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Wave climate and the wave power resource

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Summary

Wave climate statistics are reviewed from the wave power engineer's point of view. It is emphasised that there is no fixed answer to the question "What do we need to know about the wave power resource ?" but rather a feedback loop between our knowledge and the development of any particular type of wave power device.

The mean power available averages over 40 MW/km along the best oceanic coasts, such as those of western Europe; of which at least 50% is potentially economically extractable. The total resource worldwide is roughly equal to present world electricity consumption. (Electricity is not the only possible use; direct use of wave power may also be attractive, for instance for desalination.)

Current interest is more in small scale devices, perhaps 10 to 30 metres wide, which enjoy a number of advantages, including the 'point absorber effect', whereby they have an effective capture width considerably greater than their physical width. Mean outputs of 1 to 2 MW should be feasible, and even devices with outputs as low as 100 kW may be economically attractive.

The outstanding data requirement is for better information on directional spectra, for specific analyses of device performance and survival rather than for power output estimates.

Contents

1 Introduction

That there is energy stored in the oscillations of the sea surface is obvious to anyone who has experienced an ocean storm. Wave climate statistics, especially of extremes, have been of interest for a long time to ship and harbour designers, and more recently to the oil industry. That wave energy travels, and that it forms a substantial renewable energy resource, has only become widely appreciated over the last dozen years, though patents for wave power devices go back to the eighteenth century.

My intention here is to answer the question - What is the wave climate like? - from this new point of view, emphasising those aspects of the resource of most interest to the designers of wave power devices, and indicating aspects where more work is needed. The answer is necessarily somewhat open. On the one hand it is important to describe the wave climate in as simple terms as possible so as to clarify the designer's task. On the other, the development of designs, and new design ideas, often lead to new demands on our knowledge of the wave climate, and on our ability to summarise aspects which are newly seen to be important.

Some of the basic aspects of the resource were recognised early as of importance; for instance, its character, as mechanical oscillations of a medium, its characteristic frequency range (.05 to .2 Hz) and scale, and its variability, both from wave to wave and over longer time periods. One cannot consider the resource in isolation from the type of device proposed for its exploitation; a distinction which is increasingly recognised as important is that between the large and small scale resources.

The recent development of wave power studies owes its origin to the panic over energy supplies which followed the Yom Kippur war of 1973. Understandably therefore, the early emphasis was on establishing and trying to exploit the large scale resource, researching and designing installations that could make a significant contribution to a large population's electricity demands.

In the oil glut of the 1980s governmental attitudes, especially in the UK, have changed to indifference or hostility towards large scale use of renewable energy sources. Emphasis has therefore shifted to where perhaps it should have started: to the design of small scale devices, both to meet local demands, and as prototypes for the time when interest in renewable energy sources revives.

The distinction between large and small scale devices is not just political and social. A device small compared with the wavelength of sea waves can capture power on a front significantly greater than its physical width, and this provides a clear scientific reason for distinguishing between large and small scale resources in this review.

Before trying to characterise these resources, I shall first describe how wave power arises, its general characteristics, how the resource is estimated, and how these estimates can be used in the design and evaluation of devices.

2 General character of the resource

Considered in terms of the earth's energy balance, wave power is a minor byproduct of wind power, which is in turn a minor byproduct of solar power. The atmospheric circulation aids in redistributing the sun's energy input (see e.g. Battan [3]), and loses a very small proportion of this energy through drag on the sea surface, thus creating gravity surface waves. The net power input from wind to waves is seldom as much as 1 watt per square metre, compared with the mean solar radiation of 350 watts/m^2.

However, once energy has been put into gravity surface waves, in deep water it dissipates slowly (hardly at all for low waves of more than 9 seconds period) and meanwhile travels, typically at 500 to 1000 kilometres a day. Thus the power input over an area of ocean can be harvested along its boundary, with power densities higher than those of the original solar input: as we shall see, the net resource crossing a line facing prevailing oceanic winds can average 40 to 50 MW/km (or kW/m; it seems appropriate to use MW/km for the large scale resource, kW/m for density at a point or for the small scale resource).

This advantage of concentrating the earth's solar energy input is to some extent shared by wind power, as is its form as mechanical energy rather than heat. The overall wind power flux is much greater, but is spread over a height of several kilometres, whereas the wave power flux is largely contained in the top 10 to 20 metres of the sea, so that it can achieve power densities about ten times as large. Also, wave power enjoys a theoretical advantage: with wind power the medium travels, and there is a limit to the possible efficiency of extraction of $^{16}/_{27}$, ≈ 57% (see e.g. Lipman *et al* [24]); with wave power, the medium only oscillates, and 100% efficiency of extraction is theoretically possible (and can very nearly be attained in experiments).

Both wave and wind power have the practical advantage of being in general terms concentrated when and where they are wanted: in the temperate zones, and in winter rather than summer, since this is where the solar energy gradient with latitude is greatest. The wave power resource is thus concentrated around the oceanic countries, particularly those with prevailing onshore winds, such as western Europe, the (western) United States and Australia. Large scale installations could make a substantial contribution to those countries' energy requirements. Small scale installations are an exciting prospect for a number of oceanic communities presently disadvantaged by expensive power supplies. The direct use of wave power for desalination is also a possibility for some of these communities.

The main disadvantage of wave power, which it shares with wind, is its variability. There is great variation in power levels, with the passage of each wave, from day to day, and even from month to month (though, as already noted, the general pattern of seasonal variation is favourable). This poses problems for the wave statistician as well as for the engineer. Long data runs are needed to provide reliable wave climate estimates, and there are considerable problems of correlation in choosing representative samples for tank tests, and in estimating the return period of extreme wave heights. The accuracy of climatic estimates can be improved by the use of background *wind* climate information.

The waves generated by a particular wind show a directional spread about the wind's principal direction. We need to distinguish between the gross wave power level, which may average nearly 100 kW/m in mid ocean, and the net resource crossing a line which will only be a proportion (the *directionality coefficient*) of that figure. In mid ocean the directionality coefficient will typically average only around 50% even for the best (fixed) direction; near coasts the combination of sheltering and refraction (which bends waves towards the slope direction in shoaling water) leads to directionality coefficients close to 100%.

While accurate measurement of wave heights, and of the frequency domain, can be carried out routinely by a variety of techniques, notably wave buoys, the directional domain is more problematic. Directional measurements are expensive, and subject to theoretical limitations on their resolution. Although there has been substantial progress on a variety of techniques in recent years, for wave climate estimation we still have to rely on syntheses and wind-wave models, calibrated by routine non-directional measurements and using formulae for directional spread based on the best measurements that have been made.

In evaluating the small scale resource, we need to take account of the available advantages of geographically special sites, of refraction 'hot spots', and above all of the point absorber effect: namely that devices can potentially capture power from an additional 'capture width' of *wavelength*/π (or *wavelength*/2π for an axisymmetric device) - for a small device this can multiply the potential resource by a factor of five or more.

3 Wave spectra

Deep water waves can, to a good approximation, be regarded as composed of the linear superposition of a large number of simple components. Each of these basic components is a sinusoidal wave train, with period T say (typically 5 to 20 seconds), which appears to travel at its *phase velocity* $U = gT/2\pi$. The phase velocity is typically similar to the velocity of the wind generating the waves (see next Section). The wavelength is $L = UT = gT^2/2\pi$.

In fact the water particles do not (to first order) travel, but only oscillate in circular orbits, whose amplitude falls off exponentially with depth ($a_d = a\exp(-kd)$, where k is the wave number, $= 2\pi/L$).

The energy of the wave train, per unit area, is $E = \rho g H^2$, where ρ is the density of sea water, and H is the root mean square wave height : $H^2 = \frac{1}{2}a^2$. This energy is made up from equal components of kinetic energy and potential energy.

The power flux P, per unit of wave front, can be evaluated from the simple observation that, by moving in circular orbits at constant speed, water particles are transferring potential energy in the apparent direction of the waves. This shows that P is equal to the potential energy per unit area, $\frac{1}{2}E$, multiplied by the phase velocity U, i.e. $P = cH^2T$, where the constant $c = \rho g^2/4\pi$, $\approx$ 7.87 kW/m^{3}sec.

This calculation only applies to a wave train in steady state. More generally, the total energy E can be shown to travel at the *group velocity*, which in deep water is equal to half the phase velocity. (Drop a stone into a pond, and note how the group travels more slowly than the individual ripples, which first grow and then decay as they pass through the centre of the group.) Thus $P = \frac{1}{2}EU = cH^2T$, as before.

For the overall sea state, the movements, energy and power of the individual sinusoidal components add up to give a Gaussian random process, whose spectrum (strictly spectral density) $e(f,\theta)$ describes how the total energy E is distributed over frequencies f and directions θ. Thus

$$E = \iint e(f,\theta)\ df\ d\theta = \rho g H_{rms}^2 .$$

Here H_{rms}^2 is the variance of the surface elevation. A common alternative is to work in terms of the *significant wave height* H_S : this was originally defined as the height of the highest one-third of waves (measured trough to crest between successive zero crossings), which $\approx 4.0\ H_{rms}$ for a narrow spectrum, but is now normally used simply as a synonym for $4H_{rms}$.

It is often convenient to work in terms of the spectral distribution $s(f,\theta) = e(f,\theta)/E$, whose overall integral equals unity. The gross power flux is

$$P = \iint \tfrac{1}{2}U\ e(f,\theta)\ df\ d\theta = E\iint \tfrac{1}{2}U\ s(f,\theta)\ df\ d\theta = cH_{rms}^2 T_e ,$$

where $c = \rho g^2/4\pi$ as before, and the *energy period* T_e (Mollison *et al* [34]) is the average value of T with respect to the spectral distribution, i.e.

$$T_e = \iint T\ s(f,\theta)\ df\ d\theta .$$

Since $T = 1/f$, we may write $T_e = m_{-1}$, where m_n denotes the nth moment of the spectral distribution,

$$m_n = \iint f^n s(f,\theta)\ df\ d\theta .$$

(Note: it is common to work with $S(f,\theta) = H_{rms}^2 s(f,\theta)$ instead of $s(f,\theta)$; this leads to formulae $H_{rms}^2 = m_0$ and $T_e = m_{-1}/m_0$.) It is convenient to define the mean phase velocity $U_e = gT_e/2\pi$, and the corresponding wavelength $L_e = gT_e^2/2\pi$; the ratio H_{rms}/L_e is then a measure of the steepness of the sea state.

The net power flux P_Θ in direction Θ is found by multiplying the energy of each component by its group velocity resolved in direction Θ, i.e.

$$P_\Theta = cH^2_{rms} \int \int_{\Theta-\pi/2}^{\Theta+\pi/2} T \; s(f,\phi) \cos(\phi-\Theta) \; df \; d\phi \; .$$

Note the limits of integration: waves from opposite directions pass through each other, they do not cancel out. For either an individual sea state, or a wave climate average of sea states, we can define the *directionality coefficient* $d_\Theta = P_\Theta/P$.

Various measures of spectral width have been proposed. The most convenient from a theoretical point of view (see Longuet-Higgins [25]) are defined in terms of higher moments of the spectrum, including at least m_2. For wave power purposes it is best to have a parameter which is dependent much more on the more powerful low frequency end of the spectrum; one such is σ_T, the relative standard deviation of the period T (Mollison [31]), i.e.

$$\sigma_T = \sqrt{m_{-2} - m_{-1}^2} / m_{-1} = \sqrt{(m_{-2}/m_{-1}^2) - 1} \; .$$

(See Section 8 for a use of m_{-3} !)

4 Wave growth, travel and decay

The slow processes of wind and wave interactions that generate surface waves are highly nonlinear, and still not fully understood (see e.g. Leblond & Mysak [23]).

Briefly, energy and momentum are transferred from the turbulent atmospheric boundary layer to the sea through air pressure and shear stress. Energy is input mostly to the high frequency end of the wave spectrum, and subsequently transferred to lower frequencies through nonlinear wave-wave interactions (Hasselmann [17, 18]). These interactions conserve the overall energy and momentum, and there is also a transfer to higher frequencies, but this is balanced by dissipation through breaking (see Figure 4.1).

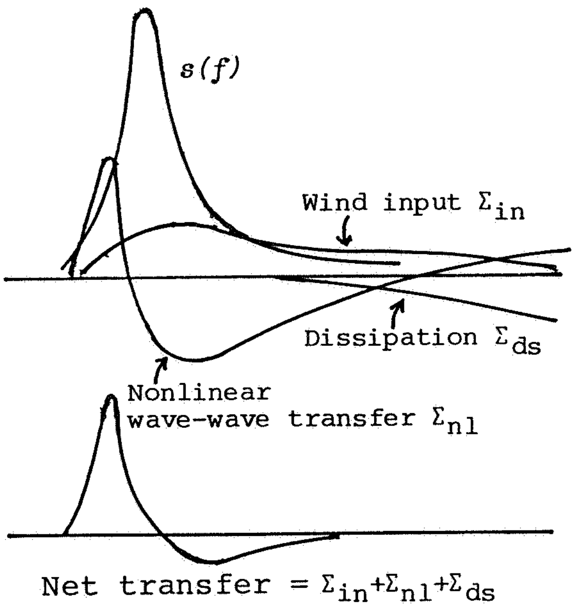

Figure 4.1
Energy transfers in the spectrum of a growing sea (after Hassselmann *et al* [19]).

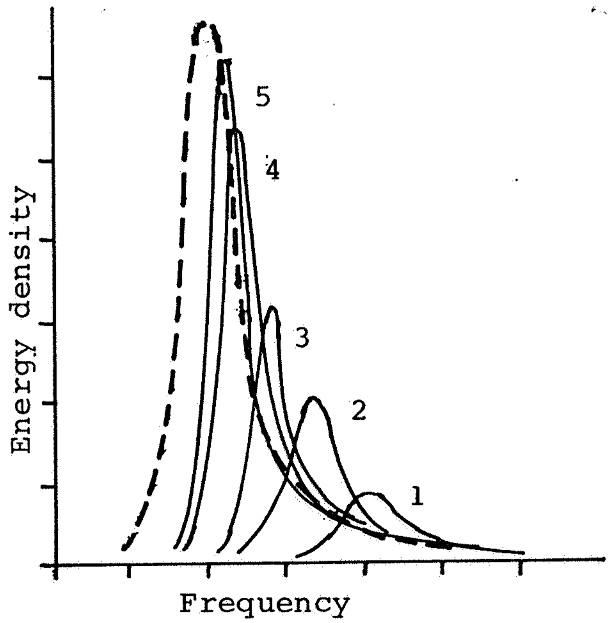

Figure 4.2
Growth of wave spectrum with fetch (after Hasselmann *et al* [19]), with tentative fully-developed spectrum (dashed curve).

The growth of waves from flat sea to near full development was studied in detail in the JONSWAP project (Hasselmann *et al* [19], Ewing [13]), confirming this general theory of wave growth. They found that wave energy density E grew approximately linearly with fetch x; thus

$$H_{rms}^2 \approx 1{\cdot}6 \times 10^{-7}\ U_{10}^2\ x/g$$

where U_{10} denotes the windspeed measured at 10 metres above the sea surface. Note that this relation, which was observed to hold for fetches up to 10^3 - 10^4 times U_{10}^2/g, does not mean that the rate of net energy input is constant, as is sometimes assumed in discussing this result. The rate of net energy input equals the rate of change of power, and thus will be proportional to T_e, which is increasing. Figure 4.2 shows the kind of growth of wave spectra observed during the JONSWAP experiments. Hasselmann *et al*'s formula fitted to the JONSWAP data gives T_e proportional to $x^{\cdot 33}$, but this, like the energy growth formula, can only hold for growing seas.

The JONSWAP experiments unfortunately tell us little about the limits to wave growth. For this we must turn to earlier work, such as the classic papers of Pierson & Moskowitz [35, 38, 37]. Following the scaling law approach of

Kitaigorodskii, and on the basis of data for apparently fully developed seas in winds of up to 40 knots (≈ 20 m/sec), they proposed a wave spectrum which can be written as

$$\int_0^f S(y)\, dy = H_{rms}^2 \exp[-\cdot 675(fT_e)^{-4}]$$

where $T_e = \cdot 625\ U_{19\cdot 5}$, $H_{rms} = \cdot 0053\ U_{19\cdot 5}^2$. This spectrum has a high frequency tail $\sim f^{-5}$ as originally proposed by Phillips [36].

Its spectral width $\sigma_{T'} \approx \cdot 28$, and its steepness, $H_{rms}/L_e \approx 1/115$, are independent of the windspeed. Its mean phase velocity is almost exactly the same as the windspeed, $U_e = \cdot 975 U_{19\cdot 5}$; this seems a suggestive coincidence, but there is no direct theoretical explanation for it.

As one would expect from dimensional considerations, the power in such a fully developed sea is proportional to the fifth power of windspeed U^5, and the fetch required to generate it is proportional to U^2, and can thus be expressed as a multiple of the wavelength. If we assume that the JONSWAP energy growth formula applies up to the stage where the Pierson-Moskowitz value of H_{rms} is attained, we find that this fetch is approximately $3000L_e$. For instance, for $U_{19\cdot 5} = 20$ m/sec, we obtain $H_{rms} = 2\cdot 12$ m, $T_e = 12\cdot 5$ secs, and the required fetch is about 700 km (the power density attained is 440kW/m, so in this example the mean rate of energy input during growth is about 0·6 watts/m^2).

Fully developed seas associated with windspeeds greater than about 20 m/sec will be rare because of the fetch required. In practice, one of the most favourable conditions is where a strong depression moves so as to stay on top of some of the waves it is creating, which will occur if the depression moves at approximately their mean group velocity (= $\frac{1}{2}U_e$). In the North Atlantic, depressions have an average eastward velocity of around 8 m/sec (Bartholomew & Herbertson [2]), and so the effective fetch of westerly seas is often significantly increased.

Nevertheless, most occurrences of extreme power levels and steepness are associated with growing seas, for which the narrower JONSWAP shape of spectrum is more appropriate ($\sigma_T \approx \cdot 2$).

The directional spread of wind seas is less accurately known than their frequency spectrum. Mitsuyasu [30] fitted a spreading function of $\cos^{2r}(\theta/2)$

form,

$$s(f,\theta) = s(f)\ c_{\theta} \cos^{2r}((\theta-\theta_0)/2)$$

where θ_0 is the wind direction, and the frequency dependent parameter r reaches a maximum of 15·85 at the peak frequency f_p; that is, the spectrum is most narrowly directional around the peak frequency. Note that this formula allows for energy travelling in all directions, unlike the alternative $\cos^r(\theta)$ $(\theta_0-\pi/2 < \theta < \theta_0 + \theta/2)$ form of spreading function which is commonly fitted to swell, appropriately enough as swell normally has a very narrow directional spread.

In the open ocean, waves decay only very slowly once their steepness is reduced below the point at which breaking losses are significant. Snodgrass *et al* [47] observed swells which had crossed the whole Pacific Ocean. Numerical models (e.g. Golding [15]) tend to assume that no losses occur. Certainly losses from viscosity are minimal: a wave of 9 seconds period would need to go three times round the world to lose even 10% of its amplitude (Crapper [10]): but losses to other kinds of ocean wave, such as internal gravity waves, are possible (see e.g. LeBlond & Mysak [23]).

As waves enter shoaling water, their direction is altered by refraction so that their principal direction is brought nearer the perpendicular to the depth contours, while their directional spread tends to be narrowed. Note that it is the net power crossing a line that is conserved, so that as the directional coefficient increases the gross power level will decrease even if there is no actual energy loss.

Over uneven bathymetry, refraction will cause focussing over a convex bottom (defocussing over a concave one) leading to 'hot spots' of above average power: Mehlum [29] gives examples of sites whose mean power is more than doubled by this effect.

It is well known that waves lose the last of their energy through breaking in shallow water. Energy losses in intermediate depths (20 to 100 m) are mainly due to bottom friction (Bagnold [1], Vitale [50], Hasselmann *et al* [19]). Mollison [33] estimated the bottom friction coefficient required to account for observed wave climate losses over the rough but gently shoaling bottom off South Uist. The estimated loss rate was (e.g.) around 1% per km in 35 m depth, and 70% of the energy was lost between 100 and 15 m depth (see also Section 7). Backscattering and reflection may also account for significant losses in appropriate circumstances (Shemdin *et al* [46]).

5 Measurement, estimation and simulation of waves

Wave buoys at present afford the most reliable and accurate wave height measurements, and hence estimates of gross power levels and frequency spectra. Calibration errors are normally only a few per cent, less than the absolute error in estimating spectral parameters from a perfect wave record of typical length (around 100 wave periods) (see e.g. Tucker *et al* [49]).

While good quality data runs over considerable periods exist for wave buoys (e.g. an over 90% data return during 1976-78 for the Institute of Oceanographic Sciences 42 m depth buoy off South Uist, NW Scotland), wave power variability from year to year is such that it is desirable to use longer term wind information when estimating the long term average power or selecting a subsample representative of the wave climate. Two methods, stratified sampling (as used by Crabb [8]) and a 'wind input' method are compared by Mollison [31]. Mollison [31] also estimated that even with 13 years of exact wave measurements, the estimate of the long term average power would only be known within 95% confidence limits of ± 6%; however this variability is unduly influenced by rare extreme storms, so that estimates of mean output power will be considerably more reliable.

Directional buoys have been available for some time (Longuet-Higgins *et al* [26]), but have only recently been deployed on a routine basis. Alternative methods include arrays of small buoys (Mehlum [28]), and radar backscattering (Shearman [45]), while satellite techniques offer great promise for the future (Tucker [48]). All these methods are to an extent indirect, since assumptions about the form of the directional spectrum are required for their analysis. Present estimates of directional wave climate mostly rely on syntheses and numerical hindcast methods.

Crabb [8] selected a representative set of 399 wave records for a site in 42 metres depth off South Uist and synthesised their directional spectra, using local wind speed and direction information for the wind sea, and meteorological chart analysis for swell. (See Mollison [31, 33] for comments on the method of selection and effect of finite depth respectively.)

The estimates presented here (Section 7) are largely based on the numerical hindcasting method developed by the UK Meteorological Office (Golding [15]), which has been in routine use for the North Atlantic since 1978. Wave climate estimates from this model for South Uist are in close agreement with Crabb's synthesis. Also a comparison with the NORSWAM hindcasting model, developed to analyse North Sea storms, showed reasonable agreement of the two models with each other and with measured wave data (Ephraums *et al* [11]).

Where estimates of wave climate are required for a new area, it will probably be best (or at least most economical) to use numerical hindcasts if available, calibrated by direct wave buoy measurements; and otherwise to use directional measurements or syntheses. Where, as will usually be the case, several possible sites are to be covered by a single measurement site, it is much preferable to measure (or hindcast) waves at a deep water site with good exposure, from which the climate at inshore and sheltered sites can be estimated using numerical wave models; whereas one cannot use measurements at an inshore or sheltered site to estimate waves anywhere else.

When it comes to reproduction of directional wave climate samples in test tanks, satisfactory methods exist using either discrete spectra (Jeffrey *et al* [22], Salter [42]) or filtered white noise (Bryden [4]). An advantage of discrete spectra is that these can be measured and thus verified with an accuracy that is not possible for more realistic continuous spectra. Two errors to avoid when using discrete spectra are: using more than one component at any frequency, which leads to non-ergodic seas (see, e.g. Jefferys [21]); and not allowing for the inherent variability in (e.g.) energy between samples from the same sea state (see Tucker *et al* [49], but note that the right variability *can* be achieved with a small number of components if their individual energies are chosen correctly, i.e. chi-squared distributed with the appropriate number of degrees of freedom).

6 Wave climate estimates in relation to device design

As described by Mollison [31] in reviewing the prediction of device performance, the detail in which we require knowledge of wave climate advances hand in hand with the development of a device design. At each stage it is important for understanding to present analyses clearly, showing the assumptions behind any conclusions on performance and estimating the errors involved.

Usually the first requirement is for the frequency distribution of the wave climate. One helpful presentation, suggested in this context by Stephen Salter, is as a 'stretchy' axis (Mollison [31]), in which the distance between points on the frequency axis is stretched to represent the proportion of power available in that frequency interval. If efficiency curves are plotted against such an axis (see e.g. Figure 8.1), the area under the curve shows the overall average efficiency in that wave climate; thus providing a good indication as to whether the frequency range(s) where the device is most efficient are well matched to the wave climate. If this presentation is to be a realistic guide to device optimisation, we must discount frequencies most associated with high power levels at which the device will be at full output, for instance by setting a height limit on the data used for calculating the stretchy axis.

Tests in wave spectra, rather than single frequency waves, are essential for the optimisation of device overload and power take-off limits (for more discussion see Mollison [31]). Except for devices with energy storage, such as Ducks and Tapchan (Salter [43], Mehlum [28]), these limits will begin to affect performance while the device is on average still far below full load.

When it comes to selecting a set of spectra for device tests, our aim should be to allow us (1) to estimate output in the overall wave climate adequately (say to within ± 5%), and (2) to evaluate correctly whether particular changes in device design or parameters constitute an improvement relative to the particular wave climate (which may require considerably greater accuracy in comparing pairs of tests).

A sample of 'real seas', however chosen, is not the best answer to this problem. We shall still need to estimate performance in other real seas, both from the same and other wave climates. We may also want to consider the performance of our device scaled up or down in size. And we will want to set error bounds on our output estimates for a particular wave climate. All these considerations argue for using a parametric description of seas, and for testing in a set of seas chosen for its coverage of a range in parameter space including the wave climates we are likely to be interested in (at the appropriate scale or scales). We should develop an interpolation formula for estimating performance in seas not tested in; the role of modelled real seas comes in evaluating whether our interpolation formula is sufficiently accurate.

The choice of an economical set of sea states for tests is a problem of multivariate optimal design, but one whose most important aspect is the choice of an appropriately economical parametric description of sea states. At worst, a description allowing five parameters (H_{rms}, T_e, σ_T, direction θ, and directional spread) each for the wind sea and any swell components should suffice. But for most devices one would expect some much smaller set of parameters to explain most of the variation in output; for instance, for a device with broad frequency response and approximate cos(ϕ-θ) directional characteristic, the two parameters P_θ and T_e might provide adequate accuracy (here θ is the direction the device faces, ϕ the direction of a component of the wave spectrum).

In any case, it is important to choose test values covering the outer limits of the range of parameter values of interest; just as it is easier to predict the past than the future, so interpolation is to be preferred to extrapolation.

For survival tests, parametric seas are again appropriate; especially as the honest engineer will be searching for the particular circumstances which test her device most severely. This will not necessarily be an extreme 100 year wave, and may be something which might well not be included in a standard test set, such as steep seas within a particular frequency band, or seas in a direction which excites resonance in a spine.

7. The large scale resource

No accurate estimates are available for the world wave power resource; Tornqvist, cited in Glendenning [14], gave worldwide estimates based on visual wave observations (Hogben & Lumb [20]), which are reproduced in Figure 7.1. (For some estimates of the U.S. resource see Hagerman [16].) Arthur, cited in Pond & Pickard [40], estimated the mean power incident on the world's coasts at 2000 GW. From these figures, the world's potentially exploitable wave power resource may be described as being of roughly the same magnitude as present world electricity consumption, i.e. around 1000 GW. Electricity is not the only possible use: direct use of wave power may also be attractive, for instance for desalination (Salter [44]); or for chemical production, for instance of ammonia or hydrogen, where floating factories might be able to harvest the mid-ocean resource.

In mid-ocean the gross power can average nearly 100 kW/m, with substantial contributions from all points of the compass. Nearer to coasts, the onshore components will be broadly similar, but the offshore components of course much reduced (Crabb [7]). Estimates of gross wave power averages for the NE Atlantic

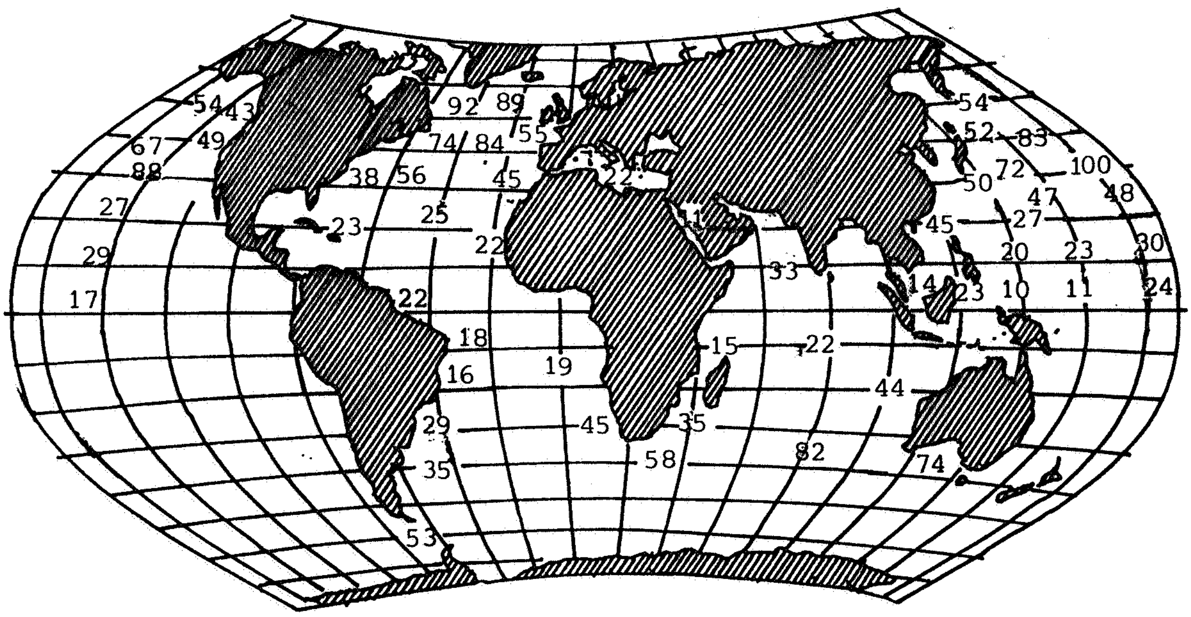

Figure 7.1
Estimates of worldwide mean gross wave power levels in kW/m, from Tornqvist's calculations based on visual wave observations.

are shown in Figure 7.2, with 'wave roses' showing the directional distribution.

In the course of the UK wave energy programme, much attention has been given to various sites off South Uist where measurements have been made by the Institute of Oceanographic Sciences (Crabb [9]), and particularly to a site in intermediate depth (42m) for which Crabb [8] produced a synthesised set of directional spectra (see Section 6). However, as pointed out by Mollison [33], this directional synthesis did not allow for changes of direction due to refraction, and so as it stands is more appropriate for the deeper (100m) site. For this deep water site, there is good agreement concerning the net resource between the adjusted IOS synthesis (46 MW/km estimated long term average, Mollison [33]) and the Met Office hindcasts (45 MW/km average for 1978-81). The latter's higher estimate of gross power (67 as against 59 kW/m) is accounted for by its not taking full account of the site's being sheltered from the east, and including 8 kW/m from that sector.

Off South Uist the sea bottom is mostly very rough, but shelves at a low mean angle (the 100 m site is about 30 km offshore); the significant power loss at intermediate and shallow water sites is thus not surprising (the net power level

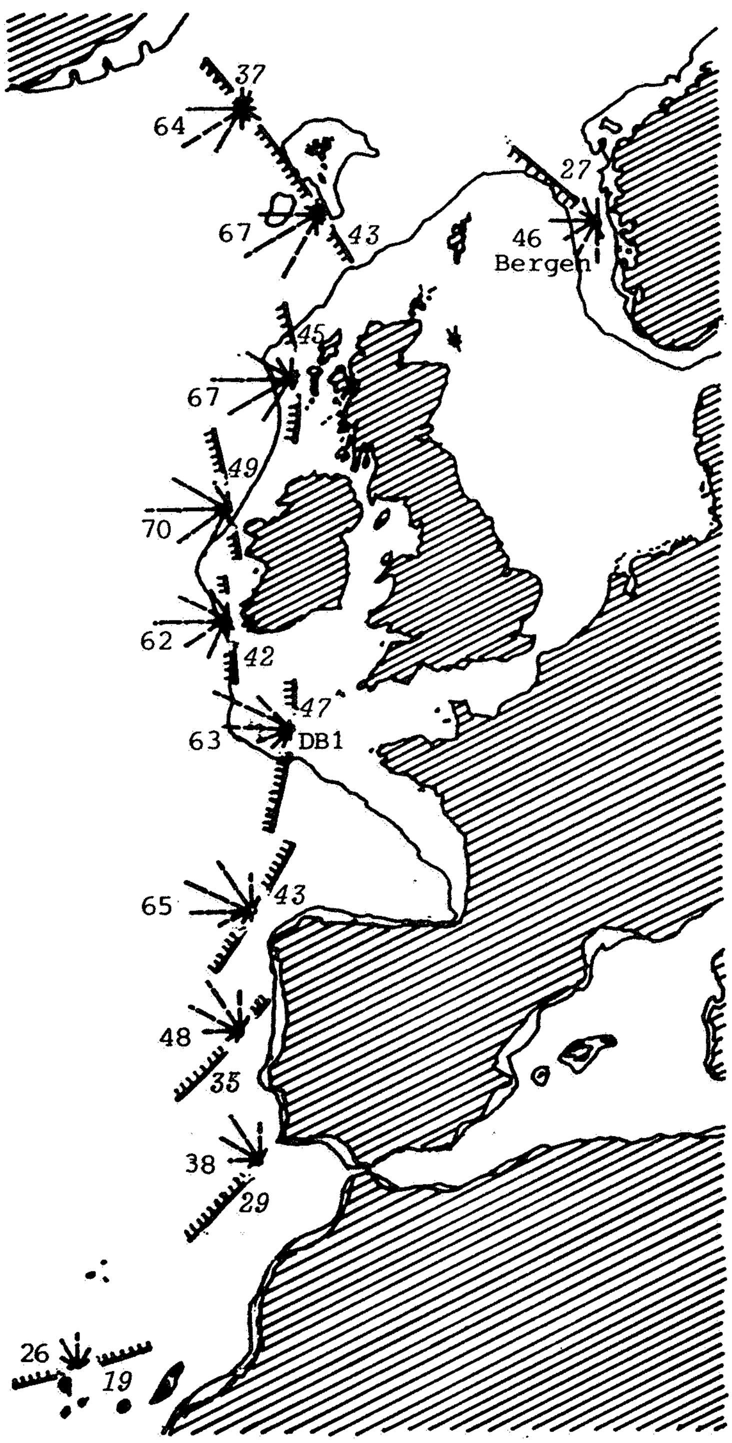

Figure 7.2
Wave power estimates for western Europe based on the UK Met Office wind-wave hindcast model (Norwegian Met Inst model for Bergen). Wave roses show mean power from each 30^0 sector ($22 \cdot 5^0$ for DB1), with marks at 5 kW/m intervals. Gross power levels are shown (in kW/m); and also net power P_Θ (numbers in *italics*) crossing lines (⊥⊥⊥⊥⊥⊥⊥) whose direction Θ maximises P_Θ for the particular site.

Figure 7.3
Tentative lines for the maxim exploitation of Europe's Atlantic wave power resource with international sea bed boundaries (– – – –).

(In both figures, the 200m depth contour is shown.)

reduces to 40 MW/km in 42m depth, 30 in 25m, 15 in 15m, Mollison [33]). Even at the 100m site, a few per cent has already been lost in crossing nearly 100 km of continental shelf (depth ≈ 125m). Along most of Europe's west coasts, deep water can be found somewhat closer to shore.

Lines in directions crossed by maximum mean power are also shown in Figure 7.2, together with the value of the maximum, which for coastal sites is typically about 75% of the gross power level. Naturally, these lines face towards the areas of the North Atlantic whence most of the power originates, rather as though they were ripples caused by large stones dropped in the middle of a pond.

Off Scotland and Ireland, these lines run roughly parallel to the coast. Portugal is less favoured, with a net resource of up to nearly 40 MW/km, but with the best direction to face (at a deep water site) being angled at about 45^0 to the coast. The resource component perpendicular to the coast is thus considerably lower, ranging from about 30 MW/km at the Spanish border to 20 MW/km off C. de S. Vicente; and following refraction in intermediate depths, one would expect the gross power level at an inshore site to be only slightly higher than these figures, *before* allowing for any energy losses. These estimates are thus in reasonable agreement with Portuguese estimates of mean gross power from inshore wave buoy measurements, of 27 kW/m in 30 m depth off Oporto (Pontes & Perdigao [41]), and of 26 kW/m off C. da Roca and 16.5 kW/m off Sines (Pires and Pessanha [39]).

Potential output power is best discussed in terms of an actual device. The obvious choice here is the Salter Duck, whose design has been aimed, more than any other, at extracting as much of the available resource as possible. Of course, this is not to deny that other present devices might be more economic, or future ones more productive.

The estimated mean output for Ducks at a deep water site off South Uist is 19 MW/km (Salter [43]); the Met Office model and IOS's data agree for this site to the nearest MW/km. (This is for a device with a peak power rating of 52 MW/km.) The best coastal Atlantic sites would appear to be off western Ireland, with estimated output about 10% higher than off South Uist (Mollison [32]).

While flat calms can occur even in winter, the general pattern of wave power is favourable, with the mean output in the six winter months (October to March) being about 50% higher, nearly 30 MW/km off the Hebrides. (Thus the load factor averages over 55% for the half of the year when demand is greatest.)

All these output figures are for electricity landed (in the case of South Uist, including transmission to central Scotland). That the mean output is less than half the 45 MW/km available in the sea can be explained by three factors of roughly equal importance: the contribution from high power levels which it is not economical to extract, the efficiency of extraction from the sea (about 75% in small waves), and the efficiency of power conversion and transmission (about 80% on average); an allowance has also been made for losses through non-availability, about 5% with economically optimal maintenance effort (Salter [43]).

While clearly there is scope for improvement on current designs, it is difficult to imagine an economic device which extracted more than about two-thirds of the mean resource at such a site. Thus it seems reasonable to summarise the large scale resource of the NE Atlantic, from Iceland to North Portugal, as having a net resource of 40-50 MW/km, of which 20-30 MW/km is potentially economically extractable.

It is not clear how this oceanic resource should be shared out. Stephen Salter has suggested that wave energy should belong to the country on whose shores it would otherwise have been lost. In terms of the law of the sea, wave power is clearly a resource of the high seas rather than of the sea bed (Chapman [5]), but it is not a 'living resource' to which the UN Convention on fishing might apply.

Figure 7.3 shows a rough allocation of Europe's Atlantic wave power resource based on the median line boundaries used for sharing sea bed resources (Chapman [5]). Using a conservative mean output estimate of 15-20 MW/km, the total potential is around 50 GW. Of this, the UK and Ireland each have about 12 GW, while the Faeroes (Denmark), France, Spain and Portugal each have between 5 and 8 GW. The UK also has the advantage of being the natural customer for the excess resource of its less populous neighbours (in particular, the 'Cape Wrath spur', stretching 800 kilometres from Scotland to Iceland).

This discussion of sharing the resource of course looks well into the future, beyond the technical, economic and political problems of exploiting fully such a large and variable source of power. The way towards that future seems presently to lie via the development of the small scale resource.

8 The small scale resource

As mentioned in the introduction, there have been political and commercial reasons for the recent shift of emphasis towards the small scale wave power resource. But at least as important has been the realisation that small scale devices have a number of advantages to balance against the economies of scale of larger installations.

Firstly, advantage can be taken of geographically special sites, for instance where deep water comes close to the shore. More specifically, parts of the shore itself may be conveniently shaped for adaptation to wave power extraction, such as natural blowholes, or clifftop reservoir sites (Mehlum [28]).

Secondly, refraction of waves over variable bottom topography creates local concentrations of above average power ('hot spots'). Whether onshore or in intermediate depths, device sites can be chosen to take advantage of such concentrations, which even averaged over the variety of wave climate conditions can amount to a doubling of the available resource (Mehlum [29]).

Both the above may operate over a scale of kilometres, and thus apply to quite large installations (tens of MW). The third advantage of small scale devices is the point absorber effect, which operates on a scale small compared with the wavelength (tens of metres, output up to one or two MW).

Evans, Newman and Budal (Evans [12]) proved that a device can absorb power from a 'capture width' wider than its own physical width, by up to L/π for a device of general shape ($L/2\pi$ for an axisymmetric device). Experiments, including those on the full scale Kvaerner Brug prototype (Malmo & Reitan [27]), confirm this.

For large scale devices the implication is that the resource can be fully exploited by a line with gaps, provided these are short compared with a wavelength; for instance, Duck units of 37 metres on a 45 metre spine extract very nearly as much power as 45 metre units.

For small scale devices, it is interesting to calculate the maximum potential gain for a single unit. If the device is of width W, its potential capture width in waves of period T is $(W + L(T)/\pi)$, where the wavelength $L(T) = gT^2/2\pi$. Thus the potential resource in a mixed sea is

$$P^* = \int (W + L/\pi)\ kT\ s(f)\ df = pW + (cgH^2_{rms}/2\pi^2) \int T^3 s(f)df$$

where $s(f)$ denotes the wave spectral distribution (with allowance for directionality if appropriate), and p is the large scale resource in kW/m. Hence

$$P^* = p(W + W^*),$$

where W^* represents the potential mean additional capture width. W^* can be expressed as $\mu_{-3}L_e/\pi$, where L_e is the wavelength corresponding to the mean wave climate period, i.e. $gT_e^2/2\pi$, and μ_{-3} is a standardised third moment of the spectrum defined by

$$\mu_{-3} = \text{"E}\left[(T/T_e)^3\right]\text{"} = m_{-3}/m_{-1}^3 .$$

For instance, for a Pierson-Moskowitz spectrum $\mu_{-3} = \Gamma(1\tfrac{3}{4})/\Gamma(1\tfrac{1}{4})^3 = 1{\cdot}234$ (note that μ_{-3} is always $\geqslant 1$).

For the average wave spectrum at the deep water South Uist site W^* is approximately 70 metres, and the additional average resource is therefore over 3 MW. Similar levels may be expected at other good oceanic sites.

An example of how the point absorber effect increases the available resource is shown in Figures 8.1 and 8.2. The first shows the performance of a 29 metres wide solo Duck, from an early test at 1:100 scale in unidirectional seas, compared with efficiency curves from narrow tank experiments simulating a close-packed spine based Duck. The horizontal 'stretchy' axis represents the frequency distribution of the large scale resource at the 42 metres depth site off South Uist (with a 1 metre cut-off on H_{rms}). The upper curve shows the theoretical limit on 'efficiency' of $100(1+L/(\pi W))\%$.

In Figure 8.2 the same performance curve is redrawn as a percentage of this upper limit, and the horizontal axis correspondingly re-stretched to represent the frequency distribution of the resource available to the isolated device. Note the 'red shift' in the wave spectrum, i.e. towards long periods, due to the proportionality of the added capture width to T^2. While this design promises a full-scale output of approximately 1 MW, there is clearly still scope for improvement, especially at these longer periods; and this possibility is being actively pursued.

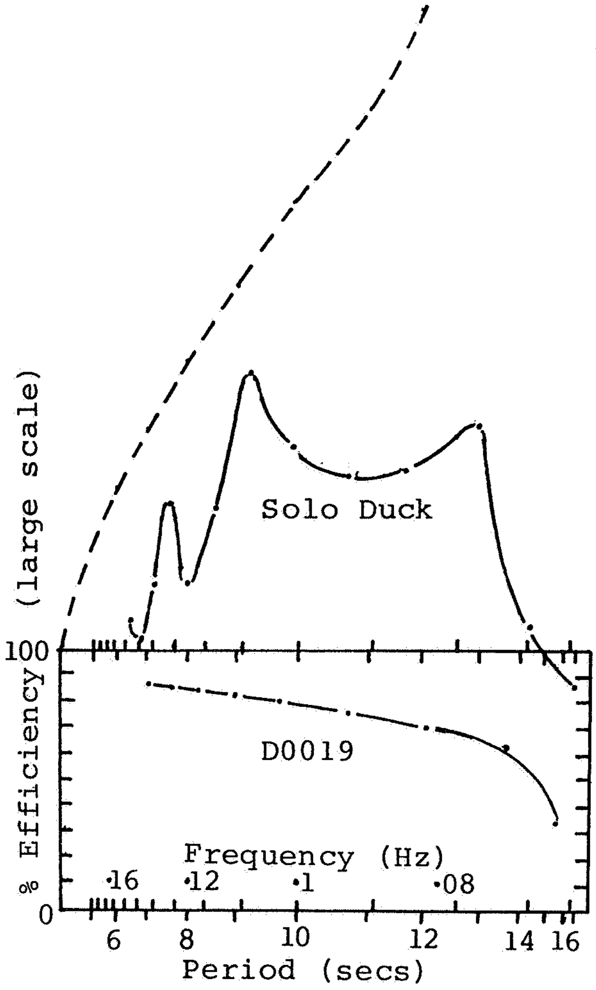

Figure 8.1
Dependence of output on frequency for early solo Duck model @ 1:100 scale, compared with efficiency curve for a spine based Duck (D0019, Dec 1979). The horizontal axis represents the distribution of the large scale resource at the 42 m depth South Uist site (with cut-off at H_{rms} = 1m). The dashed curve shows the theoretical output limit $100(1+L/(\pi W))$% for the solo device.

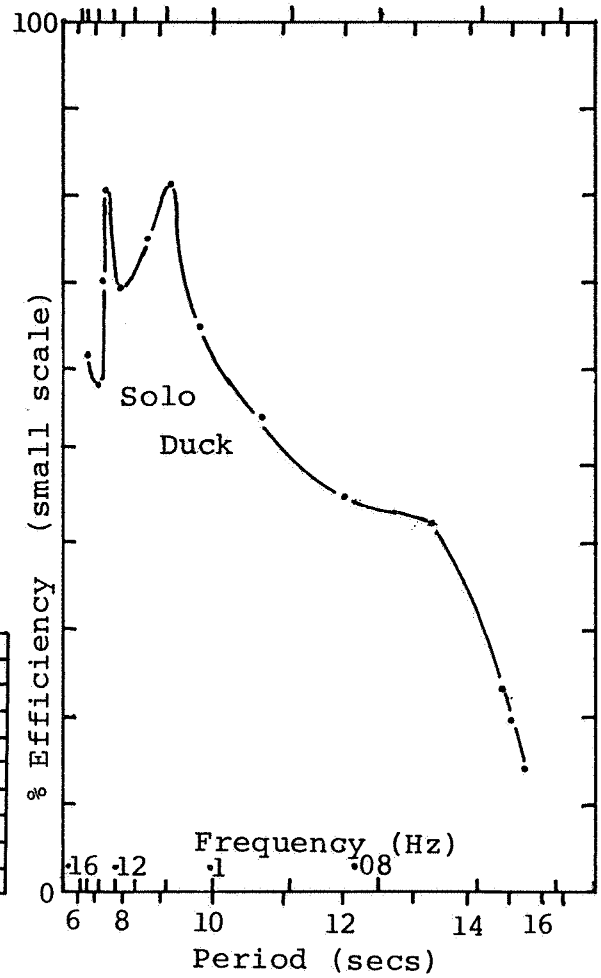

Figure 8.2
Output curve for solo Duck as in Figure 8.1, replotted as a percentage of the theoretical output limit, against a horizontal axis representing the distribution of the small scale resource at the same site. (Thus in both figures the area under the curve is the same, and represents the overall mean efficiency in this wave climate.)

For the present, we can look forward to the exciting demonstration projects of small scale devices in Norway. In an (inshore) wave climate averaging only 15 kW/m, the Kvaerner Brug prototype (Malmo & Reitan [27]) is estimated (based on a year's tests on-site) to produce 180 kW from a front of only 10 metres, i.e. with a final output 'efficiency' averaging over 100% thanks to the point absorber effect. The Tapchan (Mehlum [28]) is on a similar scale, but works by gathering waves over a wide front rather than the point absorber effect; and has the advantage of incorporating storage.

The Norwegians' achievement is considerable, in building a first generation of devices which are already economic or nearly so, in a relatively modest wave climate. This promises well for the exploitation of the best oceanic sites, such as our own Atlantic resource.

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