Macroscopic Degeneracy and Order in a 3D Plaquette Ising model

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Plan of talk

A 3D plaquette Ising model with a highly degenerate low-T phase

Consequences for FSS at first order transition

Consequences for order parameter

A 3D Plaquette Ising action

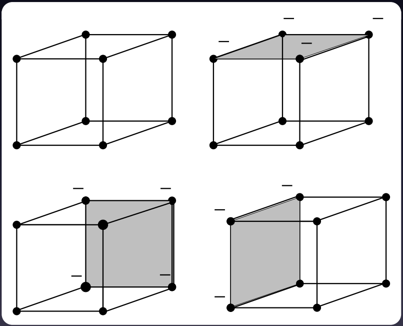
$3D$ cubic lattice, spins on *vertices*

$$H = -\sum_{\Box} \sigma_i \sigma_j \sigma_k \sigma_l$$

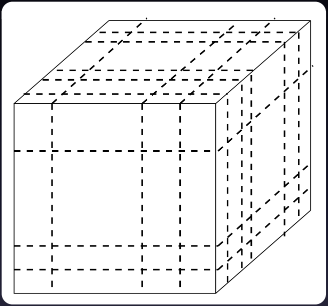
Not edges

$$H = -\sum_{\Box} U_{ij} U_{jk} U_{kl} U_{li}$$

Plaquette Hamiltonian Groundstates: Single cube



Ground states, Low T: Lattice



Persists into low temperature phase (Wegner, Pietig)

Degeneracy 2^{3L}

Consequences: FSS

First order FSS: Boundary Conditions

Pirogov-Sinai Theory (Borgs/Kotecký)

$$Z(\beta) = \left[e^{-\beta L^d f_d} + q e^{-\beta L^d f_o} \right] [1 + \ldots]$$

Fixed boundaries ($1/L$ leading term)

$$Z(\beta) = \left[e^{-\beta (L^d f_d + L^{d-1} \hat{f}_o)} + q e^{-\beta (L^d f_o + L^{d-1} \hat{f}_d)} \right]$$

First order FSS: Exponential Degeneracy

Exponential degeneracy

$$Z(\beta) = \left[e^{-\beta L^d f_d} + 2^{3L} e^{-\beta L^d f_o} \right]$$

$1/L^2$ in $d = 3$

$$Z(\beta) = \left[e^{-\beta L^d f_d} + e^{(3 \ln 2) L} e^{-\beta L^d f_o} \right]$$

FSS: Specific Heat

Probability of being in any of the states

$$p_{\mathrm{o}} \propto e^{-\beta L^d \hat{f}_{\mathrm{o}}} \text{ and } p_{\mathrm{d}} \propto e^{-\beta L^d \hat{f}_{\mathrm{d}}}$$

Time spent in the ordered states $\propto q p_{\mathrm{o}}$

$$W_{\mathrm{o}}/W_{\mathrm{d}} \simeq q e^{-L^d \beta \hat{f}_{\mathrm{o}}} / e^{-\beta L^d \hat{f}_{\mathrm{d}}}$$

Expand around β^{∞}

$$0 = \ln q + L^d \Delta \hat{e} (\beta - \beta^{\infty}) + \dots$$

Solve for specific heat peak

$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{\ln q}{L^d \Delta \hat{e}} + \dots$$

FSS: Specific Heat, degenerate case

With $q \propto 2^{3L} = e^{(3\ln 2)L}$

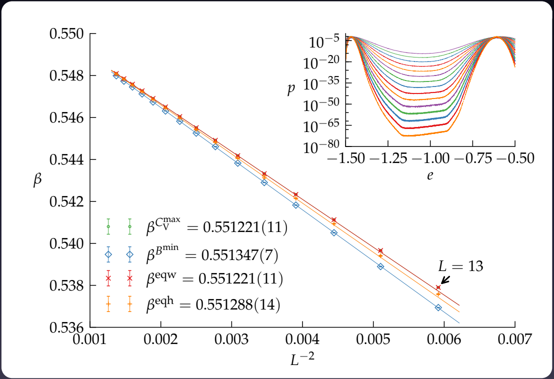
$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{\ln q}{L^d \Delta\hat{e}} + \ldots$$

become

$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{3\ln 2}{L^{d-1} \Delta\hat{e}} + \ldots$$

FSS

Plaquette Hamiltonian fits

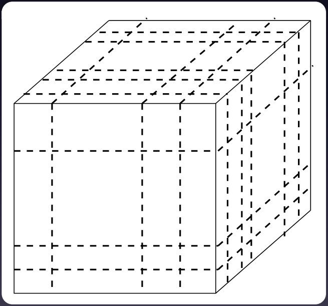


Consequences: Order parameter

Order Parameter

Magnetization won't do

Not a gauge theory



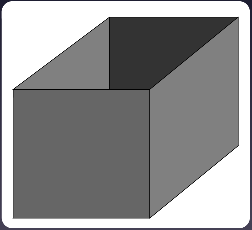
Anisotropic ("Fuki-Nuke") Model

Consider anisotropic variant

Suzuki 1973, Jonsson/Savvidy 2000, Castelnovo *et.al.* 2010

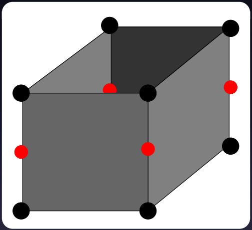
$$H_{\text{aniso}} = -J_x \sum_{\square yz} \sigma_i \sigma_j \sigma_k \sigma_l - J_y \sum_{\square xz} \sigma_i \sigma_j \sigma_k \sigma_l$$

Set $J_z = 0, J_x = J_y = 1$. No horizontal plaquettes



Anisotropic Model

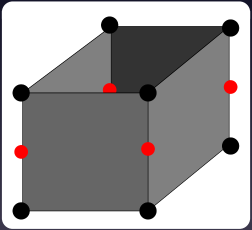
Define new spins τ from pairs of σ's



$$\tau_{x,y,z} = \sigma_{x,y,z}\sigma_{x,y,z+1}$$

Anisotropic Model = Stack of $2D$ Ising

$$H_{\mathrm{fuki-nuke}} = -\sum_{x=1}^{L}\sum_{y=1}^{L}\sum_{z=1}^{L}\left(\tau_{x,y,z}\tau_{x+1,y,z} + \tau_{x,y,z}\tau_{x,y+1,z}\right) \ ,$$



Anisotropic Model Order Parameter

Single layer

$$m_{2d,z} = \left\langle \frac{1}{L^2} \sum_{x=1}^{L} \sum_{y=1}^{L} \tau_{x,y,z} \right\rangle$$

In terms of original variables

$$m_{2d,z} = \left\langle \frac{1}{L^2} \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right\rangle$$

Add 'em up

$$m_{\text{abs}} = \left\langle \frac{1}{L^3} \sum_{z=1}^{L} \left| \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right| \right\rangle ,$$

Isotropic case

Hypothesis: Same order parameter works

Hashizume and Suzuki (2011)

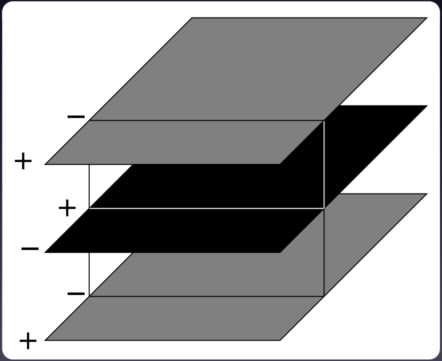
Take layers, add ’em up

$$m^z_{\mathrm{abs}} = \left\langle \frac{1}{L^3} \sum_{z=1}^{L} \left| \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right| \right\rangle$$

$$m^z_{\mathrm{sq}} = \left\langle \frac{1}{L^5} \sum_{z=1}^{L} \left(\sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z} \sigma_{x,y,z+1} \right)^2 \right\rangle$$

Isotropic case: Effect of flips

$$m^z_{\mathrm{abs}} = \left\langle \frac{1}{L^3} \sum_{z=1}^{L} \left| \sum_{x=1}^{L} \sum_{y=1}^{L} \sigma_{x,y,z}\sigma_{x,y,z+1} \right| \right\rangle$$



Isotropic case: numerical investigation

Strong first order PT

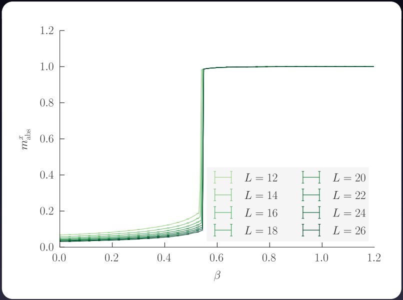
Multicanonical simulation

Correct order parameter: predicts PT point? isotropic?

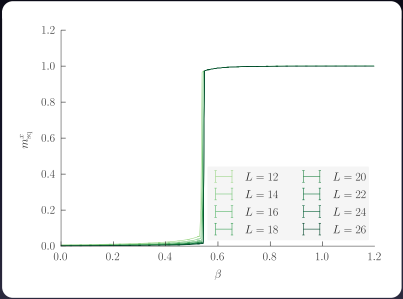
FSS properties?

Numerical results: order parameters

Order parameter m^x_{abs}



Order parameter m^x_{sq}



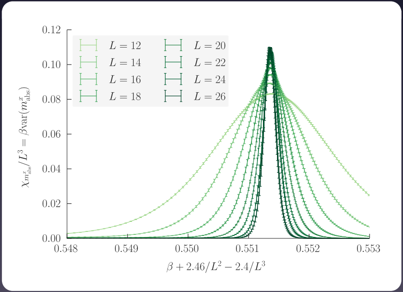
m^y and m^z identical to m^x - isotropy restored

Numerical results: scaled susceptibilities

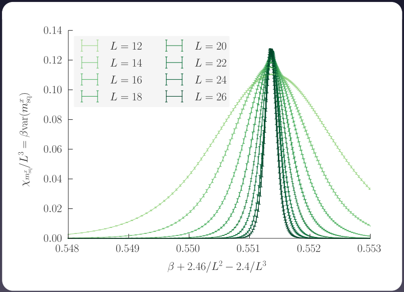
$$\chi(\beta) = \beta L^3 \left(\langle m^2 \rangle(\beta) - \langle m \rangle(\beta)^2 \right)$$

$$\beta^{\chi_{m^x_{\text{abs}}}}(L) = 0.551\,37(3) - 2.46(3)/L^2 + 2.4(3)/L^3$$

Susceptibility for m^x_{abs}



Susceptibility for m^x_{sq}



Conclusions

Exponential degeneracy gives $1/L^2$ corrections in 3D

Suzuki was right - isotropic model displays Fuki-Nuke order

OP Scaling agrees well with energetic observables

Perspectives

Cases where *ground state* is exponentially degenerate but low-T phase is not: AFM Ising on FCC

Scaling at first order Quantum PTs

References

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