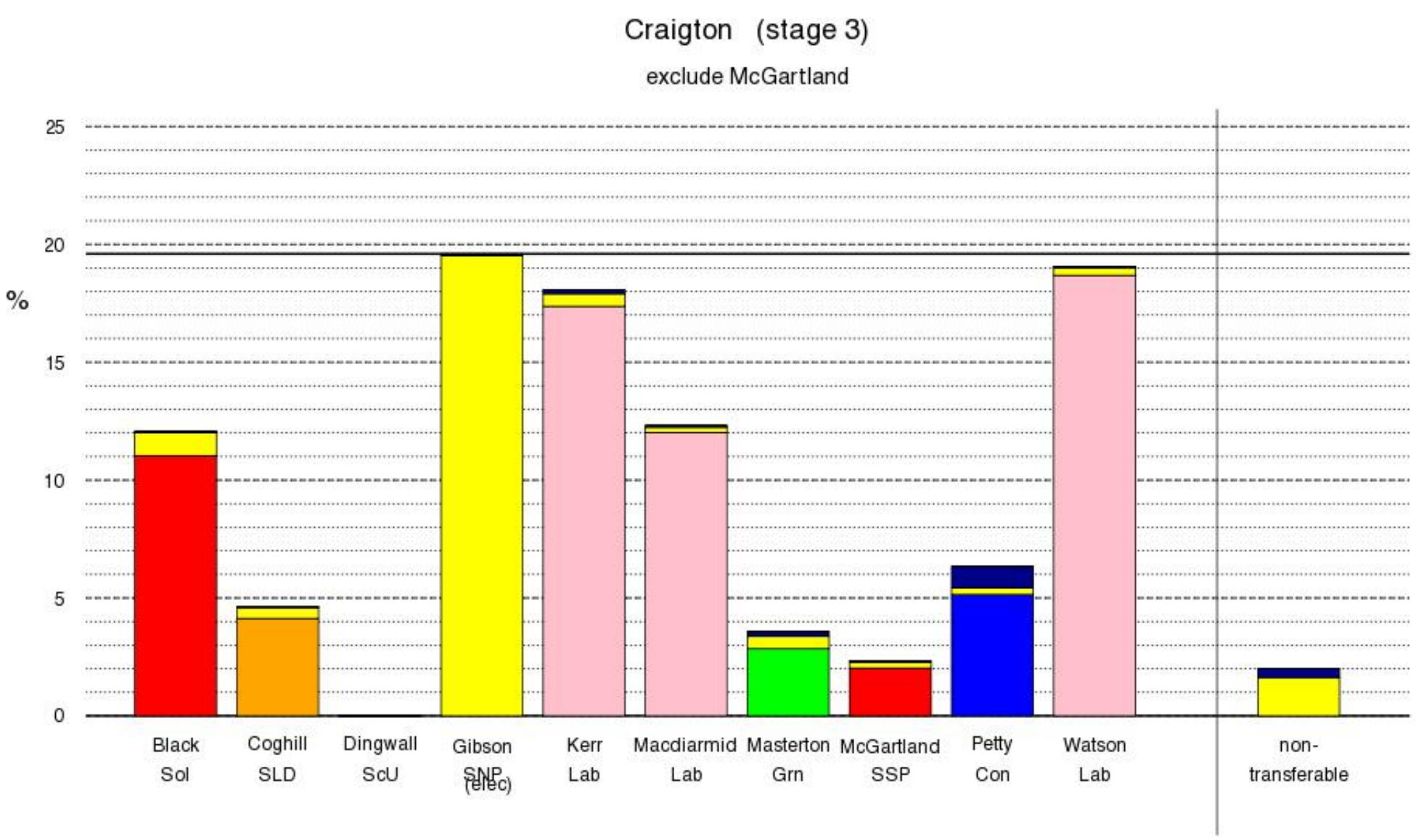
Examples: *see*

www.ma.hw.ac.uk/∼denis/STV_elections/

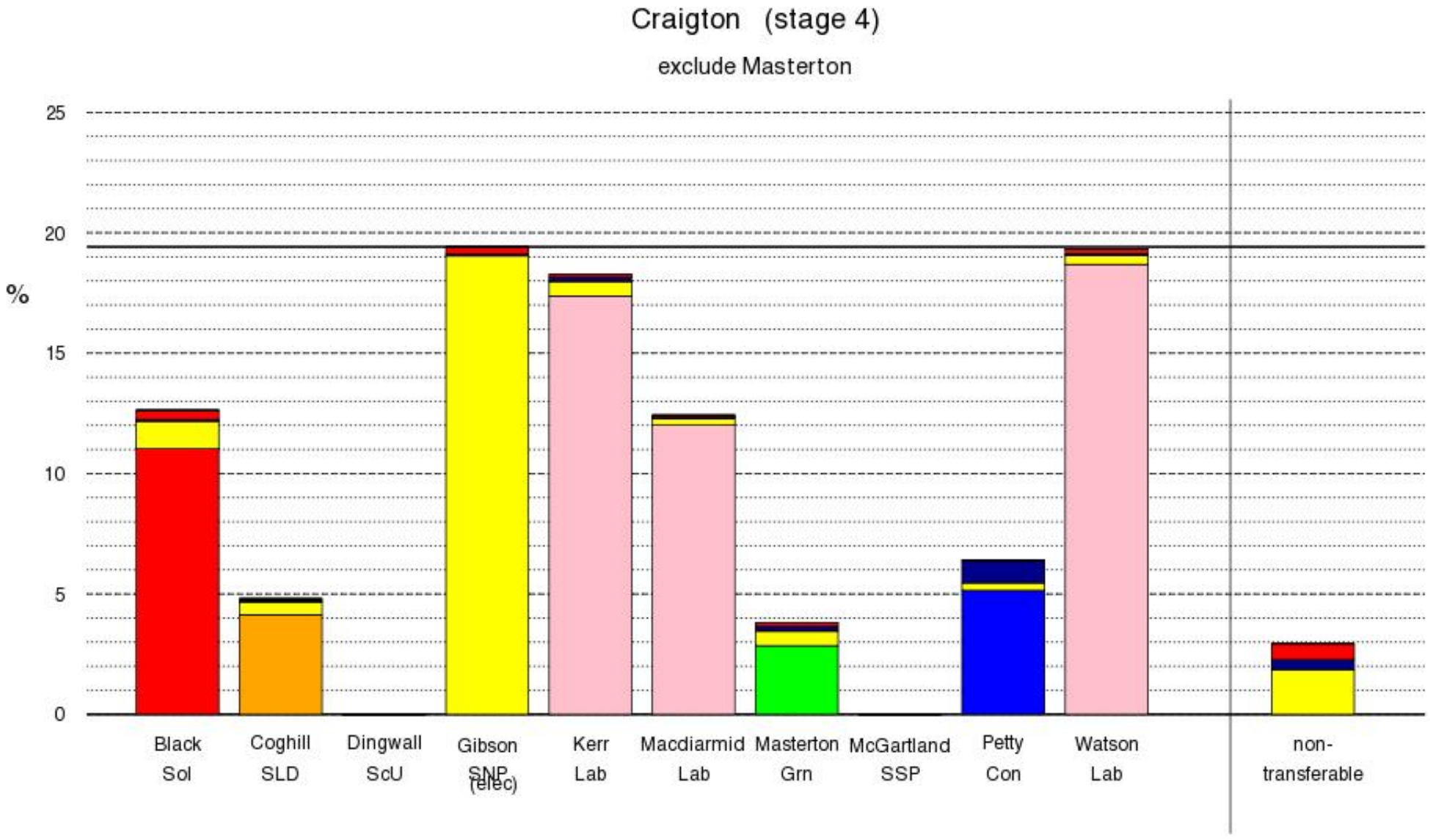

Craigton 2007 (stage 1)
%
25
20
15
10
5
0
Black Sol
Coghill SLD
Dingwall ScU
Gibson SNP (elec)
Kerr Lab
Macdia - rmid Lab
Master - ton Grn
McGart - land SSP
Petty Con
Watson Lab
non- transferable



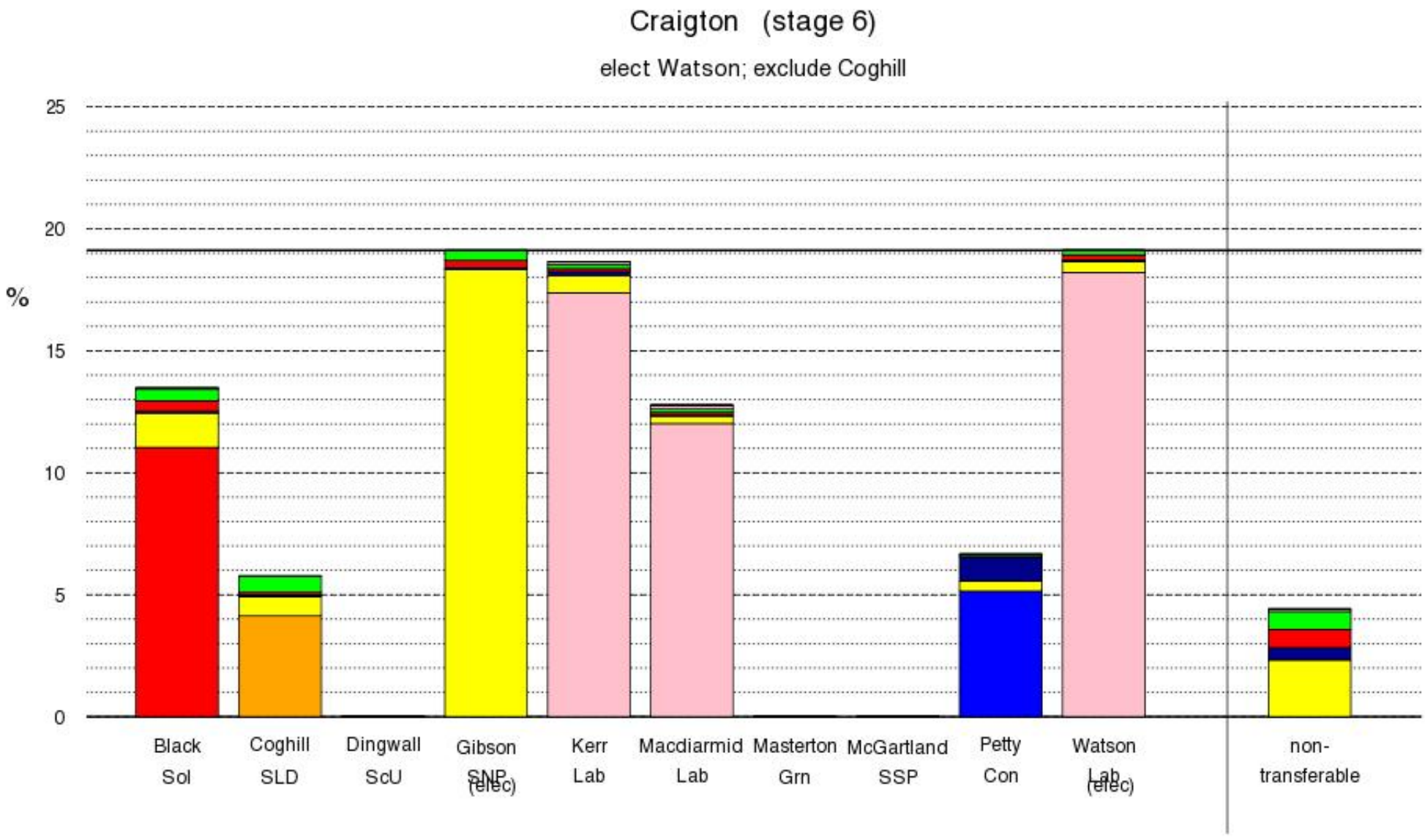
STV system: Meek



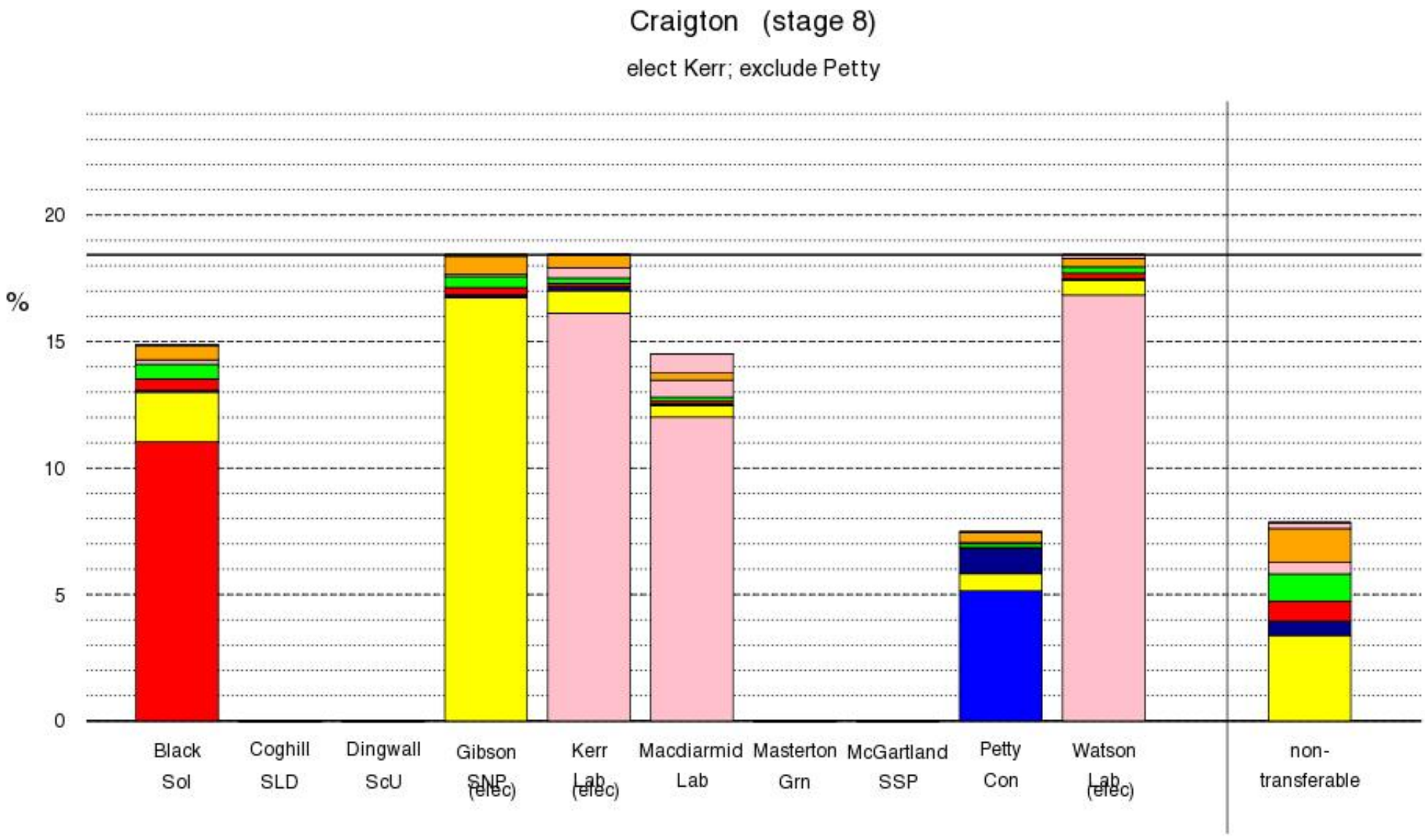
STV system: Meek



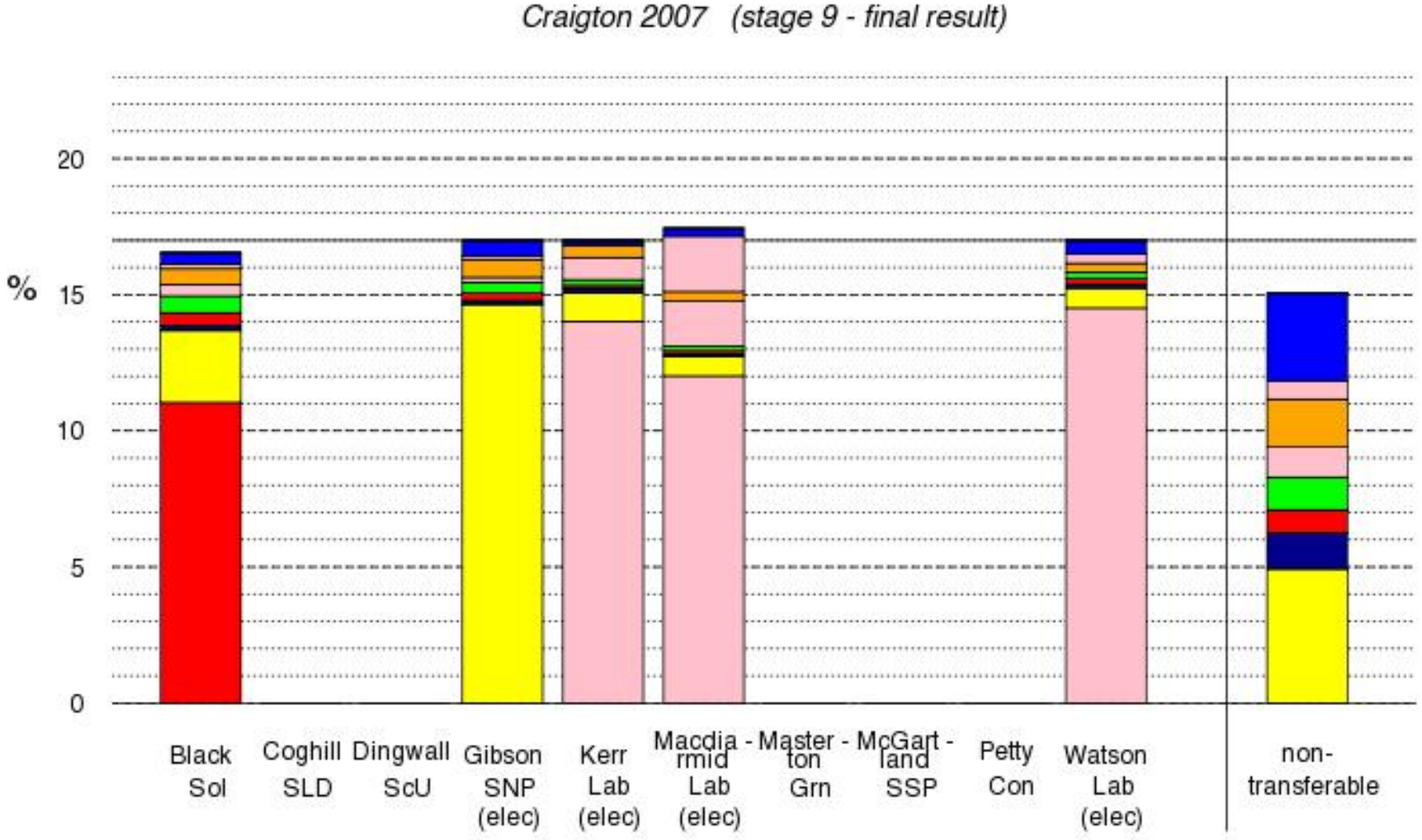
STV system: Meek



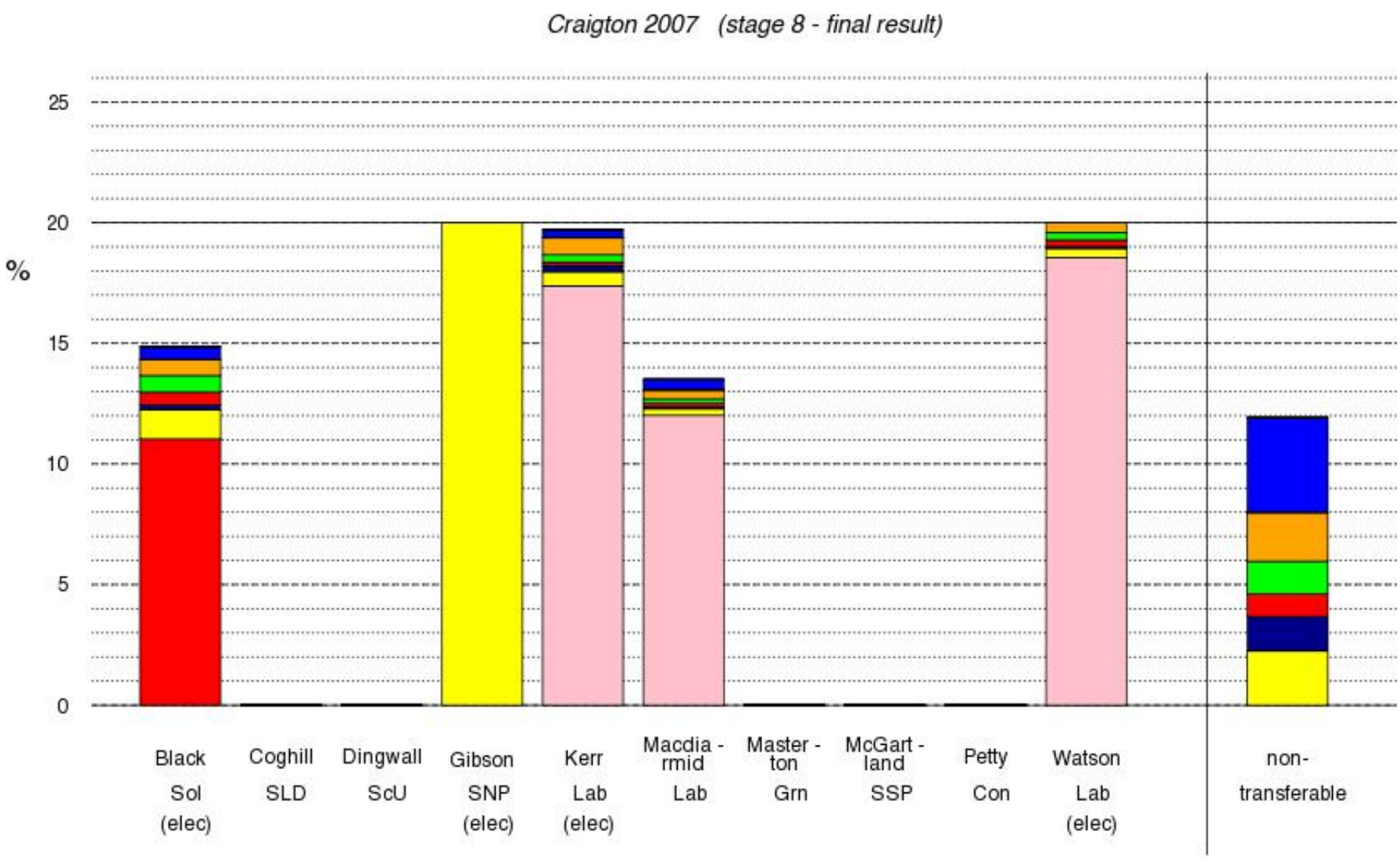
STV system: Meek



STV system: Meek



STV system: Meek



STV system: WIG (as used in this election)

At any stage of the count the state of play is described by the 'keep value' k_i of each candidate.

- can start again just from knowledge of who's still in and who's excluded

- it's straightforward to calculate what happened to your vote:

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e.g. if k = (0.5, 0, 0.8, 1, ..), a vote for 'ABCD..' will be shared (0.5

- it's straightforward to calculate what happened to your vote:

e.g. if k = (0.5, 0, 0.8, 1, ..), a vote for 'ABCD..' will be shared (0.5,0,0.4,0.1).

Thus exact STV is a very stable algorithm, and easily audited.

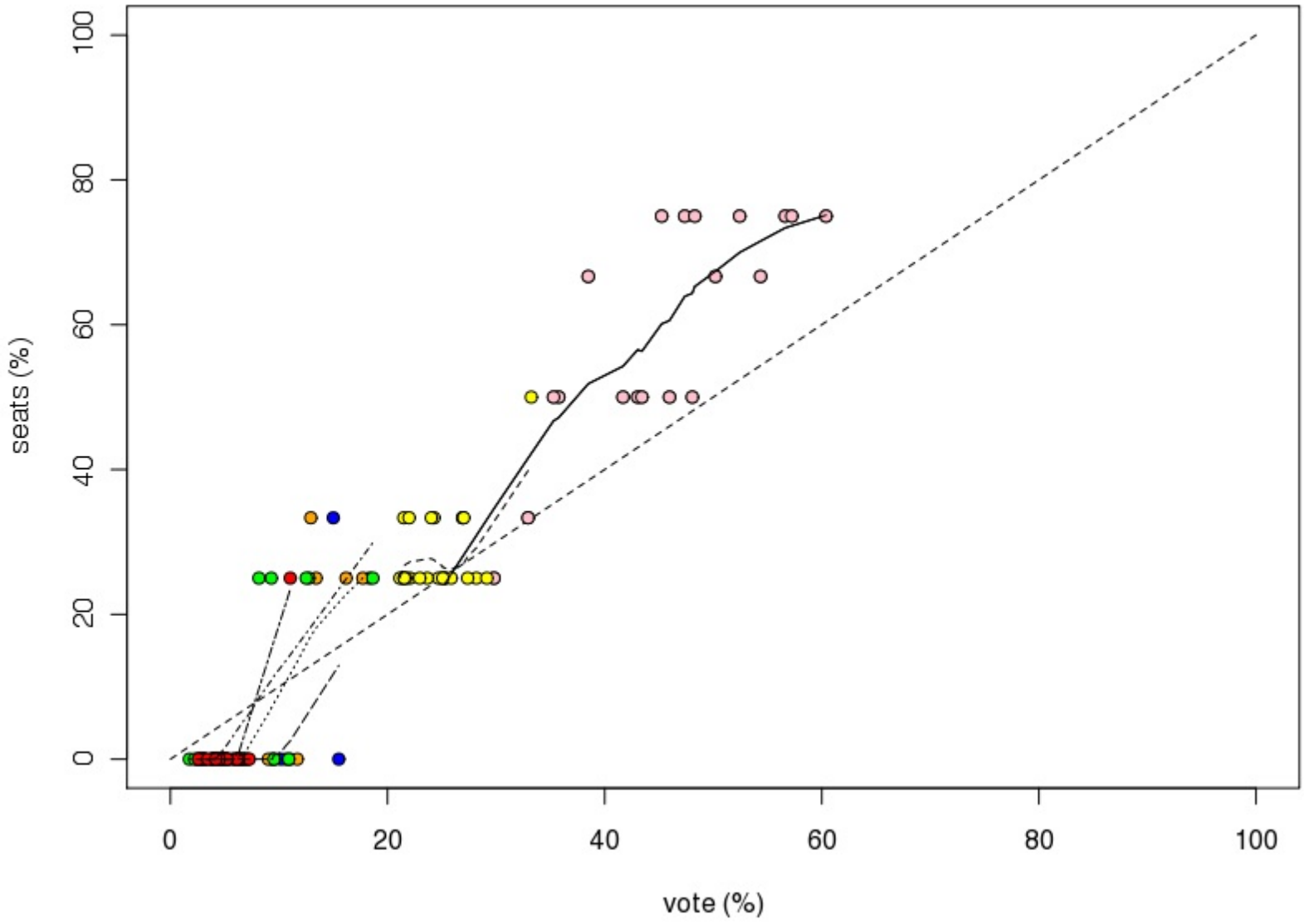
The approximate methods differ very little in practice, but are not so easy to understand or audit.

Also, they have discontinuities, and offer some scope for tactical voting.

3. STV in practice

Glasgow 2007
(wards of 3 or 4 seats)

	Con	Lab	LD	SNP	Gn	S	O
Vote %	8	44	8	25	7	5	3
Seats %	1	57	6	28	6	1	0



Seats as a function of votes under STV, Glasgow council election 2005. Parties (Con, Lab, LD, SNP, Green, Sol) are represented by (blue, pink, orange, yellow, green, red) respectively.

Wards with 3 candidates of same party:

In 12 wards, Labour had 3 candidates.

GLM model shows

significant ward and individual effects

order "123" about twice as common as others

evidence of "Condorcet cycles" – $A > B > C > A$

	1>2	2>3	3>1	sum	twist
Linn	-3.9	4.7	-2.9	-2.0	
Greater_Pollok	-5.4	-7.5	-8.5	-21.3	--
Craigton	-2.2	-10.0	4.9	-7.2	-
Govan	-17.2	2.4	22.8	8.1	+
Southside_Central	17.6	-25.8	-16.4	-24.6	--
Garscadden-Scotstounhill	10.8	-9.2	-5.5	-4.0	
Drumchapel-Anniesland	6.5	-22.2	-3.5	-19.2	--
Canal	0.5	14.8	-17.4	-2.1	
East_Centre	-1.2	7.6	2.9	9.4	+
Shettleston	-0.2	3.2	7.1	10.0	+
Baillieston	-4.1	4.4	-33.7	-33.4	---
North_East	0.5	11.7	-4.3	8.0	+

The Aardvaark effect

(idle votes for “12” or “123”)

Possible solutions include

- varying order on the ballot paper
- allowing equal prefererences

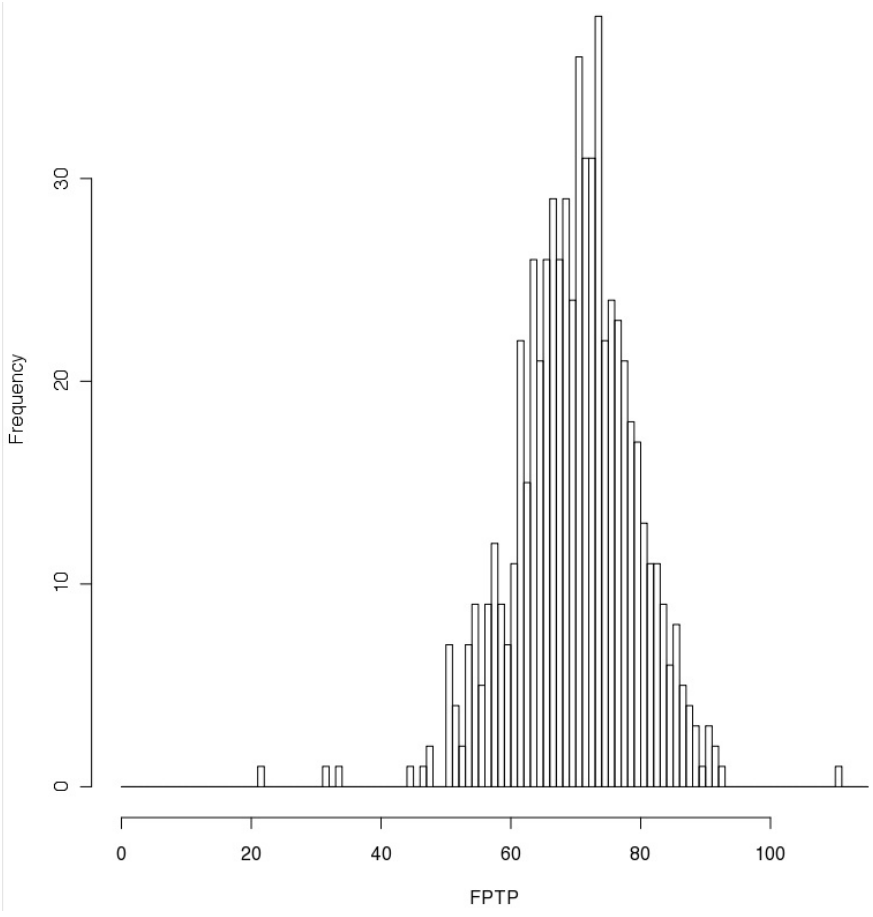
4. Design of constituencies

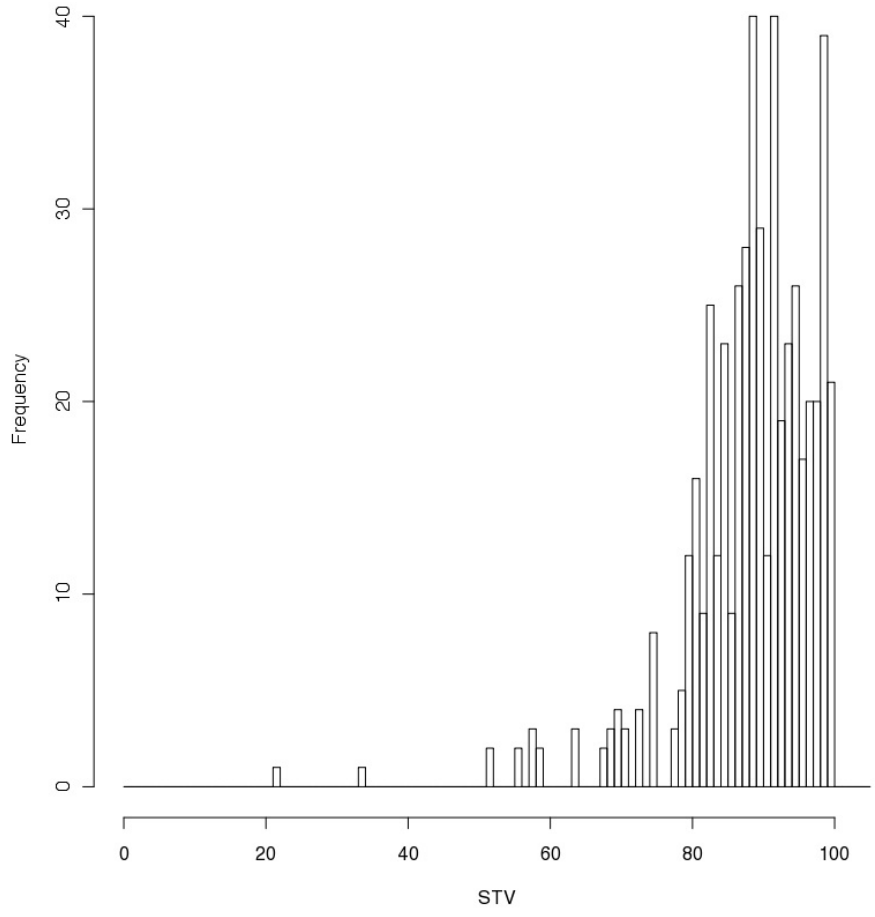
2010 proposal to use STV for the UK Parliament, with constituencies based on local government boundaries.

+ Constituencies mostly with 4 or 5 seats, giving good proportionality

+ Fixed boundaries match communities

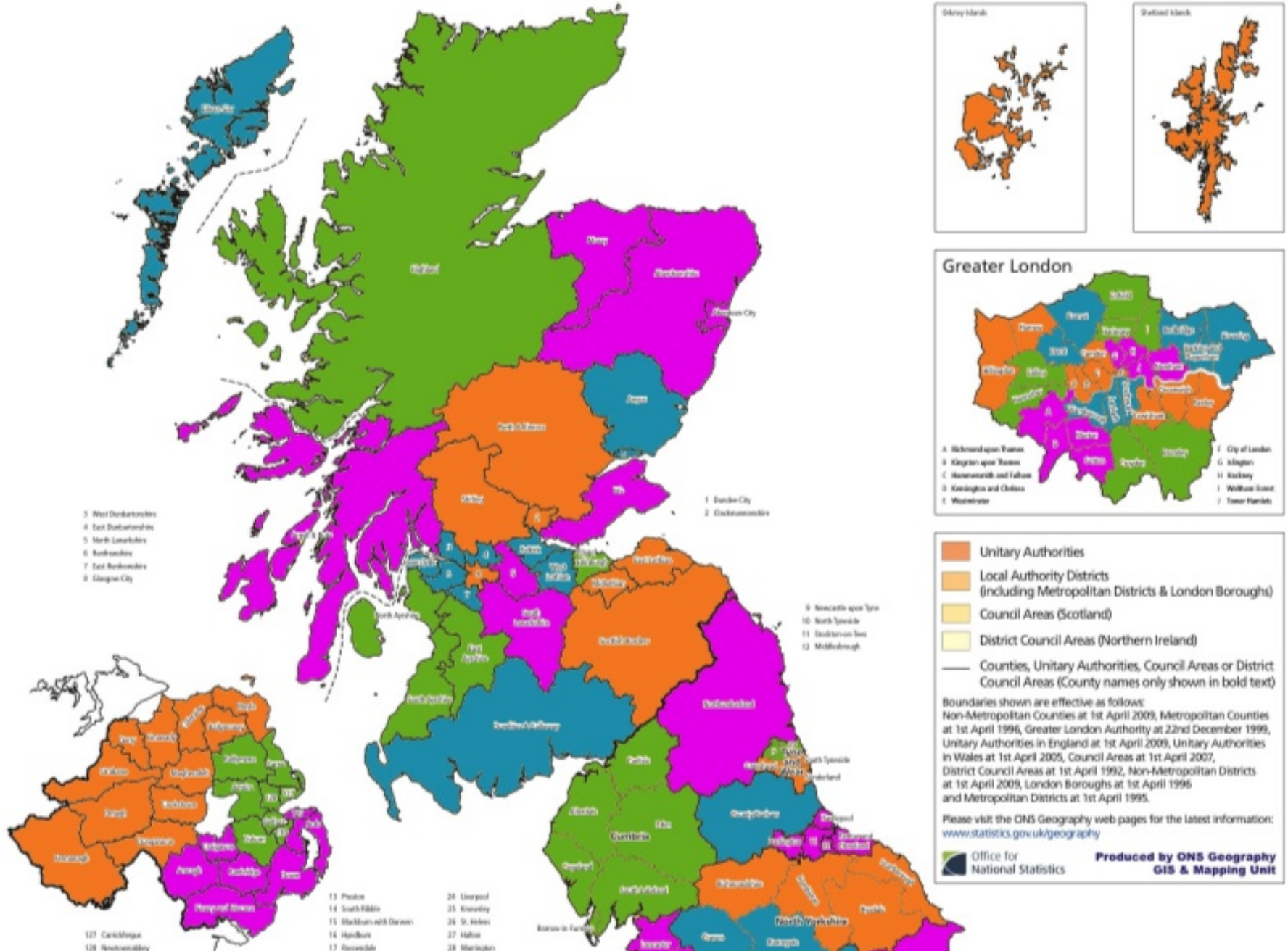
- Variability in electors/seat ("+/- 0.5/4.5")

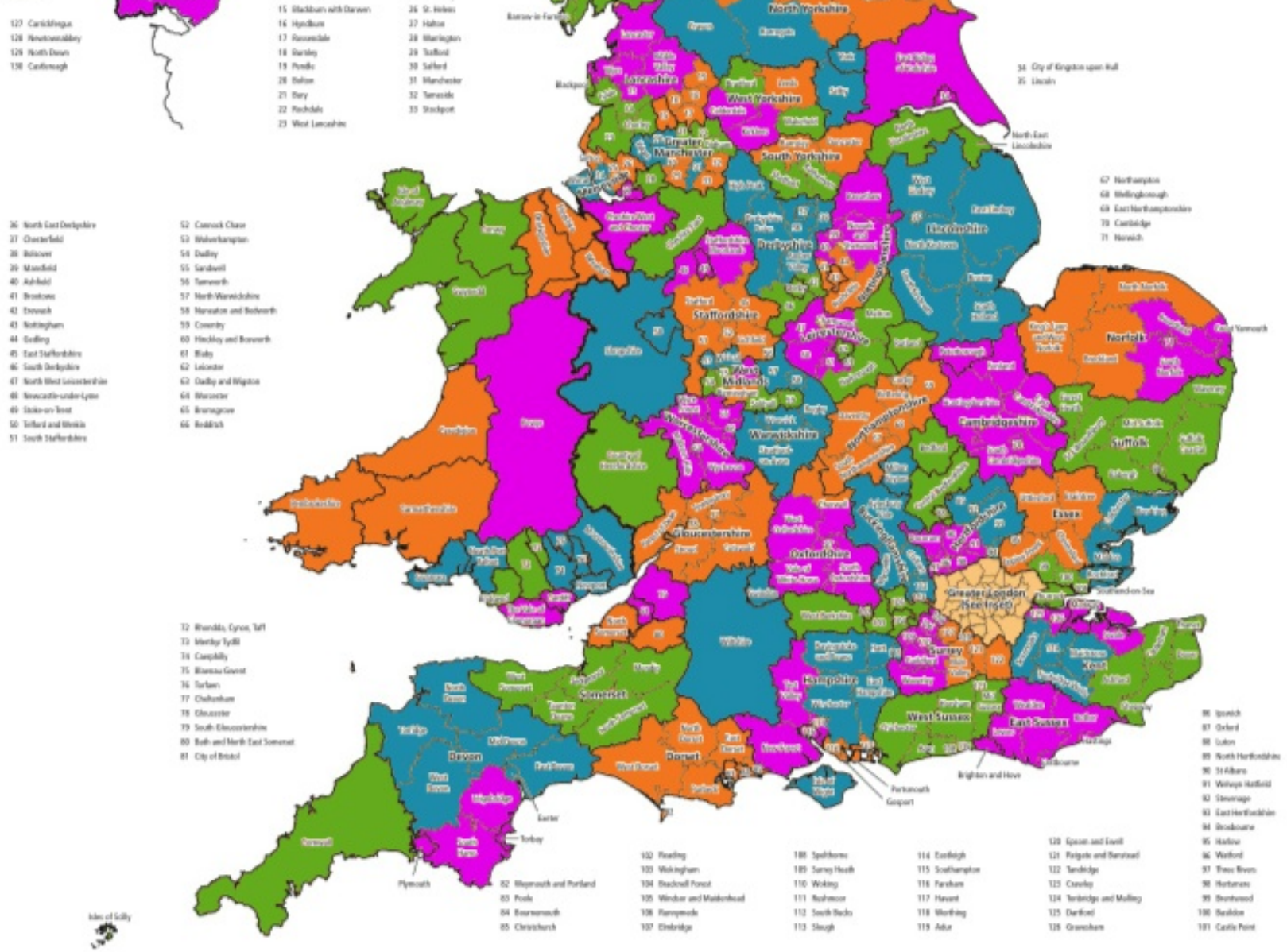

Frequency
0
10
20
30
0
20
40
60
80
100
FPTP


Frequency
0
10
20
30
40
0
20
40
60
80
100
STV

Party	Con	Lab	LD	Nat	Other
Votes	33.2	36.2	22.6	2.2	5.7
MPs (STV)	34.9	39.5	23.2	2.2	0.2
MPs (actual)	31.5	56.8	9.9	1.3	0.5

United Kingdom: Local Authority Districts, Counties and Unitary Authorities,[1] 2009





1 Council Areas in Scotland and District Council Areas in Northern Ireland are equivalent to Unitary Authorities in England and Wales, but are shown separately.

5. A multi-option referendum?

A No change

B More powers (financial autonomy?)

C Independence

Ideally, identify a *Condorcet winner*, *i.e.* an option that has a majority against either of the other two options.

In general, this requires *preferential voting*, or 3 questions (A v B, A v C, B v C).

There is the possibility of a Condorcet cycle ($A > B > C > A$ or vice versa).

Arguably, for our three options there is a direction of travel, $A \rightarrow B \rightarrow C$.

This would imply that noone has preference ACB or CAB, and allow us to use just two questions:

1. Do you want more powers? (A v B)
2. Given more powers, do you want independence? (B v C)

Testing the "Direction of travel" hypothesis requires data on order of preference, to see what percentage have the "irrational" preferences ACB or CAB.

It could also test the "chasm" hypothesis, which says that Independence is so different from the other options that there will be very few people with preference BCA.

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No major politicians admit to preference BCA, but indirect evidence from existing polls suggest it is quite common.

Comparing answers on first preferences for two (A,C) and three (A,B,C) options in a recent poll suggests:

pref.	ABC	BAC	B	BCA	CBA
%	33	19	8	11	29

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(http://www.ma.hw.ac.uk/∼denis/)

.. /STV_elections/ *examples*

.. /soft.html *"run your own election"*

.. /stv4uk/ *UK Parliament, etc.*