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NIELSEN IDENTITIES FOR GAUGE-FIXING VECTORS AND COMPOSITE EFFECTIVE POTENTIALS

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We show how to derive the Nielsen identities which govern the gauge dependence of physical quantities and the effective action for the gauge fixing vectors in both the 't Hooft gauge and in the planar gauge by using an extended set of BRS transformations. We also show that it is possible to derive the identities for an effective action which depends on composite operators.

1. Introduction

In this paper we discuss an alternative method introduced in [1] of deriving the Nielsen identities and show how it may be used to demonstrate the independence of physical quantities, for instance the minimum of the effective potential, from gauge variables such as the gauge fixing vectors (v in the 't Hooft gauge and n_μ in the planar gauge). We shall take the abelian Higgs model as our prototypical gauge theory for convenience, though the approach generalizes without difficulty to non-abelian gauge theories.

The effective potential, defined as

$$V(\bar{\phi}) = -\sum \frac{1}{n!}\tilde{\Gamma}(0,\dots 0)[\bar{\phi}-v]^n, \tag{1}$$

where $\tilde{\Gamma}$ is the n-leg 1PI generating functional at zero momentum and v is the vacuum expectation value of ϕ is an off-shell quantity and one would not expect, a priori, such an object to be gauge independent. Some explicit calculations (after Jackiw [2]) for massless scalar QED confirm these suspicions. Indeed, if we choose the gauge fixing term for the abelian Higgs model to be of the form

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi v_i\phi_i\right)^2 \tag{2}$$

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giving a lagrangian of the form

$$L = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\phi_i)(\partial^\mu\phi_i) + \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 - e\varepsilon_{ij}(\partial_\mu\phi_i)\phi_j A^\mu$$

$$+ \tfrac{1}{2}e^2A^2\phi^2 - \frac{1}{2\xi}\left(\partial_\mu A^\mu + e^\xi v_i\phi_i\right)^2 + \partial_\mu\psi^* \partial^\mu\psi - e^2\psi^*\psi\varepsilon_{ij}\xi v_i\phi_j, \qquad (3)$$

where $\varepsilon_{12} = -\varepsilon_{21} = 1$ and we have split the complex scalar field into two components, we would find that even the tree-level potential contained the unphysical quantities ξ and v [3,4],

$$V(\bar{\phi}) = \tfrac{1}{2}\xi(v_i\phi_i)^2 - \tfrac{1}{2}m^2\bar{\phi}^2 + \frac{\lambda}{4!}\phi^4. \qquad (4)$$

One possible solution to the problem is to ignore it! One could choose to shift the Higgs fields by their tree level vacuum expectation values (provided we did not choose a gauge such as (2)) and retain tadpole graphs in higher order calculations [5]. This would only work in cases where the vacuum is already determined at the tree level, which would preclude the consideration of the Coleman-Weinberg-type symmetry breaking, where it is the radiative corrections that lead to the spontaneous symmetry breakdown. It would also prevent one examining models in which the classical Higgs potential has a larger symmetry than the rest of the lagrangian (which gives rise to the so-called pseudo-Goldstone bosons [6]). These, too, require the inclusion of one-loop effects to determine the true vacuum. Finally, one might also like to check that radiative corrections do not change the minima even in the standard case.

Other solutions that have been advanced are that only in the "physical" unitary gauge does the effective potential have any significance [6] and that expressing V in terms of renormalized quantities rather than bare ones resolves the problem [7]. Alternatively one may work with renormalization group equations rather than the effective potential itself [8], or use a modified definition of the effective potential which is automatically gauge invariant [9,10]. The correct approach when dealing with the standard effective potential was, however, first proposed by Nielsen [11], and we follow his approach, drawing heavily on the later work of Aitchison and Fraser [12].

If one considers the effective potential for a gauge theory such as (3) it will depend explicitly on ξ and other gauge fixing variables, so we write it as $V(\bar{\phi}, \xi)$. The vacuum is determined by the condition

$$\frac{\partial V}{\partial \bar{\phi}} = 0, \qquad (5)$$

and spontaneous symmetry breaking occurs when (5) has a non-zero solution, say

$\bar{\phi}_0(\xi)$. This situation would be gauge invariant if, under a small change in ξ, the value of V at the minimum (which is after all a physical quantity) remained constant;

$$V(\bar{\phi}_0 + \delta\bar{\phi}_0, \xi + \delta\xi) = V(\bar{\phi}_0, \xi) = V_{\min}. \tag{6}$$

We could write this as

$$\left.\frac{\partial V}{\partial \bar{\phi}}\right|_{\xi} \frac{\mathrm{d}\bar{\phi}}{\mathrm{d}\xi} + \frac{\partial V}{\partial \xi} = 0, \tag{7}$$

which states that the total differential of V with respect to ξ at the minimum is zero.

One may derive similar identities for other physical quantities, such as the mass of the Higgs particles and the vector mesons and these are considered in [11] and [12]. In this paper we shall consider only the identity for the minimum of the effective potential itself, but the method applies to the Nielsen identities for the other physical quantities, which are obtained by differentiating the effective potential identity with respect to the appropriate semi-classical fields.

Nielsen's remarkable result was to derive, using the BRS [13] invariance of the theory, a set of identities of precisely the form above, thus guaranteeing the gauge invariance of physical quantities. They were

$$\xi \frac{\partial V}{\partial \xi} + C(\bar{\phi}, \xi) \frac{\partial V}{\partial \bar{\phi}} = 0, \tag{8a}$$

$$\xi \frac{\partial m^2}{\partial \xi} + C(\bar{\phi}, \xi) \frac{\partial m^2}{\partial \bar{\phi}} = 0 \qquad \text{if } \frac{\partial V}{\partial \bar{\phi}} = 0, \tag{8b}$$

where we have denoted the mass of any of the physical particles in the theory by m. A similar equation would also apply for physical couplings such as the electromagnetic coupling e. The object $C(\phi, \xi)$ is obtained as an explicit field-theoretic expression and could be calculated in some expansion scheme. We note that eq. (8a) exceeds our requirements, as it does not just apply at the minimum of V. Along the characteristics of V, the curves in the $\bar{\phi}$, ξ plane for which

$$\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}\xi} = \frac{C(\bar{\phi}, \xi)}{\xi}, \tag{9}$$

V is a constant. The beauty of Nielsen's results is that they enable one to calculate in closed form the change in $\bar{\phi}_0$ that compensates for the gauge variation.

2. Derivation of the Nielsen identities

We present in this section an alternative derivation of the Nielsen identities which is based on the work of Piguet and Sibold [14]. It sets the Nielsen identities in their proper context in a group of identities that control the gauge dependence of the generating functionals in a gauge theory. The trick we use is to enlarge the BRS transforms to act on the gauge parameter as well and to use the auxiliary field method of gauge-fixing that was first promoted by Kugo and Ojima [15]. The auxiliary field approach has the great advantage (to mathematicians!) of making all the BRS transformations nilpotent, which suggests a possible geometric interpretation for the BRS variation in terms of exterior derivations.

Piguet and Sibold introduce a BRS variation on the gauge parameter ξ, where ε is the anticommuting BRS parameter

$$\delta\xi = \varepsilon\chi\,, \qquad \chi \text{ grassmannian}, \tag{10}$$

and show that under this extended set of BRS transformations the Slavnov-Taylor identity becomes (in a Yang-Mills theory)

$$S(\Gamma) + \chi\frac{\partial\Gamma}{\partial\xi} = 0, \tag{11}$$

where S is the usual Slavnov operator

$$S(\Gamma) = \mathrm{Tr}\int \mathrm{d}^4x \left[\frac{\delta\Gamma}{\delta\rho^\mu}\frac{\delta\Gamma}{\delta\overline{A}_\mu} + \frac{\delta\Gamma}{\delta\sigma}\frac{\delta\Gamma}{\delta\overline{\psi}} + \overline{B}\frac{\delta\Gamma}{\delta\overline{\psi}^*}\right]. \tag{12}$$

In the above σ and ρ^μ are sources for the BRS variations of ψ and A_μ respectively and B is the auxiliary field which allows us to write the gauge-fixing part of the lagrangian in the form

$$L_{\mathrm{gf}} = \tfrac{1}{2}\xi B^2 + B\left(\partial_\mu A^\mu\right) + \tfrac{1}{2}\chi\psi^* B + \partial_\mu\psi^* D^\mu\psi\,. \tag{13}$$

One can easily check that this is invariant under the BRS variations

$$\delta A_\mu = \varepsilon\,\partial_\mu\psi\,, \qquad \delta\psi = 0\,, \qquad \delta\psi^* = \varepsilon B\,, \qquad \delta\xi = \varepsilon\chi\,. \tag{14}$$

We also note that in the absence of the last transformation and the corresponding $\frac{1}{2}\chi\psi^* B$ term in the lagrangian, eliminating B by gaussian integration in the path integral (it has no kinetic terms) gives the standard Fermi gauge-fixing,

$$L_{\mathrm{gf}} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \partial_\mu\psi^* D^\mu\psi\,. \tag{15}$$

The effective action precursor of the Nielsen identity is then simply obtained by differentiating (13) w.r.t. χ and then setting $\chi = 0$ (taking care with the sign of anticommuting quantities in the process)

$$S\left(-\frac{\partial \Gamma}{\partial \chi}\right)+\frac{\partial \Gamma}{\partial \xi}=0. \tag{16}$$

In [1] we showed how to recast the gauge fixing term in the abelian Higgs model into a form which is similar to (13) and which is invariant under the new BRS variation in (10). We then performed explicitly the steps leading to (16). We show below how the method of extended BRS transformations may be used to demonstrate the independence of physical quantities from the gauge-fixing vector v in the 't Hooft gauge.

In the older approach [12] we would have found that

$$v_i \frac{\partial L}{\partial v_i} = -e v_i \phi_i \left(\partial_\mu A^\mu + e\xi v_i \phi_i\right) - e^2 \psi^* \psi \varepsilon_{ij} v_i \phi_j, \tag{17}$$

which can be generated from the BRS transform of the operator

$$0 = e\psi^* \phi_i \xi v_i. \tag{18}$$

In our approach we introduce the new BRS transform

$$\delta v_i = \varepsilon \rho_i. \tag{19}$$

In addition to the usual BRS transforms for the abelian Higgs model,

$$\delta\psi^* = \varepsilon B, \qquad \delta\psi = 0, \qquad \delta B = 0, \qquad \delta A_\mu = \varepsilon\, \partial_\mu \psi, \qquad \delta\phi_i = \varepsilon e \varepsilon_{ij} \psi \phi_j. \tag{20}$$

The ρ_i have the appropriate group properties but are anticommuting objects. We would modify the gauge fixing term to be invariant under this new transformation

$$L_{\mathrm{gf}} = \tfrac{1}{2}\xi B^2 + B\left(\partial_\mu A^\mu + e\xi v_i \phi_i\right) + \partial_\mu \psi^*\, \partial^\mu \psi - e\xi \psi^* \rho_i \phi_i$$
$$- e^2 \xi \psi^* \psi \varepsilon_{ij} v_i \phi_j. \tag{21}$$

The Nielsen identity is then, symbolically

$$S\left(-\frac{\partial \Gamma}{\partial \rho_i}\right)+\frac{\partial \Gamma}{\partial v_i}=0, \tag{22}$$

where

$$-\frac{\partial \Gamma}{\partial \rho_i} = \Gamma(e\xi\psi^*\phi_i) = \Gamma(O). \tag{23}$$

We can write this explicitly as

$$\int d^4x \left(\frac{\delta\Gamma}{\delta \bar{A}_\mu(x)} \partial_\mu \bar{\psi}(x) + \frac{\delta\Gamma}{\delta K_i(x)} \frac{\delta\Gamma}{\delta \bar{\phi}_i(x)} + \bar{B}(x) \frac{\delta\Gamma}{\delta \bar{\psi}^*(x)} \right) + \rho_i \frac{\delta\Gamma}{\delta v_i} = 0. \quad (24)$$

If we integrate out the auxiliary field we will replace it by its minimum value in the exponent of the path integral, which is given by the solution to its equation of motion

$$\frac{\partial L}{\partial B} = \xi B + \left(\partial_\mu A^\mu + e\xi v_i \phi_i \right). \quad (25)$$

Using (25) we can substitute for B in (24) and we find that

$$\int d^4x \left(\frac{\delta\Gamma}{\delta \bar{A}_\mu(x)} \partial_\mu \bar{\psi}(x) + \frac{\delta\Gamma}{\delta K_i(x)} \frac{\delta\Gamma}{\delta \bar{\phi}_i(x)} - \tfrac{1}{\xi} \left(\partial_\mu \bar{A}^\mu(x) + e\xi v_i \bar{\phi}_i(x) \right) \frac{\delta\Gamma}{\delta \bar{\psi}^*(x)} \right)$$

$$+ \rho_i \frac{\partial\Gamma}{\partial v_i} = 0. \quad (26)$$

We now differentiate w.r.t. ρ and set $\rho = 0$ to get the equivalent of (16)

$$\int d^4x\, d^4z \left(\frac{\delta\Gamma(O(z))}{\delta \bar{A}_\mu(x)} \partial_\mu \bar{\psi}(x) + \frac{\delta\Gamma(O(z))}{\delta K_i(x)} \frac{\delta\Gamma}{\delta \bar{\phi}_i(x)} + \frac{\delta\Gamma}{\delta K_i(x)} \frac{\delta\Gamma(O(z))}{\delta \bar{\phi}_i(x)} \right.$$

$$\left. - \frac{1}{\xi} \left(\partial_\mu \bar{A}^\mu(x) + e\xi v_i \bar{\phi}_i(x) \right) \frac{\delta\Gamma(O(z))}{\delta \bar{\psi}^*(x)} \right)$$

$$- \frac{\partial\Gamma}{\partial v_i} = 0. \quad (27)$$

Specializing to x-independent $\bar{\phi}$ and setting the other classical fields to zero gives

$$\frac{\partial V}{\partial v_i} - \int d^4x \frac{\delta\Gamma(O(x))}{\delta K_i(0)} \frac{\partial V}{\partial \bar{\phi}_i} = - \frac{e\xi v_i \bar{\phi}_i \xi}{\Omega} \int d^4x\, d^4z \frac{\delta\Gamma(O(x))}{\delta \bar{\psi}^*(z)}. \quad (28)$$

This is identical to the result that one obtains in the older approach and, provided that $v_i \bar{\phi}_i = 0$ a point discussed in detail in [12], is of the correct form for a Nielsen identity.

We have made no attempt at discussing the renormalizability of the extended actions. However, Piguet and Sibold give an extensive discussion of the renormalizability properties of a Yang-Mills theory with the extra terms in the gauge fixing and show that the theory is essentially unchanged in comparison with the usual case. Similar considerations would apply to the extended actions in our abelian Higgs model.

3. Nielsen identities in the axial gauge

In order to further demonstrate the utility of the Piguet-Sibold approach we shall give a derivation of the Nielsen identities in the axial gauge. There are two points to bear in mind for such a derivation: the first is that there are various possible ways of implementing an axial gauge in the path integral with gauge fixings of the form

$$\tfrac{1}{2}\xi B^a \left(f^{ab}\right)^{-1} B^b + B^a\left(n_\mu A^{\mu a}\right). \tag{29}$$

The most obvious choice with $f^{ab} = -\delta^{ab}$ is pathological for $\xi \neq 0$ because the propagator is of the form

$$\frac{1}{ip^2}\left[g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{\left(n_\mu p^\mu\right)} + \frac{p_\mu p_\nu n^2}{\left(n_\mu p^\mu\right)^2} + \xi \frac{p_\nu p_\nu p^2}{\left(n_\mu p^\mu\right)^2}\right], \tag{30}$$

which goes as O(1) for large p^2. This invalidates the usual power counting arguments used in demonstrating the renormalizability of the theory [16]. Another possible choice is the planar gauge with

$$f^{ab} = \left(\frac{\partial^2}{n^2}\right)^{ab}. \tag{31}$$

The propagator takes a particularly simple form in this gauge

$$\frac{1}{ip^2}\left[g_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{n_\mu p^\mu}\right]. \tag{32}$$

We note that the auxiliary field method of gauge-fixing used in (30) would obviate the need for Nielsen-Kallosh ghosts if one were working in a background field formalism, where f^{ab} would be equal to $D^2(A)^{ab}/n^2$, where $D(A)$ is the background field covariant derivative [17,18]. If we had written the gauge fixing in the usual form

$$-\frac{1}{2\xi}\left(n_\mu a^{\mu a}\right) f^{ab} \left(n_\mu a^{\mu a}\right), \tag{33}$$

where a_μ is the quantum field, we would have needed to add the terms

$$\omega^a f^{ab} \omega^b + \tfrac{1}{2}\gamma^a f^{ab} \gamma^b, \tag{34}$$

where ω is a complex anticommuting ghost and γ is a real commuting ghost to reproduce the required $\sqrt{\det f}$ factor that ensures the invariance of the measure. This is automatically reproduced upon the integration of the B fields.

The second point to note is that the axial gauges, like the Fermi gauges, will suffer from infrared divergences. However, it is argued by Thompson and Wu [16] that these may be regulated in the context of dimensional regularization and do not affect the veracity of the identities. With the preceding provisions in mind we write the gauge-fixing term in the planar gauge for an abelian Higgs model as

$$L_{\mathrm{gf}} = \tfrac{1}{2}\xi B\left(\frac{n^2}{\partial^2}\right)B + B(n_\mu A^\mu) - \psi^*(n_\mu \partial^\mu)\psi . \tag{35}$$

We now choose to extend the set of BRS transforms acting on our lagrangian by including

$$\delta\xi = \varepsilon\chi , \qquad \delta n_\mu = \varepsilon\rho_\mu , \tag{36}$$

where both χ and the components of ρ_μ are anticommuting objects. In order to maintain the BRS invariance of the gauge fixing term under these new transformations we must add the following

$$\tfrac{1}{2}\chi\psi^*\left(\frac{n^2}{\partial^2}\right)B - \psi^*(\rho_\mu A^\mu) - \xi\psi^*\left(\frac{n_\mu\rho^\mu}{\partial^2}\right)B . \tag{37}$$

We now consider separately the change in Γ under a change in ξ and n_μ. In the first case we have the equation

$$S\left(-\frac{\partial\Gamma}{\partial\chi}\right) + \frac{\partial\Gamma}{\partial\xi} = 0 \qquad \text{with } \delta n_\mu = 0 , \tag{38}$$

and in the second we have

$$S\left(-\frac{\partial\Gamma}{\partial\rho_\mu}\right) + \frac{\partial\Gamma}{\partial n_\mu} = 0 \qquad \text{with } \delta\xi = 0 . \tag{39}$$

We find that the corresponding operator insertions are given by

$$\left.\frac{\partial\Gamma}{\partial\chi}\right|_{\chi=0} = \Gamma\left(\tfrac{1}{2}\psi^*\left(\frac{n^2}{\partial^2}\right)B\right) , \tag{40}$$

$$\left.\frac{\partial\Gamma}{\partial\rho_\mu}\right|_{\rho_\mu=0} = \Gamma\left(\psi^* A^\mu + \xi\psi^*\left(\frac{n^\mu}{\partial^2}\right)B\right) . \tag{41}$$

If we now eliminate the auxiliary field B in the usual manner we can see that

$$\left.\frac{\partial \Gamma}{\partial \chi}\right|_{\chi=0} = \Gamma\left(-\frac{1}{2\xi}\psi^*(n_\mu A^\mu)\right), \tag{42}$$

$$\left.\frac{\partial \Gamma}{\partial \rho_\mu}\right|_{\rho_\mu=0} = \Gamma\left(\psi^*\left(g^{\mu\sigma} - \frac{n^\mu n^\sigma}{n^2}\right) A^\sigma\right). \tag{43}$$

This allows us to obtain the Nielsen identity in its usual form for ξ

$$\xi\frac{\partial V}{\partial \xi} + C(\bar{\phi}, \xi)\frac{\partial V}{\partial \bar{\phi}} = 0. \tag{8a}$$

C is now given by

$$C(\bar{\phi}, \xi) = i\hbar\int \mathrm{d}^4x\, \langle 0| T\left(\frac{i}{\hbar}\right)^2\Big[-\tfrac{1}{2}\psi^*(x)\big(n_\mu A^\mu(x)\big)e\psi(0)\phi_2(0)$$

$$\times \exp\left(\frac{i}{\hbar}S_{\mathrm{eff}}[\Phi, \phi]\right)\Big]|0\rangle. \tag{44}$$

In a similar manner we can write the equation for n_μ dependence

$$\frac{\partial V}{\partial n_\mu} + D^\mu(\bar{\phi}, \xi)\frac{\partial V}{\partial \bar{\phi}} = 0, \tag{45}$$

with D^μ given by

$$D^\mu(\bar{\phi}, \xi) = i\hbar\int \mathrm{d}^4x\, \langle 0| T\left(\frac{i}{\hbar}\right)^2\Big[\left(g^{\mu\sigma} - \frac{n^\mu n^\sigma}{n^2}\right)\psi^*(x)A^\sigma(x)e\psi(0)\phi_2(0)$$

$$\times \exp\left(\frac{i}{\hbar}S_{\mathrm{eff}}[\Phi, \phi]\right)\Big]|0\rangle. \tag{46}$$

We note that C and D^μ do not receive any contributions at the one-loop level because there is no mixed A/ϕ propagator in the planar gauge. The two-loop contribution is given by a graph of the form fig. 1.

4. A Nielsen identity for an effective action containing composite operators

As well as the usual effective action which has a scalar field as its argument one might also like to consider a generalization which depends not only on $\bar{\phi}(x)$ but

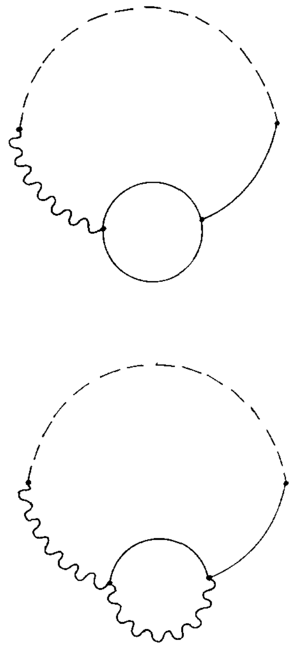

Fig. 1. The two-loop contribution to C. The dotted line represents the ghost propagator, the straight line the scalar propagator and the wavy line the gauge boson propagator.

also on a possible expectation value for $T\phi(x)\phi(y)$, which we shall call $G(x, y)$. The physical vacuum for such an action would then be given by

$$\frac{\delta\Gamma[\bar{\phi}, G]}{\delta\bar{\phi}} = \frac{\delta\Gamma[\bar{\phi}, G]}{\delta G} = 0. \tag{47}$$

Such a formalism is useful in the study of dynamical symmetry breaking where an object such as $T\phi(x)\phi(y)$ may develop a vacuum expectation value [19].

To develop the formalism consider (after [19]) the following action with a compound source.

$$Z[J, L] = \int [\mathrm{D}\phi] \exp\left[\frac{i}{\hbar}\left(S[\phi] + J_i\phi_i + K_i Q_i + \tfrac{1}{2}\phi_i L_{ij}\phi_j\right)\right], \tag{48}$$

where we have lapsed into condensed notation and where the Q_i's are the BRS variations of the fields.

More explicitly

$$\tfrac{1}{2}\phi_i L_{ij}\phi_j = \tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\phi_i(x)L_{ij}(x,y)\phi_j(y). \tag{49}$$

We now define $W[J, L]$ by

$$W[J,L] = -i\hbar \ln Z[J,L] \tag{50}$$

and the classical field $\bar{\phi}$ as

$$\frac{\delta W[J,L]}{\delta J_i(x)} = \bar{\phi}_i(x). \tag{51}$$

We also have the new relation

$$\frac{\delta W[J,L]}{\delta L_{ij}(x,y)} = \tfrac{1}{2}\left[\bar{\phi}_i(x)\bar{\phi}_j(y) + \hbar G_{ij}(x,y)\right]. \tag{52}$$

To obtain the effective action we perform the Legendre transform

$$\Gamma[\bar{\phi},G] = W[J,L] - \int \mathrm{d}^4x\,\bar{\phi}_i(x)J_i(x) - \tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\bar{\phi}_i(x)L_{ij}(x,y)\bar{\phi}_j(y)$$
$$- \tfrac{1}{2}\hbar\int \mathrm{d}^4x\,\mathrm{d}^4y\,G_{ij}(x,y)L_{ji}(y,x). \tag{53}$$

We observe from the above definition that

$$\frac{\delta\Gamma}{\delta\bar{\phi}_i(x)} = -J_i(x) - \int \mathrm{d}^4x\,L_{ij}(x,y)\bar{\phi}_j(y), \tag{54}$$

$$\frac{\delta\Gamma}{\delta G_{ij}(x,y)} = -\tfrac{1}{2}\hbar L_{ij}(x,y). \tag{55}$$

The standard effective action corresponds to $\Gamma[\bar{\phi},G]$ evaluated at $L=0$, or equivalently $\Gamma[\bar{\phi},G]$ for the values of G which satisfy $\delta\Gamma/\delta G = 0$. One may show that $\Gamma[\bar{\phi},G]$ is the generating functional for 2-particle-irreducible diagrams with lines representing $\hbar G(x,y)$ [19] and it can be computed in a similar manner to $\Gamma[\bar{\phi}]$ by considering the vacuum graphs for the shifted lagrangian, but this time retaining only the 2PI graphs.

We now consider our canonical example, the abelian Higgs model, for which we can write the generating functional more explicitly as

$$Z[J,L] = \int[\mathrm{D}A_\mu]\ldots[\mathrm{D}\phi_i]\exp\left[\frac{i}{\hbar}\int \mathrm{d}^4x\left[L + K_i e\psi\varepsilon_{ij}\phi_j + J_\mu A^\mu\right.\right.$$
$$+ J_i\phi_i + \psi^*\eta + \eta^*\psi + J_b B$$
$$\left.\left. + \tfrac{1}{2}\int \mathrm{d}^4y\,\phi_i(x)L_{ij}(x,y)\phi_j(y)\right]\right]. \tag{56}$$

We have again chosen the gauge fixing so that it is invariant under the transformation $\delta\xi = \chi$. We use the invariance of the measure and gauge-fixed lagrangian under a BRS transformation to obtain

$$\int d^4x \int [D\phi] \Big(J_\mu \partial^\mu \psi - \eta B + J_i e \psi \varepsilon_{ij} \phi_j + \tfrac{1}{2} \int d^4y\, e \psi(x) \varepsilon_{i1} \phi_1(x) L_{ij}(x, y) \phi_j(y) + \tfrac{1}{2} \int d^4y\, e \psi(y) \varepsilon_{j1} \phi_i(x) L_{ij}(x, y) \phi_1(y) \Big) \times \exp\left(\frac{i}{\hbar} S_{\mathrm{eff}}[\Phi, \phi] \right) = \frac{\hbar}{i} \chi \frac{\partial Z}{\partial \xi}, \tag{57}$$

where B is the auxiliary field and $[D\phi]$ denotes the measure for all the fields. To express the terms containing L more succinctly we write them as

$$e \int d^4x \left(\varepsilon_{i1} \psi(x) \phi_1(x) f_i^1(x) \right) + e \int d^4y \left(\varepsilon_{j1} \psi(y) \phi_1(y) f_j^1(y) \right), \tag{58}$$

where

$$f_i^1(x) = \int d^4y\, L_{ij}(x, y) \phi_j(y),$$

$$f_j^2(y) = \int d^4x\, L_{ij}(x, y) \phi_i(x). \tag{59}$$

We now note that the invariance of the source term containing L_{ij} under the exchange of x and y implies that $L_{ij}(x, y) = L_{ji}(y, x)$, so f^1 and f^2 are equal, as are both the terms in (58). We can thus rewrite (57) as

$$\int d^4x \int [D\phi] \Big(J_\mu \partial^\mu \psi - \eta B + J_i e \psi \varepsilon_{ij} \phi_j + \int d^4y\, e \psi(x) \varepsilon_{i1} \phi_1(x) L_{ij}(x, y) \phi_j(y) \Big) \times \exp\left(\frac{i}{\hbar} S_{\mathrm{eff}}[\Phi, \phi] \right) = \frac{\hbar}{i} \chi \frac{\partial Z}{\partial \xi}. \tag{60}$$

This may be transformed, as in the standard case, to an operator identity acting on Z

$$\int d^4x \left(J^\mu \partial_\mu \frac{\delta}{\delta \eta^*} - \eta \frac{\delta}{\delta J_b} + J_i \frac{\delta}{\delta K_i} + \frac{\hbar}{i} \int d^4y\, \frac{\delta}{\delta K_i} L_{ij} \frac{\delta}{\delta J_j} \right) Z[J, L] = \chi \frac{\partial Z}{\partial \xi}. \tag{61}$$

This translates to the following identity on W,

$$\int \mathrm{d}^4x \left(J^{\mu} \frac{\delta W}{\delta \eta} - \eta \frac{\delta W}{\delta J_b} + J_i \frac{\delta W}{\delta K_i} + \int \mathrm{d}^4y \frac{\delta W}{\delta K_i} L_{ij} \frac{\delta W}{\delta J_j} + \frac{\hbar}{i} \int \mathrm{d}^4y \, L_{ij} \frac{\delta^2 W}{\delta K_i \delta J_j} \right)$$

$$= \chi \frac{\partial W}{\partial \xi} . \qquad (62)$$

When one Legendre transforms to get the equivalent identity for Γ the third and fourth terms on the l.h.s. combine by virtue of (54) to give

$$\int \mathrm{d}^4x \left(\frac{\delta \Gamma}{\delta \bar{A}_{\mu}} \partial_{\mu} \bar{\psi} + \frac{\delta \Gamma}{\delta \bar{\psi}^*} \bar{B} - \frac{\delta \Gamma}{\delta \bar{\phi}_i} \frac{\delta \Gamma}{\delta K_i} + \frac{2}{i} \int \mathrm{d}^4y \, \mathrm{d}^4z \, \frac{\delta \Gamma}{\delta G_{ij}} \left(\frac{\delta^2 \Gamma}{\delta \bar{\phi}_j \delta \bar{\phi}_k} \right)^{-1} \frac{\delta^2 \Gamma}{\delta K_i \delta \bar{\phi}_k} \right)$$

$$= \chi \frac{\partial \Gamma}{\partial \xi} . \qquad (63)$$

If one now considers a translationally invariant effective potential defined from the effective action by

$$\int V(\bar{\phi}, G, \xi) = -\Gamma[\phi, G, \xi]|_{\text{translationally invariant}}$$

one finds (symbolically) a Nielsen identity of the following form

$$\xi \frac{\partial V}{\partial \xi} + D(\bar{\phi}, G, \xi) \frac{\partial V}{\partial \bar{\phi}} + \int E(\bar{\phi}, G, \xi) \frac{\partial V}{\partial G} = 0, \qquad (64)$$

where V, D and E are to be calculated from 2PI vacuum graphs with the internal lines set equal to $\hbar G$. The identity shows how both $\bar{\phi}$ and G change to compensate for a change in the gauge parameter and maintain the gauge independence of physical quantities.

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