Why (your) mass isn't gauge dependent

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Nielsen Identities

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Apologies and some justifications

It's been a *VERY* long time since I've done PPT related stuff..

The topic is something that has been picked over for many years

The Higgs (particle) is back on the agenda

Plan of Talk

Symmetry breaking

Higgs mechanism

Quantum corrections, gauge fixing, effective potential

The problem (and the solution)

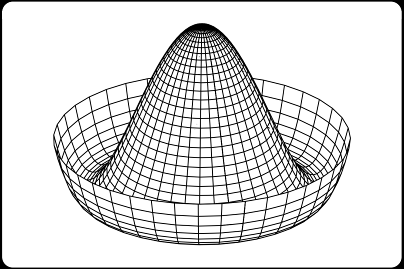
Other models - Standard, gravity...

Form of Potential

Spontaneously broken symmetry

$$V(|\phi|^2) = -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4!}|\phi|^4$$

$$\phi = (\phi_1 + i\phi_2)$$



Goldstone bosons and all that

Complex scalar field theory

$$L(x) = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi^* - V(|\phi|^2)$$

Minimum: $\phi_0 = \left(\frac{6\mu^2}{\lambda}\right)^{1/2}$

Take: $\phi(x) = \phi_0 + \phi_1(x) + i\phi_2(x)$

Lagrangian: $\frac{1}{2}(\partial_\mu\phi_1)^2 - \mu^2\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \ldots$

Abelian Higgs

Scalar electrodynamics - Lagrangian

$$L(x) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi_i D^\mu\phi_i - V(\phi^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ \ , \ \ \phi^2 = \phi_1^2 + \phi_2^2$$

$$D_\mu = \partial_\mu + ieA_\mu$$

Same symmetry breaking as before

$$\frac{1}{2}(\partial_\mu\phi_1)^2 - \mu^2\phi_1^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \frac{1}{2}e^2\phi_0^2 A_\mu A^\mu + e\phi_0 A^\mu\partial_\mu\phi_2 + \cdots$$

Abelian Higgs II

ϕ_2 is surplus to requirements

Gauge invariance to the rescue

$$\begin{aligned} \phi(x) &= \phi(x) \exp[i\theta(x)] \\ A_\mu(x) &= A_\mu(x) + \partial_\mu \theta(x) \end{aligned}$$

Unitary gauge

$$\phi(x) = v + \phi_1(x)$$

ϕ_2 is “gauged away”

Quantum Abelian Higgs

Scalar electrodynamics - Lagrangian in gory detail

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i-e\epsilon_{ij}(\partial_\mu\phi_i)\phi_j A^\mu+\frac{1}{2}e^2A_\mu A^\mu\phi^2+V(\phi^2)$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \quad , \quad \phi^2=\phi_1^2+\phi_2^2$$

$$\epsilon_{ij}=\left(\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right)$$

Gauge fixing/regularization

Add (R_ξ) gauge fixing terms

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i-e\epsilon_{ij}(\partial_\mu\phi_i)\phi_jA^\mu+\frac{1}{2}e^2A_\mu A^\mu\phi^2+V(\phi^2)$$

$$+\frac{1}{2}\xi B^2+B(\partial_\mu A^\mu+e\xi v_i\phi_i)+\partial_\mu\psi^*\partial^\mu\psi-e^2\xi\psi^*\psi\epsilon_{ij}v_i\phi_j$$

Use dimensional regularization/$\overline{\text{MS}}$ subtraction

$v_i=\epsilon_{ij}\phi_{i0}$ is ’t Hooft gauge

The usual setup

Functional integral

$$Z[J] = \int [D\Phi] \exp\left(i \int d^4x \, [L(x) + J\Phi] \right)$$

Connected generating functional

$$W[J] = -i\hbar \ln Z[J]$$

One particle irreducible generating functional

$$\Gamma[\bar{\Phi}] = W[J] - J\bar{\Phi}$$

Feynman diagrams 0

Some useful definitions before we start...

$$v_i = v\, e_i \quad , \quad e_i = (0,1)$$
$$\phi_{i0} = \phi_0\, \eta_i \quad , \quad \eta_i = (1,0)$$

$$m_1^2 = \frac{1}{2}\lambda\phi^2 - \mu^2$$
$$m_2^2 = \frac{1}{6}\lambda\phi^2 - \mu^2$$

$$D_N = k^4 - k^2(m_2^2 - 2e^2\xi v\phi) + e^2\phi^2(e^2\xi^2 v^2 + \xi m_2^2)$$

Feynman diagrams I

Propagators for the theory

Ghost:

$$\frac{i}{k^2 - e^2 \epsilon_{ij} \xi v_i \phi_j}$$

Scalar:

$$\frac{i(k^2 - \xi e^2 \phi^2)}{D_N} (\delta_{ij} - \eta_i \eta_j) + \frac{i \eta_i \eta_j}{k^2 - m_1^2}$$

Gauge:

$$-iC \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - iD \frac{k_\mu k_\nu}{k^2}$$

$$C = \frac{1}{k^2 - e^2 \phi^2}\ ,\ D = \frac{\xi (k^2 - m_2^2 - e^2 \xi v^2)}{D_N}$$

Feynman diagrams II

Propagagators in the 't Hooft gauge

Ghost:

$$\frac{i}{k^2 - e^2 \xi \phi^2}$$

Scalar:

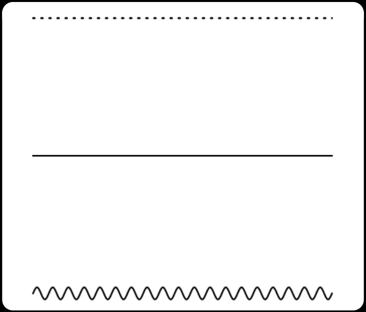
$$\frac{i}{(k^2 - m_2^2 - \xi e^2 \phi^2)} (\delta_{ij} - \eta_i \eta_j) + \frac{i \eta_i \eta_j}{k^2 - m_1^2}$$

Gauge:

$$-i \frac{1}{k^2 - e^2 \phi^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - i \frac{\xi}{(k^2 - e^2 \xi \phi^2)} \frac{k_\mu k_\nu}{k^2}$$

Feynman diagrams III

Graphically:



Feynman diagrams IV

Vertices for the theory

Scalar/Scalar/Gauge:

$$-e\epsilon_{ij}(k_1 + k_2)^\mu$$

Scalar/Ghost/Ghost:

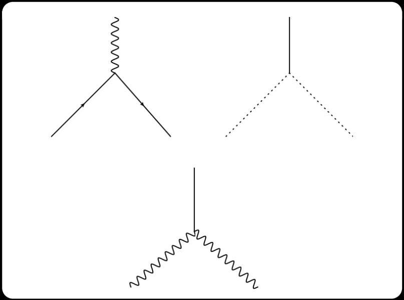
$$-ie^2\xi v_i\epsilon_{ij}$$

Scalar/Gauge/Gauge:

$$2ie^2\phi_i g_{\mu\nu}$$

Feynman diagrams V

Vertices for the theory



Effective Potential I

Quantum equivalent of classical potential

Should use this to discuss corrections to classical symmetry breaking picture

Done perturbatively using loop (i.e. $\hbar$) expansion

$$V = i \sum_n \frac{1}{n!} \Gamma_{i_1 \ldots i_n}(0, \ldots, 0) \phi_{i_1} \cdots \phi_{i_n}$$

Effective Potential II

$$
\begin{aligned}
V^{(1)} &= i \int d^4k \Big[\ln(k^2 + e^2 \xi v \phi) - \frac{3}{2} \ln(-k^2 + e^2 \phi^2) \\
&- \frac{1}{2} \ln(k^2 - m_1^2) - \frac{1}{2} \ln D_N \Big]
\end{aligned}
$$

$$
\xi \frac{\partial V^{(1)}}{\partial \xi} = -\frac{1}{2} i e \xi \phi m_2^2 \int d^4k \frac{(2v + \phi)k^2 - v \xi e^2 \phi^2}{(k^2 + e^2 \xi v \phi) D_N}
$$

Ugly, gauge parameter dependent

And the mass too...

Masses, 2nd derivatives of (effective) potentials

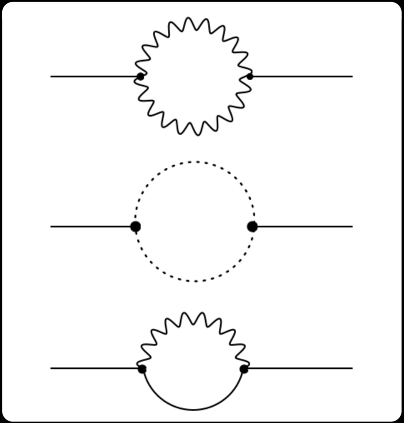
Choose $\lambda \sim O(e^4)$ (Coleman-Weinberg)

$$m^{2(1)} = m_1^2 + \Sigma^{(1)}(0) + m_1^2 \frac{\partial \Sigma^{(1)}(p^2)}{\partial p^2}$$

$$m_1^2 = \frac{1}{2}\lambda\phi^2 - \mu^2$$

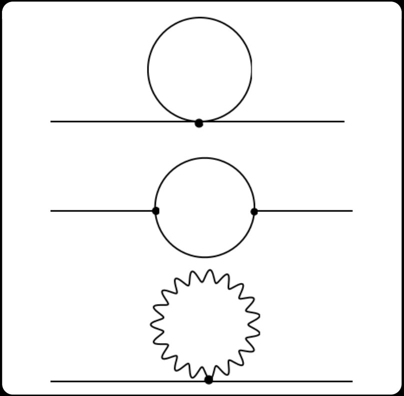
Included Diagrams

Work that can't be avoided



Excluded Diagrams

Work that *can* be avoided



And finally...

Maybe not so ugly, but still gauge parameter dependent:

$$\xi \frac{\partial m^{2(1)}}{\partial \xi} = \frac{e^2 \xi \lambda \phi_0^2}{32\pi^2} \ln \left[\frac{e^2 \xi \phi_0^2}{M^2} \right]$$

One loop correction to effective potential and Higgs mass look like they depend on ξ

What's going on here?

Effective potential and mass shouldn't be gauge parameter dependent

Nielsen (1975) - don't forget about *implicit* dependence

All would be well if

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$
$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0$$

What's going on here?

Why is this a fix?

Minimum of effective potential at some $\bar{\phi}$

$$\frac{\partial V(\phi,\xi)}{\partial\phi} = 0$$

V_{min} stays the same under $\xi \to \xi + d\xi$

$$V(\bar{\phi} + \delta\bar{\phi}, \xi + d\xi) = V(\bar{\phi}, \xi) = V_{min}$$

$$\frac{\partial V}{\partial\xi} + \frac{d\bar{\phi}}{d\xi}\frac{\partial V}{\partial\bar{\phi}} = 0$$

Nielsen Identities

The man himself N. K. Nielsen - not H. B. Nielsen

Proved requisite identities exist using BRST symmetry

(Standard) BRST symmetry I

"Modern" way of dealing with gauge invariance

Valid for the gauge-fixed Lagrangian

$$\delta^2 = 0$$

$$\delta\psi^* = \epsilon B \ \ , \ \ \delta\psi = 0 \ \ , \ \ \delta B = 0 \ \ , \ \ \delta A_\mu = \epsilon\partial_\mu\psi$$

$$\delta\phi_i = \epsilon e \epsilon_{ij}\psi\phi_j$$

(Standard) BRST symmetry II

The BRST identities are encapsulated in:

$$S(\Gamma) = \int d^4x \left(\frac{\delta \Gamma}{\delta \rho^\mu} \frac{\delta \Gamma}{\delta A_\mu} + \frac{\delta \Gamma}{\delta K_i} \frac{\delta \Gamma}{\delta \phi_i} + \frac{\delta \Gamma}{\delta \sigma} \frac{\delta \Gamma}{\delta \psi} + B \frac{\delta \Gamma}{\delta \psi^*} \right) = 0$$

ρ^μ, K_i, σ sources for δA_μ, $\delta \phi_i$ and $\delta \psi$

Nothing that looks like $\frac{\partial \Gamma}{\partial \xi} \to \frac{\partial V}{\partial \xi}$

Trick, Piguet and Sibold

Extended BRST symmetry

Introduce a BRST transform on the gauge *parameter*

$$\delta\xi = \epsilon\chi$$

This requires extra terms in the Lagrangian too

$$+\frac{1}{2}\chi\psi^* B + e\chi\psi^* v_i \phi_i$$

The BRST identity now reads

$$S(\Gamma) + \chi\frac{\partial\Gamma}{\partial\xi} = 0$$

Extended BRST symmetry II

Differentiate w.r.t. χ, substitute for B, set everything but ϕ to zero

$$\int d^4x \left(\frac{\delta\Gamma(O(x))}{\delta K_i} \frac{\delta\Gamma}{\delta\phi_i} - \frac{e\xi v_i \phi_i}{\xi} \frac{\delta\Gamma(O(x))}{\delta\psi^*} \right) - \frac{\partial\Gamma}{\partial\xi} = 0$$

Take constant field limit

$$\xi \frac{\partial V}{\partial\xi} - \int d^4x \frac{\delta\Gamma(O(x))}{\delta K_i} \frac{\partial V}{\partial\phi} = 0$$

Explicit expression for $C(\phi, \xi)$

$$-\int d^4x \frac{\delta\Gamma(O(x))}{\delta K_i}$$

Some observations

We have seen that

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$

is true

Also true for other physical quantities *with the same* $C(\phi, \xi)$

Applies in non-Abelian theories (e.g. standard model)

Also in gravity

Matters of gravity...

Suppose you are made of scalar, bosonic matter

And you are happy to be described by 1-loop quantum gravity

$$
\begin{aligned}
L(x) &= -\frac{2}{\kappa^2}\sqrt{-g}(R+\Lambda) + L_{gf} \\
&+ \sqrt{-g}[(\nabla_\mu \phi)(\nabla_\nu \phi) g^{\mu\nu} - \frac{1}{2} m^2 \phi^2]
\end{aligned}
$$

Then there is a Nielsen identity

$$
\xi \frac{\partial m^2}{\partial \xi} + \eta^{\mu\nu} \tau(\phi, \xi) \frac{\partial m^2}{\partial h_{\mu\nu}} = 0
$$

Interpreted as

$$
\begin{aligned}
\xi &\rightarrow \xi + d\xi \\
\eta^{\mu\nu} &\rightarrow (1 + \tau d\xi) \eta^{\mu\nu}
\end{aligned}
$$

Matters of higher derivative gravity I

A renormalizable, asymptotically free theory of gravity exists (Stelle 1977)

Unfortunately, it is also non-unitary

$$L(x) \;=\; \sqrt{-g}\left[\frac{\gamma}{\kappa^2}R + \frac{\Lambda}{\kappa^4} + \beta R^2 - \frac{1}{\alpha^2}(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2)\right]$$

Spin two propagator

$$D(q^2) = \frac{2}{\alpha^2\gamma}\left[\frac{1}{q^2} - \frac{1}{q^2 - (\alpha^2\gamma/\kappa^2)}\right]$$

Matters of higher derivative gravity II

Massive ghost decays

General recipe

$$D(q^2) = (D_0(q^2)^{-1} - \Sigma(q^2))^{-1}$$

Large N

$$D(q^2) = \frac{2}{\alpha^2\gamma}\left[\frac{1}{q^2[1 - ANq^2\ln(q^2/L^2)]}\right]$$

Poles at $q^2 = 0$, $q = r\exp(i\theta)$ with $r \sim \gamma/N\kappa^2$

Loop corrections $\partial M^2/\partial\xi \neq 0$

Summary

At first sight it looks as if physical quantities depend on gauge parameters in a loop expansion

They don't really, you are saved by:

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0$$
$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0$$

$C(\phi, \xi)$ explicitly calculable from (extended) BRST identities

A reference or ten

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THE END :)