Maths Masterclass

The Game of Life

The game of Life is played on a grid. Each square in the grid is called a cell. Notice that each cell has eight neighbours.

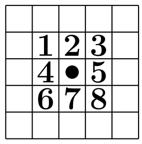


Figure 1

Each cell is alive or dead. The pattern of cells changes each generation according to given rules. The game gets its name from its resemblance to changing groups of living things which can grow, move and die out. The rules which determine the state of a given cell in the next generation are:-

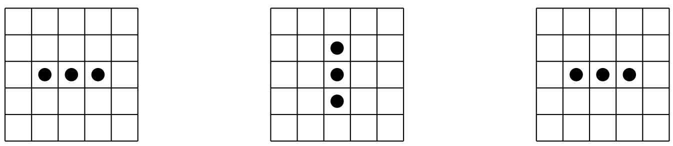
1. **Survivals:** If a live cell has 2 or 3 neighbours it will live to the next generation.
2. **Birth:** If an empty (dead) cell hes exactly 3 neighbours, the cell will be born in the next generation.
3. **Death:** Each cell with 4 or more live neighbours dies (from overpopulation). Each cell with 1 neighbour or no neighbours dies (from isolation).

Summary of the rules

- Exactly 3 neighbours for a birth.
- 2 or 3 live neighbours for survival.

We choose a pattern of live and dead cells for generation 0. The fate of future generations is then determined by applying the rules.

An example is shown in *Figure 2.* The dead cells are left blank, while the live cells have a circle in them. The notes will help you follow the progression from generation to generation.



Generation 0 Generation 1 Generation 2

Figure 2

Generation 0 — 1 The two end live cells have just one neighbour each and so will die. The middle cell has 2 living neighbours and will therefore survive. The empty cells above and below the middle cell each have 3 live neighbours, so it will be born in generation 1.

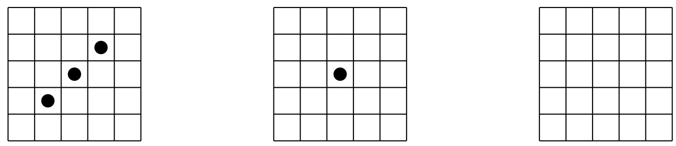
Generation 1 — 2 The pattern is the same as before, (turn it on its side) so the same reasoning applies.

Since generation 2 is exactly the same as generation 0 the pattern will repeat itself every two generations.

There are many possibilities after you choose generation 0:-

1. The pattern may repeat as in *Figure 2.*

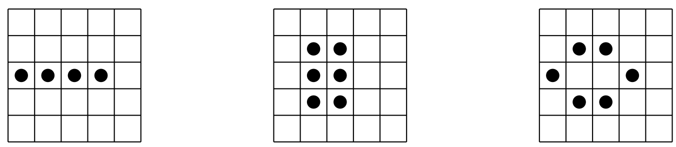
2. The pattern may die leaving you with an empty grid.



Generation 0 Generation 1 Generation 2

Figure 3.

3. It may stop at a still life as in *Figure 4*.



Generation 0 Generation 1 Generation 2

Figure 4

These patterns remain the same from generation to generation. An assortment of still life patterns is shown in *Figure 5.*

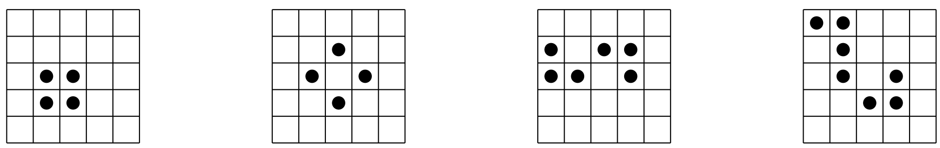
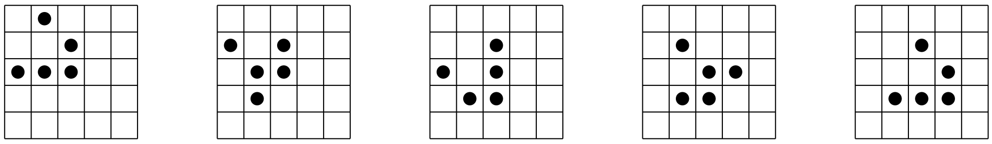


Figure 5 : Varieties of Still Life.

4. There are patterns which move. An example of this is the so- called glider. Every four generations it regains its original shape, except that it has moved one square diagonally.



Generation 0 Generation 1 Generation 2 Generation 3 Generation 4

Figure 6: The Glider Pattern

5. There are patterns which continue indefinately and which are always growing. Examples of this are too complicated to show here and were in fact found by computer. The idea behind one of the examples is that the pattern ejects a new glider every 30 generations.

6. There are patterns for which we cannot predict what will happen. This may seem strange at first since **Life is but a game** and there surely can't be any difficult mathematical problems here. However it is an astonishing fact that any well stated mathematical problem can be reduced to a problem about **Life!** Thus predicting future generations for some initial patterns can be arbitrarily difficult which justifies the title of this lecture:-

LIFE IS UNPREDICTABLE

Reference: Martin Gardner: Wheels, Life and Other Mathematical Amusements, 1983, W.H. Freeman.

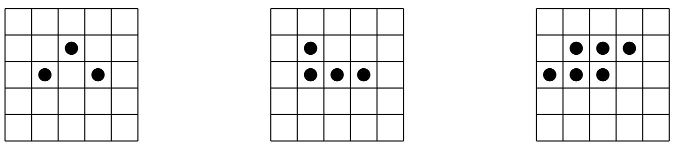
PROBLEMS

1. Check that *Figures 2* and *3* are correct.

2. Give an example of an initial pattern with 7 live cells which dies in generation 4. (Hint: look at *Figure 3*)

3. The blinker in *Figure 2* is an example of a pattern that repeats itself every 2 generations. Find an example of a pattern containing 6 live cells which repeat every 2 generations.

4. Investigate what happens to each pattern in *Figures 7(a), (b)* and *(c)*.



(a) *(b)* *(c)*

Figure 7

5. In this problem we use different rules for the change in patterns from generation to generation. We think of each cell as being a clump of vegetation. Each clump is either green *(G)*, infested with insects *(I)* or defoliated *(D)*. Thus each cell has one of three states (in Life each cell can have only two states, alive or dead).

   Changes in the state of a cell are governed by

   - An infested cell becomes defoliated.
   - A green cell becomes infested if any of its 4 nearest neighbours is infested. (In *Figure 1*, the four nearest neighbours to the centre cell are numbered 2, 4, 5 and 7.
   - A defoliated cell becomes green.

   Figure 8 shows an example of an infestation propagating out from a single infested cell. Note that the blank cells all represent green cells *(G)*. Check that *Figure 8* is correct.

		I		

Generation 0

		I		
	I	*D*	*I*	
		I		

Generation 1)

		I		
	I	*D*	*I*	
I	*D*	*G*	*D*	*I*
	I	*D*	*I*	
		I		

Generation 2

Figure 8

   Find also the next 3 generations when the initial pattern is given by *Figure 9*

		I		
		D		

Figure 9

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