COLEMAN–WEINBERG, NIELSEN AND DAISIES

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In a recent paper, Nielsen suggested a method for obtaining higher-order corrections in the Coleman–Weinberg model We show a simple explicit calculation that this is consistent with his identities and corresponds, as he noted, to a "daisy" expansion

In a recent paper [1], Nielsen considered the application of his identities to models in which there is a mixing of orders in the loop expansion, either due to high temperature effects [2] or the fine-tuning of parameters [3,4] In brief, the identities which control the gauge-dependence of the effective potential in gauge theories (and quantities derived from it) are of the form shwon below [5,6],

$$\xi\, \partial V/\partial\xi + C(\Phi, \xi)\, \partial V/\partial\Phi = 0\,, \tag{1}$$

where V is the effective potential, ξ is a gauge parameter, Φ is the semiclassical field and $C(\Phi, \xi)$ is a field-theoretic expression which may be calculated in some expansion scheme The identity states that under a change in the gauge parameter $\xi\to\xi+\delta\xi$ the semiclassical field undergoes a compensating change $\Phi\to\Phi+C(\Phi,\xi)\delta\xi/\xi$ and that the values of physical quantities are preserved We can rewrite (1) as

$$\mathrm{d}V/\mathrm{d}\xi = 0\,, \tag{2}$$

where $\mathrm{d}/\mathrm{d}\xi$ denotes the total variation with respect to ξ, both explicit and implicit (via Φ, which is calculated using a gauge-fixed lagrangian)

The identities are a direct consequence of the BRS invariance of the theory [7,8] and, barring pathologies in our choice of gauge-fixing, we would expect them to hold in general [6,9] However, as Nielsen pointed out, we are constrained by our ignorance to work in some approximation scheme In many cases in field theory a loop expansion (which is equivalent to an expansion in $\hbar$) is feasible and in such cases we can expand (1) order by order in $\hbar$ We note that because C is derived from the effective action with an operator insertion it receives its first contribution at one-loop order We thus find, for the two lowest orders

$$\xi\, \partial V^{(0)}/\partial\xi = 0\,, \tag{3}$$

and

$$\xi\, \partial V^{(1)}/\partial\xi + C^{(1)}(\Phi, \xi)\, \partial V^{(0)}/\partial\Phi = 0\,, \tag{4}$$

where the superscripts denote the order in $\hbar$ In a standard loop expansion (3) will be trivially satisfied as $V^{(0)}$ is just the classical potential which will be independent of the gauge-fixing In general, both terms in (4) will contribute [6] but in some cases, such as a gravitational theory where the background satisfies the classical equations of motion, the second term is zero and we find a one-loop effective potential which is gauge-parameter independent [10,11]

However, we can run into problems with models which mix orders in the loop expansion In his paper [1], Nielsen contrasted the Coleman–Weinberg model (massless scalar QED) [3], where it is possible to find a gauge-independent approximation scheme, and self-consistent dimensional reduction in Kaluza–Klein gravity [4], where it is not In this paper, we show by explicit calculation that the Coleman–Weinberg model in a Pauli–Feynman type gauge

$$\mathscr{L}_{\text{gauge-fixing}} = B(\partial_\mu A^\mu) + \tfrac{1}{2}\xi B^2 \tag{5}$$

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(where B is an auxiliary field which may be integrated out to give the usual gauge-fixing) does satisfy the Nielsen identities in the "daisy" resummation scheme that Nielsen suggested We also observe that this is not the case in the 't Hooft/R_ξ gauge because the lowest-order equivalent of (3) is, in fact, gauge-dependent

We take the lagrangian of the theory to be

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\mathrm{D}_\mu \Phi_i)(\mathrm{D}^\mu \Phi_i) - V_{\mathrm{cl}}(\Phi^2)$$

$$+B(\partial_\mu A^\mu) + \tfrac{1}{2}\xi B^2 - C^* \Box C , \tag{6}$$

where $\mathrm{D}_\mu \Phi_i = \partial_\mu \Phi_i + e\epsilon_{ij} A_\mu \Phi_j$, i runs from 1 to 2 and V_{cl} is the classical potential The ghosts, although free, are retained to facilitate the derivation of the identity If we take the spontaneous symmetry breaking to be in the 1-direction a standard calculation [6] shows that the one-loop effective potential is given by

$$V^{(1)} = -\frac{1}{2}\int \mathrm{d}^4 k\, \{3\ln(-k^2 + e^2\bar{\Phi}^2)$$

$$+\ln[(k^2)^2 - 2(k^2 - \xi e^2 \bar{\Phi}^2)\, \partial V_{\mathrm{cl}}/\partial \bar{\Phi}^2]$$

$$+\ln(k^2 - 2\partial V_{\mathrm{cl}}/\partial\bar{\Phi}^2 - 4\bar{\Phi}^2\, \partial^2 V_{\mathrm{cl}}/\partial^2 \bar{\Phi}^2)\} \tag{7}$$

We can perform the integral above, using dimensional regularization and minimal subtraction, to find

$$V^{(1)} = (1/64\pi^2)\{3e^4\bar{\Phi}^4[\ln(e^2\bar{\Phi}^2/M^2) - \tfrac{3}{2}]$$

$$+ m_1^4[\ln(m_1^2/M^2) - \tfrac{3}{2}] + k_1^4[\ln(k_1^2/M^2) - \tfrac{3}{2}]$$

$$+ k_2^4[\ln(k_2^2/M^2) - \tfrac{3}{2}]\} , \tag{8}$$

where k_1^2 and k_2^2 are the roots of the quadratic expression in k^2 in the second term in (7) and m_1^2 is the V_{cl} part in the third term If $V_{\mathrm{cl}} = (\lambda/4!)\Phi^4$, we have explicitly

$$m_1^2 = \tfrac{1}{2}\lambda\bar{\Phi}^2 , \quad m_2^2 = \tfrac{1}{6}\lambda\bar{\Phi}^2 , \tag{9}$$

and

$$k_1^2 = \tfrac{1}{2}m_2^2 + \tfrac{1}{2}m_2(m_2^2 - 4\xi e^2\bar{\Phi}^2)^{1/2} ,$$

$$k_2^2 = \tfrac{1}{2}m_2^2 - \tfrac{1}{2}m_2(m_2^2 - 4\xi e^2\bar{\Phi}^2)^{1/2} \tag{10}$$

The Coleman–Weinberg scheme is obtained when we choose $\lambda \sim \mathrm{O}(e^4)$, so only the first term in (8), arising from transverse gauge bosons, contributes and the effective potential is given by

$$V' = (1/64\pi^2)\{3e^4\bar{\Phi}^4[\ln(e^2\bar{\Phi}^2/M^2) - \tfrac{3}{2}]\}$$

$$+(\lambda/4!)\,\bar{\Phi}^4 , \tag{11}$$

where we have used a prime to denote lowest order in λ Fortuitously this is gauge-invariant so we have the equivalent of (3),

$$\xi\, \partial V'/\partial\xi = 0 \tag{12}$$

Had this not been so we would have been stuck, because there is no O(1) contribution to C to balance out the gauge-parameter dependence The next approximation, according to Nielsen, is to substitute V' for V_{cl} in (7) To see that this does, indeed, correspond to an infinite resummation of daisy diagrams, we consider the expressions for the resummed gauge-boson/scalar propagator and for the physical scalar propagator As a lemma, we note that only the transverse part of the bosonic loop contributes to the daisies because the longitudinal part of the loop is given by an integral of the form

$$e^2\int \mathrm{d}^4 k\, \frac{(k^2 - m_2^2)\xi}{(k^2)^2 - 2(k^2 - e^2\xi\bar{\Phi}^2)m_2^2} , \tag{13}$$

which is of $\mathrm{O}(e^2\lambda)$, whereas the transverse part is given by

$$e^2 I_{\mathrm{loop}} = 3e^2\int \mathrm{d}^4 k\, \frac{1}{k^2 - e^2\bar{\Phi}^2} , \tag{14}$$

which is of $\mathrm{O}(e^4)$ and hence of lower order if $\lambda \sim \mathrm{O}(e^4)$

Taking the physical scalar ($\Phi_1\Phi_1$) propagator first, we see that it will be given by a sum of diagrams of the form shown in fig 1 This gives

$$D_{11} = \sum_{n=0}^{\infty} \frac{1}{(k^2 - m_1^2)} \left(\frac{\mathrm{i}e^2 I_{\mathrm{loop}}}{k^2 - m_1^2}\right)^n , \tag{15}$$

or

$$D_{11} = \frac{1}{(k^2 - m_1^2 - \mathrm{i}e^2 I_{\mathrm{loop}})} \tag{16}$$

If we now write $V' = V_{\mathrm{cl}} + V_{\mathrm{loop}}$, we see that

$$e^2 I_{\mathrm{loop}} = -2\mathrm{i}\, \partial V_{\mathrm{loop}}/\partial\bar{\Phi}^2 , \tag{17}$$

so

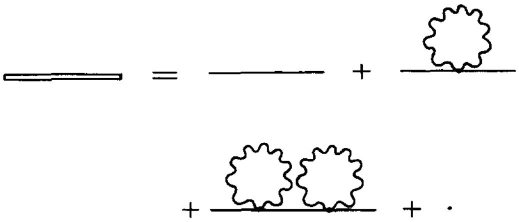

Fig 1 The Φ_1/Φ_1 propagator in the resummed theory is given by a series of the form shown The single solid line represents the standard Φ_1/Φ_1 propagator and the wavy line the transverse part of the gauge-boson propagator

$$D_{11}=\frac{1}{k^2-2\,\partial V'/\partial\bar{\Phi}^2-4\bar{\Phi}^2\,\partial^2 V_{\rm cl}/\partial^2\bar{\Phi}^2} \tag{18}$$

We see that we have replaced only the first derivative term with V', which is to be expected, as the second derivative term contains an extra power of e^2 for $V_{\rm loop}$ The contribution to the effective potential of (18) is

$$V''_{\Phi_1\Phi_1}=-\frac{1}{2}\int \mathrm{d}^4k\,[\ln(k^2-2\,\partial V'/\partial\bar{\Phi}^2 -4\bar{\Phi}^2\,\partial^2 V_{\rm cl}/\partial^2\bar{\Phi}^2)]\,, \tag{19}$$

where we have denoted our resummed approximation by V''

If we now consider the resummed Φ_2A_μ propagator we see that it may be written as a sum of diagrams of the form shown in fig 2 This gives

$$D_{2\mu}=\sum_{n=0}^{\infty}\frac{-ie\xi k_\mu\Phi}{(k^2)^2-2(k^2-\xi e^2\bar{\Phi}^2)\,\partial V_{\rm cl}/\partial\bar{\Phi}^2} \times\frac{ie^2 I_{\rm loop}(k^2-\xi e^2\bar{\Phi}^2)}{(k^2)^2-2(k^2-\xi e^2\bar{\Phi}^2)\,\partial V_{\rm cl}/\partial\bar{\Phi}^2}\,, \tag{20}$$

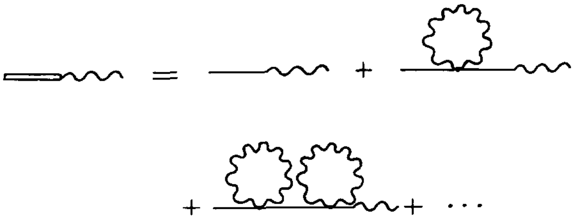

Fig 2 The Φ_2/A_μ propagator in the resummed theory is given by a similar series to fig 1 In this the solid line represents the $\Phi_2\Phi_2$ propagator and the mixed Φ_2/A_μ propagator is given by a straight line adjoined to a wavy line

where the first part comes from the mixed propagator at the end and the second from the loops and $\Phi_2\Phi_2$ propagators We can rewrite this, using (17) again, as

$$D_{2\mu}=\frac{-ie\xi k_\mu\Phi}{(k^2)^2-2(k^2-\xi e^2\bar{\Phi}^2)\,\partial V'/\partial\bar{\Phi}^2}\,, \tag{21}$$

which gives the following contribution to the effective potential

$$V''_{\Phi_2A_\mu}=\frac{-1}{2}\int \mathrm{d}^4k\,\{\ln[(k^2)^2 -(k^2-\xi e^2\Phi^2)2\,\partial V'/\partial\bar{\Phi}^2]\} \tag{22}$$

The transverse gauge-boson propagator receives no corrections and its contribution to the effective potential stays unchanged,

$$V''_{A_\mu A_\nu}=-\frac{3i}{2}\int \mathrm{d}^4k\,\ln(-k^2+e^2\bar{\Phi}^2) \tag{23}$$

We now write $V''=V''_{\Phi_1\Phi_1}+V''_{\Phi_2A_\mu}+V''_{A_\mu A_\nu}$ To verify the Nielsen identity in this approximation scheme note that

$$\xi\frac{\partial V''}{\partial\xi}=\frac{-ie^2\xi\bar{\Phi}}{2} \times\int \mathrm{d}^4k\,\frac{\partial V'/\partial\bar{\Phi}^2}{(k^2)^2-2(k^2-\xi e^2\bar{\Phi}^2)\,\partial V'/\partial\bar{\Phi}^2}\,, \tag{24}$$

which will be of the required form if we can identify the integral expression with $C(\bar{\Phi},\xi)$ In the Pauli–Feynman gauge, C is given by [6]

$$\int \mathrm{d}^4x\, i\hbar\langle 0|\mathrm{T}(i/\hbar)^2 \times[-\tfrac{1}{2}C^*(x)\partial_\mu A^\mu(x)\,eC(0)\Phi_2(0)]\,|0\rangle \tag{25}$$

with a lowest-order contribution of the form shown in fig 3 where we use the *resummed* Φ_2/gauge-boson propagator This gives, in momentum space,

$$C=\frac{ie}{2}\int \mathrm{d}^4k\,\frac{1}{k^2} \times\frac{-ik^2e\xi\bar{\Phi}}{(k^2)^2-2(k^2-\xi e^2\bar{\Phi}^2)\,\partial V'/\partial\bar{\Phi}^2}\,, \tag{26}$$

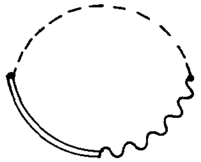

Fig 3 The lowest-order contribution to C in the resummed scheme is as shown The dotted line represents the ghost propagator and we have used the resummed Φ_2/A_μ propagator

which corresponds exactly to (24)

We have thus seen that the "daisy" approximation scheme is consistent with the Nielsen identities in the Pauli–Feynman gauge In an R_ξ gauge the situation is different With a gauge-fixing of the form

$$\mathscr{L}_{\text{gauge-fixing}} = B(\partial_\mu A^\mu + e\xi\epsilon_{ij}\langle\Phi_i\rangle\Phi_j) + \tfrac{1}{2}\xi B^2 , \quad (27)$$

the one-loop effective potential is given by an expression of the form (8) but with

$$k_1^2 = m_2^2 + \xi e^2\Phi^2 , \quad k_2^2 = e^2\xi\Phi^2 , \quad (28)$$

and the addition of an extra term

$$-2e^4\xi^2\Phi^4[\ln(e^2\xi\Phi^2/M^2) - \tfrac{3}{2}] \quad (29)$$

Thus the $O(\lambda)$ expression is gauge-parameter dependent and, by the argument after eq (12), we cannot use the $O(\lambda)$ effective potential in this gauge as the starting point for our approximation scheme We note in passing that the problem would persist in a high-temperature approximation, where we use the expansion [2]

$$\frac{1}{2\beta}\sum_{n=-\infty}^{\infty}\int d^3\boldsymbol{k}\,\{\ln[\boldsymbol{k}^2 + (2n\pi/\beta)^2 + \mu^2]\}$$

$$= -\pi^2/90\beta^4 + \mu^2/24\beta^2 \quad (30)$$

The term (29) in the effective potential would give rise to a gauge-dependent μ^2 term

To summarize, a prerequisite for an expansion scheme which satisfies the Nielsen identities is a lowest-order approximation to the effective potential which is gauge-parameter independent [the equivalent of (12)] We achieved this in the Coleman–Weinberg model by a judicious choice of gauge but in some schemes, such as self-consistent dimensional reduction in Kaluza–Klein gravity, this may not be possible [1] Given this starting point we can then "sum the daisies" to obtain a higher-order approximation

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