Planar Order in a 3D Plaquette Ising model

Marco Mueller, Wolfhard Janke, Des Johnston
MECO40, Esztergom, March 2015

Plan of talk

A 3D plaquette Ising model with a highly degenerate low-T phase

Consequences for order parameter (main focus here)

Consequences for FSS at first order transition (in passing)

A 3D Plaquette Ising action

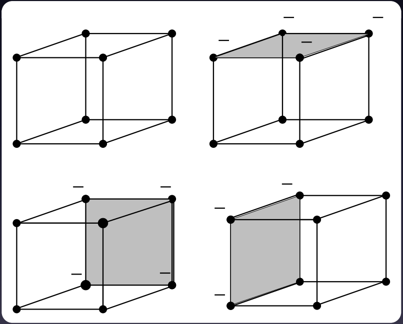
$3D$ cubic lattice, spins on *vertices*

$$H = -\sum_{\Box} \sigma_i \sigma_j \sigma_k \sigma_l$$

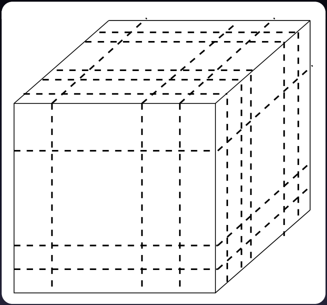
Not edges

$$H = -\sum_{\Box} U_{ij} U_{jk} U_{kl} U_{li}$$

Plaquette Hamiltonian Groundstates: Single cube



Ground states, Low T: Lattice



Persists into low temperature phase (Wegner, Pietig)

Degeneracy 2^{3L}

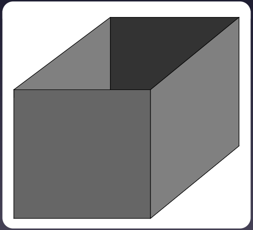
Anisotropic ("Fuki-Nuke") Model

Consider anisotropic variant

Suzuki 1973, Jonsson/Savvidy 2000, Castelnovo *et.al.* 2010

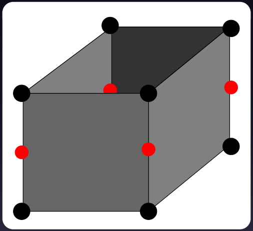
$$H_{\text{aniso}} \;=\; -J_x \sum_{\square yz} \sigma_i \sigma_j \sigma_k \sigma_l - J_y \sum_{\square xz} \sigma_i \sigma_j \sigma_k \sigma_l$$

Set $J_z = 0, J_x = J_y = 1$. No horizontal plaquettes



Anisotropic Model

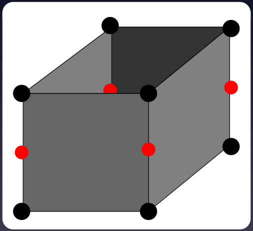
Define new spins τ from pairs of σ's



$$\tau_{x,y,z} = \sigma_{x,y,z}\sigma_{x,y,z+1}$$

Anisotropic Model = Stack of $2D$ Ising

$$H_{\text{fuki-nuke}} = - \sum_{x=1}^{L} \sum_{y=1}^{L} \sum_{z=1}^{L} \left(\tau_{x,y,z} \tau_{x+1,y,z} + \tau_{x,y,z} \tau_{x,y+1,z} \right) \ ,$$



Anisotropic Model Order Parameter

Single layer

$$m_{2d,z} = \left\langle \frac{1}{L^2} \sum_{x=1}^{L} \sum_{y=1}^{L} \tau_{x,y,z} \right\rangle$$

In terms of original variables

$$m_{2d,z} = \left\langle \frac{1}{L^2} \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right\rangle$$

Isotropic case

Hypothesis: Same order parameter works

Hashizume and Suzuki (2011)

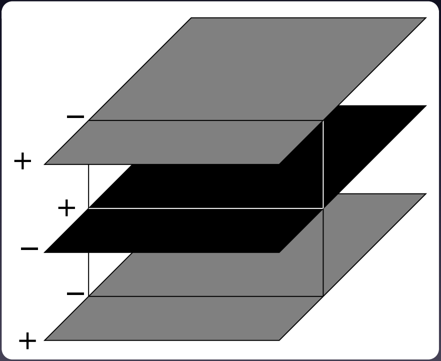
Take layers, add 'em up

$$m^z_{\mathrm{abs}} = \left\langle \frac{1}{L^3} \sum_{z=1}^{L} \left| \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right| \right\rangle$$

$$m^z_{\mathrm{sq}} = \left\langle \frac{1}{L^5} \sum_{z=1}^{L} \left(\sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right)^2 \right\rangle$$

Isotropic case: Effect of flips

$$m^z_{\mathrm{abs}} = \left\langle \frac{1}{L^3} \sum_{z=1}^{L} \left| \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z}\sigma_{x,y,z+1} \right| \right\rangle$$



Isotropic case: numerical investigation

Strong first order PT

Multicanonical simulation

Correct order parameter: predicts PT point? isotropic?

FSS properties?

FSS for 1st order PTs

Pirogov-Sinai Theory (Borgs/Kotecký)

$$Z(\beta) = \left[e^{-\beta L^d f_d} + q e^{-\beta L^d f_o}\right] [1 + \ldots]$$

Fixed boundaries ($1/L$ leading term)

$$Z(\beta) = \left[e^{-\beta(L^d f_d + L^{d-1} \hat{f}_o)} + q e^{-\beta(L^d f_o + L^{d-1} \hat{f}_d)}\right]$$

Exponential degeneracy ($1/L^2$ in $d = 3$)

$$Z(\beta) = \left[e^{-\beta L^d f_d} + 2^{3L} e^{-\beta L^d f_o}\right]$$

FSS for 1st order PTs: degenerate case

With $q \propto 2^{3L} = e^{(3\ln 2)L}$

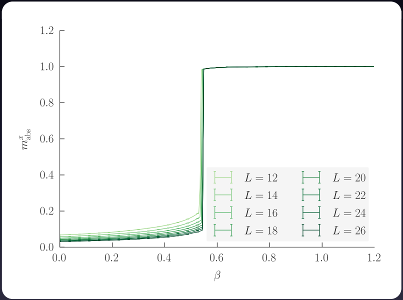
$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{\ln q}{L^d \Delta\hat{e}} + \dots$$

become

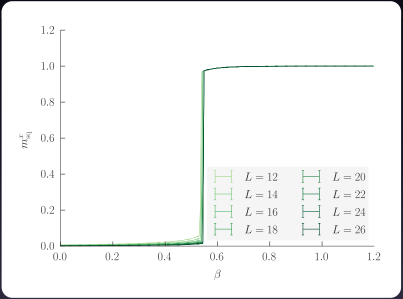
$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{3\ln 2}{L^{d-1} \Delta\hat{e}} + \dots$$

Numerical results: order parameters

Order parameter m^x_{abs}



Order parameter m^x_{sq}

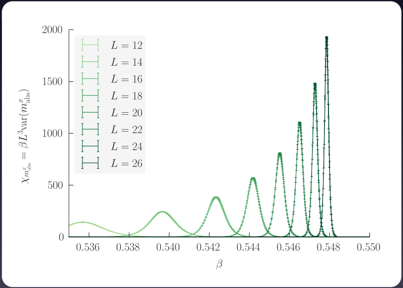


m^y and m^z identical to m^x - isotropy restored

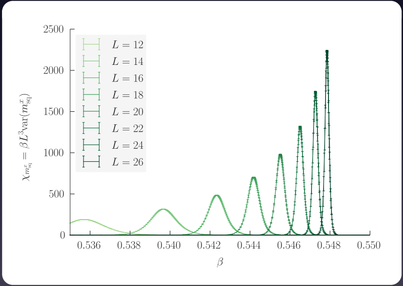
Numerical results: susceptibilities

$$\chi(\beta) = \beta L^3 \left(\langle m^2\rangle(\beta) - \langle m\rangle(\beta)^2\right)$$

Susceptibility for m^x_{abs}



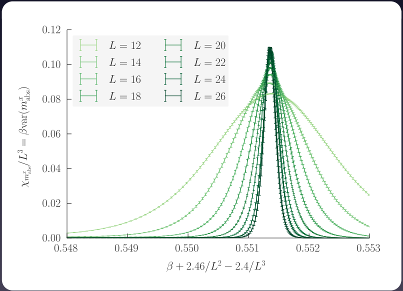
Susceptibility for m^x_{sq}



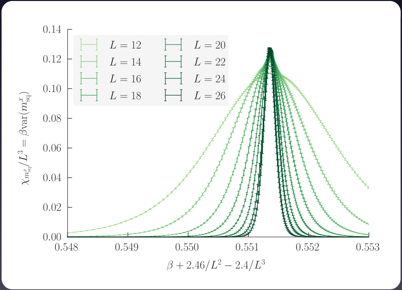
Numerical results: scaled susceptibilities

$$\beta^{\chi_{m^x_{\mathrm{abs}}}}(L) = 0.551\,37(3) - 2.46(3)/L^2 + 2.4(3)/L^3$$

Susceptibility for m^x_{abs}



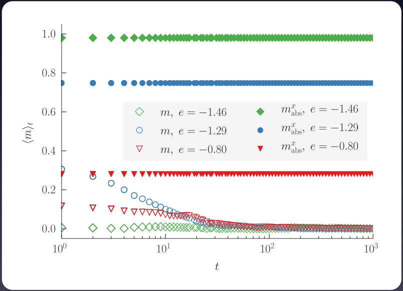
Susceptibility for m^x_{sq}



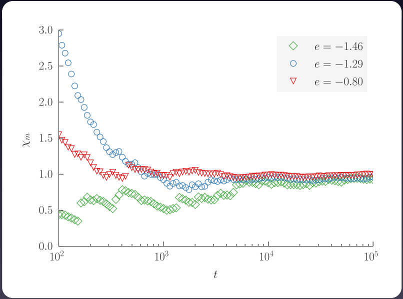
Flipping

Running averages, magnetization and *standard* susceptibility, various starting configs (green ordered, blue intermediate, red disordered):

Magnetization



Susceptibility



Conclusions

Suzuki was right - isotropic model displays Fuki-Nuke order

....which is a curious "$2.5d$" order

Scaling is non-standard (c/o degeneracy)

Results agree well with energetic observables (Spec heat, Binder etc)

References

Y. Hashizume and M. Suzuki, Int. J. Mod. Phys. B **25** (2011) 73; Int. J. Mod. Phys. B **25** (2011) 3529.

M. Mueller, W. Janke and D. A. Johnston, Phys. Rev. Lett. **112** (2014) 200601.

M. Mueller, D. A. Johnston and W. Janke, Nucl. Phys. B **888** (2014) 214.

M. Mueller, W. Janke and D. A. Johnston, Nucl. Phys. B **894** (2015) 1.