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NIELSEN IDENTITIES IN THE 'tHOOFT GAUGE

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ABSTRACT

We derive Nielsen identities for the gauge invariance of the effective potential and the physical Higgs mass in the 'tHooft gauge and verify them to one loop level. In addition to the standard derivation we also show how they may be derived by considering a B.R.S. transformation which acts on the gauge parameter.

A. Introduction

The Abelian Higgs Model forms a useful laboratory for exploring the gauge dependence of the effective potential. A recent, comprehensive paper by Aitchison and Fraser[1] (hereafter A/F) provided explicit one loop calculations in a restricted class of 'tHooft like gauges, extending the earlier more formal results of Nielsen[2], who worked in a Fermi gauge. The identities derived covered both the gauge dependence of the effective potential and that of the physical Higgs meson mass. They are of the general form shown below.

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0 \qquad (1)$$

$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0 \quad \text{AT } \phi = \phi_0 \qquad (2)$$

V: effective potential

m : physical Higgs mass

ϕ: c-number classical field

ξ: gauge parameter

ϕ_0: value of ϕ at effective potential minimum

The object $C(\phi, \xi)$ is a field theoretic expression which can be calculated in some expansion scheme. The content of these identities is simply that the implicit (via ϕ) and explicit gauge dependence of V and m^2 cancel out, which means that the minima of the effective potential and the physical Higgs mass are gauge parameter independent. In their paper A/F worked with the gauge fixing term:

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e v_i \Phi_i\right)^2 \qquad (3)$$

Φ: quantum scalar field A^{μ} : quantum gauge field

They did not use a 'tHooft gauge because they were unable to derive the appropriate Nielsen identity and they conjectured that the derivation was impossible for any gauge fixing term that contained explicit ξ dependence inside the brackets. We show here that this is possible for gauges of the form:

$$(4)\quad -\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\,\xi v_i \Phi_i\right)^2$$

This contains, as a special case, the 'tHooft gauge :

$$(5)\quad -\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\,\xi\,\epsilon_{ij}\,\phi_{jo}\,\Phi_i\right)^2 \qquad \epsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

One must exercise a modicum of care in using the 'tHooft guage because the minimum field expectation value that is being calculated is introduced into the Lagrangian in the gauge fixing term. The field shift that is carried out to calculate the effective potential via Jackiw's method[3] is not to be confused with the ϕ_{jo} appearing in the gauge fixing term. It is only at the potential minimum that the two are identical.

B. Derivation of the Nielsen Identity

The reader is encouraged to consult A/F's paper, as we follow their methods closely, but we summarize the relevant details in appendix A. The Lagrangian we consider is:

$$(6)\quad \mathcal{L}(x) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\Phi_i)(\partial^\mu\Phi_i) - e\,\epsilon_{ij}(\partial_\mu\Phi_i)\Phi_j A^\mu + \frac{1}{2}e^2A^2\Phi^2 - \frac{1}{2\xi}\left(\partial_\mu A^\mu + e\,\xi v_i\Phi_i\right)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \partial_\mu\psi^*\partial^\mu\psi - e^2\psi^*\psi\,\epsilon_{ij}\,\xi v_i\Phi_j$$

$$\Phi^2 = \Phi_1^2 + \Phi_2^2 \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The B.R.S.[4] transforms for this Lagrangian are:

$$(7)\quad \delta A_\mu = \epsilon\,\partial_\mu\psi \qquad \delta\psi = 0$$

$$\delta\psi^* = -\frac{\epsilon}{\xi}\left(\partial_\mu A^\mu + e\,\xi v_i\Phi_i\right)$$

$$\delta\Phi_i = \epsilon\, e\,\epsilon_{ij}\,\psi\,\Phi_j$$

The Nielsen identities are derived by performing a B.R.S. transform on the augmented generating functional (appendix A)

$$(8)\quad \tilde{Z}_K = \int[D\Phi_\alpha]\exp\left(i\int d^4x\left(\mathcal{L}(x) + K_\alpha Q_\alpha + J_\alpha\Phi_\alpha + hO\right)\right) = \int[D\Phi_\alpha]\exp(i\tilde{S}_K)$$

$\Phi_\alpha(x)$: generic field

$J_\alpha(x)$: generic source

$Q_\alpha(x)$: B.R.S. charge

~: denotes the presence of O

k: denotes the presence of $K_\alpha Q_\alpha$

h(x): source for O

We choose the operator O as :

$$(9)\quad O = -\frac{1}{2}\psi^*\left(\partial_\mu A^\mu - e\,\xi v_i\Phi_i\right)$$

Consider the B.R.S. transform of O : $\delta O = \epsilon\,\bar{O}$

$$\delta O = \epsilon\left[\frac{1}{2\xi}\left((\partial_\mu A^\mu)^2 - e^2\xi^2(V_i\Phi_i)^2\right) + \frac{1}{2}\psi^*\Box\psi - \frac{1}{2}e^2\xi\psi^*\psi\epsilon_{ij}V_i\Phi_j\right] \tag{10}$$

Using the equations of motion for the ghost field:

$$\delta O = \epsilon\left[\frac{1}{2\xi}\left((\partial_\mu A^\mu)^2 - e^2\xi^2(V_i\Phi_i)^2\right) + \frac{1}{2}\psi^*\eta - e^2\xi\psi^*\psi\epsilon_{ij}V_i\Phi_j\right] \tag{11}$$

One now compares this with :

$$\xi\frac{\partial\mathcal{L}}{\partial\xi} = \left[\frac{1}{2\xi}(\partial_\mu A^\mu)^2 - e^2\xi^2(V_i\Phi_i)^2) - e^2\xi\psi^*\psi\epsilon_{ij}V_i\Phi_j\right] \tag{12}$$

We see that, to within a term which vanishes when we consider the effective potential, $\xi\frac{\partial\mathcal{L}}{\partial\xi}$ has been expressed in terms of a B.R.S. transformed operator. This enables us to follow A/F's proof through to arrive at the embryonic first Nielsen identity.

$$\xi\frac{\partial V(\phi_i,\xi)}{\partial\xi} - \int d^4x\,\frac{\delta\Gamma(O(x))}{\delta K_j(0)}\,\frac{\partial V(\phi_i,\xi)}{\partial\phi_j} = \frac{-e\xi V_i\phi_i}{\Omega}\int d^4x\int d^4z\,\frac{\delta\Gamma(O(x))}{\delta\psi_c^*(z)} \tag{13}$$

Ω: spacetime volume

ϕ_i: classical field

$\Gamma(O)$: 1PI generating functional with one insertion of O.

With a gauge of the form above (4) there are, however, two stationary points[5] of the effective potential,only one of which satisfies $V_i\phi_i = 0$

which would cast (13) into the required form. At tree level we have:

$$\phi_{io} = \epsilon_{ij}\frac{V_j}{|V|}\left(\frac{6\mu^2}{\lambda}\right)^{\frac{1}{2}} \tag{14}$$

and

$$\phi_{io} = \frac{V_i}{|V|}\left[\frac{6}{\lambda}(\mu^2 - \xi e^2 V^2)\right]^{\frac{1}{2}}$$

However, as has been elucidated by Fukuda and Kugo[6], the second solution is spurious, in that it does not correspond to a vanishing expectation value of the gauge field. This can be gauged away only at the expense of an x-dependent vacuum expectation value for ϕ_i, which takes us out of the context of the effective potential.

Fukuda and Kugo have further shown that the direction of spontaneous symmetry breaking is unchanged by higher order corrections. Thus by choice we can take:

$$V_i = V e_i \qquad e_i = (0,1)$$
$$\phi_{io} = \phi_o n_i \qquad n_i = (1,0) \tag{15}$$

discarding the spurious solution and considering the effective potential as a function of $\phi = \phi_1$ only. Writing: $C(\phi,\xi) = -\int d^4x\,\frac{\delta\Gamma(O(x))}{\delta K(0)}$
We arrive at :

$$\xi\frac{\partial V}{\partial\xi}(\phi,\xi) + C(\phi,\xi)\frac{\partial V}{\partial\phi}(\phi,\xi) = 0 \tag{1}$$

C. <u>One-Loop Verification of Effective Potential Identity</u>

Expanding (1) order by order in $\hbar$:

$$\xi\frac{\partial V^{(1)}}{\partial\xi} + C^{(1)}(\phi,\xi)\frac{\partial V^{(0)}}{\partial\phi} = 0 \tag{16}$$

The superscripts denote the order in h. We can calculate both $V^{(1)}$ and $C^{(1)}$ by Jackiw's functional method. Starting with the latter.

$$C(\phi,\xi) = -i\hbar\int d^4x \langle 0|T\left(\frac{i}{\hbar}\right)^2\left[-\frac{1}{2}\psi^*(x)(\partial.A(x)-e\xi v.\Phi(x)\right) \quad (17)$$
$$.e\psi(0)\Phi(0)\exp\left(\frac{i}{\hbar}S_{eff}[\phi,\Phi]\right)|0\rangle$$

$$S_{eff}[\phi,\Phi] = S_{\mathcal{L}}[\phi+\Phi] - S_{\mathcal{L}}[\phi] - \int d^4x\, \Phi(x)\frac{\delta S_{\mathcal{L}}}{\delta\Phi}\bigg|_{\Phi=\phi} \quad (18)$$

The one loop term is:

$$C^{(1)}(\phi,\xi) = i\hbar\int d^4x \langle 0|T\left(\frac{i}{\hbar}\right)^2\frac{1}{2}\psi^*(x)(\partial.A(x)-e\xi v.\Phi(x))e\psi(0)\Phi(0)|0\rangle \quad (19)$$

We use the propagators in appendix B to evaluate this :

$$C^{(1)} = \frac{ie}{2}\int d^4k\left[\frac{i}{k^2+e^2\xi v\phi}\cdot\frac{-ik^2(\xi\phi+\xi v)}{D_N} + \right. \quad (20)$$
$$\left.\frac{-ie^2\xi v}{(k^2+e^2\xi v\phi)}\cdot\frac{i(k^2-\xi e^2\phi^2)}{D_N}\right]$$
$$D_N = k^4 - k^2(m_2^2 - 2e^2\xi v\phi) + e^2\phi^2(e^2\xi^2v^2 + \xi m_2^2)\ ;\ m_2^2 = \frac{1}{6}\lambda\phi^2 - \mu^2$$

We simplify this to:

$$C^{(1)} = \frac{ie^2\xi}{2}\int d^4k\,\frac{(2v+\phi)k^2 - e^2\xi v\phi^2}{(k^2+e^2\xi v\phi)D_N} \quad (21)$$

We now consider the one loop effective potential :

$$V^{(1)}(\phi,\xi) = i\int d^4k\left[\ln(k^2+e^2\xi v\phi) - \frac{3}{2}\ln(-k^2+e^2\phi^2)\right. \quad (22)$$
$$\left.-\frac{1}{2}\ln(k^2-m_1^2) - \frac{1}{2}\ln D_N\right]\quad ;\quad m_1^2 = \frac{1}{2}\lambda\phi^2-\mu^2$$

So :

$$\xi\frac{\partial V^{(1)}}{\partial\xi} = i\int d^4k\left[\frac{e^2\xi v\phi}{k^2+e^2\xi v\phi}\right. \quad (23)$$
$$\left.-\frac{1}{2}\frac{[2k^2e^2\xi v\phi + 2e^4\phi^2\xi^2v^2 + e^2\phi^2m_2^2\xi]}{D_N}\right]$$

which equals:

$$\xi\frac{\partial V^{(1)}}{\partial\xi} = -\frac{ie\xi\phi m_2^2}{2}\int d^4k\,\frac{(2v+\phi)k^2 - v\xi e^2\phi^2}{(k^2+e^2\xi v\phi).D_N} \quad (24)$$

Now note that :

$$\frac{\partial V^{(0)}}{\partial\phi} = m_2^2\phi \quad (25)$$

Adding the terms up:

$$\xi\frac{\partial V^{(1)}}{\partial\xi} + C^{(1)}(\phi,\xi)\frac{\partial V^{(0)}}{\partial\phi} = 0 \quad (26)$$

This verifies the first Nielsen identity in a gauge of the form:

$$-\frac{1}{2\xi}(\partial_\mu A^\mu + e\xi v_i\Phi_i)^2 \quad (4)$$

D. <u>One-Loop Verification of Mass Identity</u>

The mass[2] to one loop is given by :

$$m^{2(1)} = m_1^2 + \Sigma^{(1)}(m_1^2) \quad (27)$$

$-i\Sigma$ is the part of the one-loop Higgs self energy tensor which is the coefficient of n_in_j (the projector onto physical Higgs space). To simplify calculations, following A/F we expand in powers of e^2 and λ, choosing $\lambda \sim O(e^4)$. This allows us to write to order $e^2\lambda$:

$$m^{2(1)} = m_1^2 + \Sigma^{(1)}(0) + m_1^2\frac{\partial\Sigma^{(1)}(p^2)}{\partial p^2}\bigg|_{p^2=0} \quad (28)$$

i.e.

$$(29)\qquad m^{2(1)} = \frac{\partial^2 V^{(1)}}{\partial \phi^2} + m_1^2 \frac{\partial \Sigma^{(1)}(p^2)}{\partial p^2}\Bigg|_{p^2=0}$$

We now choose to work in the 'tHooft gauge proper:

$$(30)\qquad \underline{V} = (0, -\phi_0) \qquad \underline{\phi_0} = (\phi_0, 0)$$

The expansion scheme and the choice of gauge eliminate the graphs of Fig. 1 and the Higgs-photon mixing graphs respectively. This leaves the three graphs of Fig. 2 to calculate, which we have done here using the $\overline{MS}$ subtraction scheme with M as the arbitrary renormalisation mass

$$(31)\qquad \sum_{(2a)}^{(1)\prime}(0)\Bigg|_{\phi=\phi_0} = \frac{e^2\xi}{16\pi^2}\left[\frac{3}{\xi-1}\ln\xi - \frac{11}{6}\right]$$

$$\sum_{(2b)}^{(1)\prime}(0)\Bigg|_{\phi=\phi_0} = \frac{e^2\xi}{16\pi^2}\left[\frac{1}{6}\right]$$

$$\sum_{(2c)}^{(1)\prime}(0)\Bigg|_{\phi=\phi_0} = \frac{e^2\xi}{16\pi^2}\left[\frac{2}{3} + \ln\frac{e^2\xi\phi_0^2}{M^2} - \frac{3}{\xi-1}\ln\xi\right]$$

(32) Adding the terms up :

$$\sum_{\text{TOTAL}}^{(1)\prime}(0)\Bigg|_{\phi=\phi_0} = \frac{e^2\xi}{16\pi^2}\left[\ln\frac{e^2\xi\phi_0^2}{M^2} - 1\right]$$

Working to $O(\hbar)$ ϕ_0 is the classical minimum of the potential.

Now note:

$$m_1^2\Big|_{\phi=\phi_0} = \frac{1}{3}\lambda\phi_0^2 \qquad \frac{\partial m_1^2}{\partial\phi}\Bigg|_{\phi=\phi_0} = \lambda\phi_0 = \frac{\partial m^{2(0)}}{\partial\phi}\Bigg|_{\phi=\phi_0}$$

$$(33)\qquad \frac{\partial^2 V^{(1)}}{\partial\phi^2}\Bigg|_{\substack{V=-\phi_0\\ \phi=\phi_0}} = \frac{e^2\xi\lambda\phi_0^2}{32\pi^2}\left[\frac{1}{3}\ln\frac{e^2\xi\phi_0^2}{M^2} - \frac{1}{3}\right]$$

So :

$$(34)\qquad \xi\frac{\partial m^{2(1)}}{\partial\xi}\Bigg|_{\phi=\phi_0} = \frac{e^2\xi\lambda\phi_0^2}{32\pi^2}\left[\ln\frac{e^2\xi\phi_0^2}{M^2}\right]$$

In our expansion scheme and gauge :

$$(35)\qquad C^{(1)}(\phi,\xi)\Big|_{\substack{V=-\phi_0\\ \phi=\phi_0}} = -\frac{e^2\xi\phi_0}{32\pi^2}\ln\frac{e^2\xi\phi_0^2}{M^2}$$

Once again the terms (34)/(35) sum to zero, verifying the mass identity to one loop.

$$(36)\qquad \xi\frac{\partial m^{2(1)}}{\partial\xi} + C^{(1)}(\phi,\xi)\frac{\partial m^{2(0)}}{\partial\phi}\Bigg|_{\phi=\phi_0} = 0$$

E. Alternative Derivation of Identities

Some recent work by Piguet and Sibold[7] makes it possible to derive the Nielsen identities within the framework of a set of BRS transformations which also operate on ξ, the gauge parameter. This removes the rather ad hoc introduction of the operator O and places the Nielsen identities in the wider context of a set of identities derived in ref. 7. They showed that, in a pure Yang-Mills theory :

$$(37)\quad S(\Gamma) + \chi \frac{\partial \Gamma}{\partial \xi} = 0$$

where :

$$\delta \xi = \epsilon \chi \qquad (\chi \text{ Grassmann variable})$$

$$S(\Gamma) = \mathrm{Tr}\int d^4x \left(\frac{\delta \Gamma}{\delta \rho^\mu} \frac{\delta \Gamma}{\delta A_{\mu c}} + \frac{\delta \Gamma}{\delta \sigma} \frac{\delta \Gamma}{\delta \psi_c} + B \frac{\delta \Gamma}{\delta \psi^*_c} \right)$$

ρ^μ : source for δA_μ

σ : source for $\delta \psi$

The auxiliary field B is introduced in order that the gauge fixing, in a Fermi gauge, may be written in the form :

$$(38)\quad \mathcal{L}_{G.F} = \frac{\xi}{2} B^2 + B(\partial . A) + \frac{1}{2}\chi \psi^* B + \partial_\mu \psi^* D_\mu \psi$$

To obtain the effective action precursor of the Nielsen identity we differentiate (37) w.r.t. χ and set χ to 0 :

$$(39)\quad S\left(-\frac{\partial \Gamma}{\partial \chi}\right) + \frac{\partial \Gamma}{\partial \xi} = 0$$

We now repeat the process in more detail for the Abelian Higgs model in the 'tHooft gauge which we have been considering, translating the gauge fixing and B.R.S. transforms into the language of Piguet and Sibold :

$$(40)\quad \mathcal{L}_{G.F} = \frac{\xi}{2} B^2 + B(\partial_\mu A^\mu + e\xi v_i \Phi_i) + \partial_\mu \psi^* \partial^\mu \psi - e^2 \xi \psi^* \psi \epsilon_{ij} v_i \Phi_j + \frac{1}{2}\chi \psi^* B + e\chi \psi^* v_i \Phi_i$$

If we denote, as before, the insertion of an operator P in Γ as $\Gamma(P)$:

$$(41)\quad \frac{\partial}{\partial \chi}\Gamma = \Gamma\left(\frac{1}{2}\psi^*_c B_c + e\psi^*_c v_i \phi_{ic}\right)$$

Use $\frac{\delta \mathcal{L}}{\delta B} = 0$ to eliminate the auxiliary field B :

$$(42)\quad \frac{\delta \mathcal{L}}{\delta B} = \xi B + (\partial . A + e\xi v . \Phi) + \frac{1}{2}\chi \psi^*$$

$$B = -\frac{1}{\xi}(\partial . A + e\xi v . \Phi) - \frac{1}{2\xi}\chi \psi^*$$

This allows us to rewrite $\frac{\partial}{\partial \chi}\Gamma$:

$$(43)\quad \frac{\partial}{\partial \chi}\Gamma = \Gamma\left(-\frac{1}{2\xi}(\partial . A_c - e\xi v_i \phi_{ic})\right) = \Gamma(O')$$

The operator insertion is precisely $1/\xi$ times that of the operator O which we had to construct in the previous derivation. The B.R.S. transforms for our theory are :

$$(44)\quad \delta \psi^* = \epsilon B \qquad \delta \psi = 0 \qquad \delta B = 0$$

$$\delta A_\mu = \epsilon \partial_\mu \psi \qquad \delta \Phi_i = \epsilon e \epsilon_{ij} \psi \Phi_J$$

$$\delta \xi = \epsilon \chi$$

Identity (37) becomes :

$$(45)\quad \int d^4x\left(\frac{\delta\Gamma}{\delta A_{\mu c}}\partial_\mu\psi_c + \frac{\delta\Gamma}{\delta K_i}\frac{\delta\Gamma}{\delta\phi_{ic}} + B\frac{\delta\Gamma}{\delta\psi_c^*}\right) + \chi\frac{\partial\Gamma}{\partial\xi} = 0$$

Substitute for B :

$$(46)\quad \int d^4x\left(\frac{\delta\Gamma}{\delta A_{\mu c}}\partial_\mu\psi_c + \frac{\delta\Gamma}{\delta K_i}\frac{\delta\Gamma}{\delta\phi_{ic}} - \frac{1}{\xi}(\partial.A_c + e\xi v_i\phi_{ic})\frac{\delta\Gamma}{\delta\psi_c^*} - \frac{1}{2\xi}\chi\psi^*\frac{\delta\Gamma}{\delta\psi_c^*}\right) + \chi\frac{\partial\Gamma}{\partial\xi} = 0$$

Differentiate w.r.t. χ and set $\chi = 0$:

$$(47)\quad \int d^4z\int d^4x\left(\frac{\delta\Gamma(O')}{\delta A_{\mu c}}\partial_\mu\psi_c + \frac{\delta\Gamma(O')}{\delta K_i}\frac{\delta\Gamma}{\delta\phi_{ic}} + \frac{\delta\Gamma}{\delta K_i}\frac{\delta\Gamma(O')}{\delta\phi_{ic}} - \frac{1}{\xi}(\partial.A_c + e\xi v_i\phi_{ic})\frac{\delta\Gamma(O')}{\delta\psi_c^*} - \frac{1}{2\xi}\frac{\delta\Gamma}{\delta\psi_c^*}\psi_c^*\right) - \frac{\partial\Gamma}{\partial\xi} = 0$$

Multiplying through by ξ, specializing to x-independent ϕ_c and setting the other fields to 0 gives :

$$(13)\quad \xi\frac{\partial V}{\partial\xi} - \int d^4x\,\frac{\delta\Gamma(O_x)}{\delta K_j(0)}\frac{\partial V}{\partial\phi_x} = -\frac{e\xi v_i\phi_{ic}}{\Omega}\int d^4x\,d^4z\,\frac{\delta\Gamma(O_x)}{\delta\psi_c^*(z)}$$

This rederives the first Nielsen identity.

References

(1) I.J.R. Aitchison and C.M. Fraser: Ann. Phys. (NY) 156 (1984) 1.

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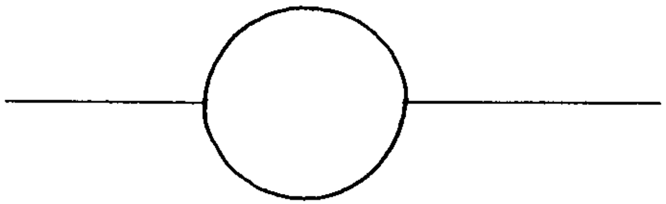
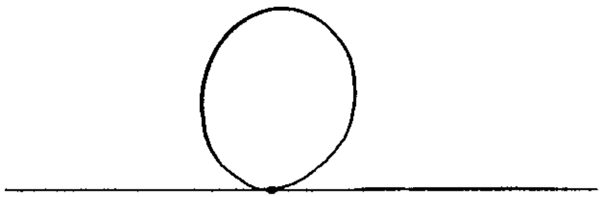
(7) O. Piguet and K. Sibold: preprint MPI-PAE/Th (May 1984).

Fig. 1. Graphs Removed by Expansion Scheme

(a) Too high order in λ



(b) p^2-independent



(c) p^2-independent

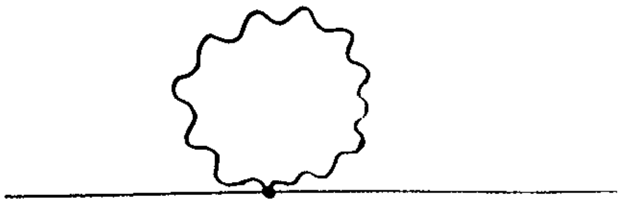
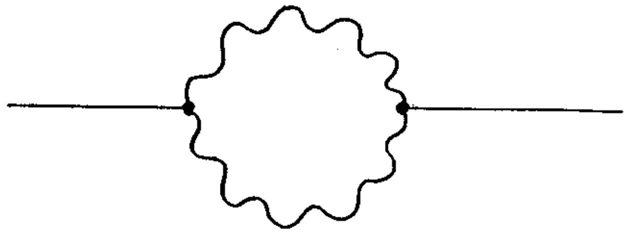
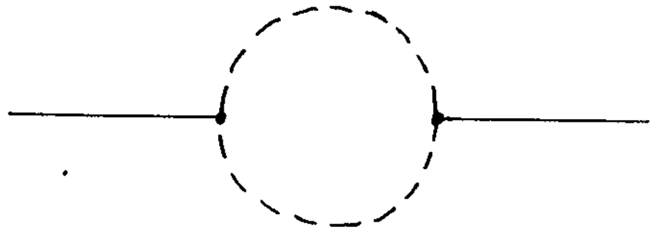


Fig. 2. Graphs to be Calculated

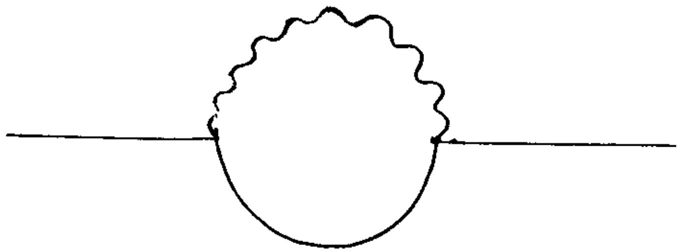
(a)



(b)



(c)



Appendix A

The first term in the Nielsen identity (1) is $\xi\frac{\partial V}{\partial\xi}$, so we consider $\xi\frac{\partial\Gamma}{\partial\xi}$, noting that by virtue of it being an explicit differentiation:

$$\frac{\partial\Gamma}{\partial\xi} = \frac{\partial W}{\partial\xi} \tag{A1}$$

$\xi\frac{\partial\Gamma}{\partial\xi}$ generates 1.P.I. Green's functions with the insertion

$$\int d^4x\left(\frac{1}{2\xi}\left((\partial.A)^2 - e^2\xi^2(v.\Phi)^2\right) - e^2\xi\psi^*\psi\epsilon_{ij}v_i\Phi_j\right) \tag{A2}$$

We cannot quite generate this insertion from a B.R.S. transformed operator but the extra $1/2\phi^*\eta$ term vanishes when we consider the effective potential.

We now denote the insertion by $\bar{O}(x)$ and write:

$$\int[D\Phi_\alpha]\bar{O}(x)\exp\left(i\int d^4x(\mathcal{L}+J_\alpha\Phi_\alpha)\right) = \frac{\delta}{\delta h(x)}\int d^4z\int[D\Phi_\alpha]h(z)\bar{O}(z)\exp\left(i\int d^4x(\mathcal{L}+J_\alpha\Phi_\alpha)\right) \tag{A3}$$

We write the $\hat{Z}_k$ of (8) more explicitly:

$$\hat{Z}_k = \int[DA_\mu][D\psi][D\psi^*][D\Phi_i]\exp(i\hat{S}_k) \tag{A4}$$

$$\hat{S}_k = \int d^4x\left(\mathcal{L} + K_i e\psi\epsilon_{ij}\Phi_j + J_\mu A^\mu + J_i\Phi_i + \eta^*\psi + \psi^*\eta + hO\right)$$

Carry out a B.R.S. transform on $\hat{Z}_k$:

$$\int d^4z[DA_\mu]\cdot\cdot[D\Phi_i]\left(J_\mu\partial^\mu\psi - \eta\frac{1}{\xi}(\partial.A + e\xi v.\Phi) + J_i e\psi\epsilon_{ij}\Phi_j + h\bar{O}\right)\exp(i\hat{S}_k) = 0 \tag{A5}$$

i.e.

$$\int d^4z\left(J_\mu\partial^\mu\frac{\delta}{\delta\eta^*} - \eta\frac{1}{\xi}\left(\partial_\mu\frac{\delta}{\delta J_\mu} + e\xi v_i\frac{\delta}{\delta J_i}\right) + J_i\frac{\delta}{\delta K_i}\right)\tilde{Z}_k = -i\int d^4z[DA_\mu]\cdot\cdot[D\Phi_i]h(z)\bar{O}(z)\exp(i\tilde{S}_k) \tag{A6}$$

Legendre transforming the L.H.S.:

$$\hat{\Gamma}_k = \hat{W}_k - \int d^4x\left(J_\mu A^\mu_c + \eta^*\psi_c + \psi^*_c\eta + J_i\phi_{ic}\right)$$

$$\tilde{W}_k = -i\hbar\ln\tilde{Z}_k$$

A_c etc.: classical fields.

$$\int d^4z\left(-\frac{\delta\hat{\Gamma}_k}{\delta A_{\mu c}}\partial_\mu\psi_c + \frac{\delta\hat{\Gamma}_k}{\delta\psi^*_c}\cdot\frac{1}{\xi}(\partial.A_c + e\xi v.\phi_c) - \frac{\delta\hat{\Gamma}_k}{\delta\Phi_{ic}}\frac{\delta\hat{\Gamma}_k}{\delta K_i}\right) = -\frac{1}{\tilde{Z}_k}\int d^4z[DA_\mu]\cdot\cdot[D\Phi_i]h(z)\bar{O}(z)\exp(i\tilde{S}_k) \tag{A7}$$

Functionally differentiate W.R.T. h(x) then set h = 0, which removes the tildes.

($\Gamma_k(O(x)) = \Gamma_k$ with O insertion)

$$\int d^4z\left(-\frac{\delta\Gamma_k(O(x))}{\delta A_{\mu c}}\partial_\mu\psi_c + \frac{\delta\Gamma_k(O(x))}{\delta\psi^*_c}\frac{1}{\xi}(\partial.A_c + e\xi v.\phi_c) - \frac{\delta\Gamma_k(O(x))}{\delta\phi_{ic}}\frac{\delta\Gamma_k}{\delta K_i} - \frac{\delta\Gamma_k}{\delta\phi_{ic}}\frac{\delta\Gamma_k(O(x))}{\delta K_i}\right) = -\frac{1}{Z_k}\int[DA_\mu]\cdot\cdot[D\Phi_i]\bar{O}(x)\exp(iS_k) \tag{A8}$$

But:

$$\frac{1}{Z_k}\int[DA_\mu]\cdot\cdot[D\Phi_i]\bar{O}(x)\exp(iS_k) = \xi\frac{\partial W_k}{\partial\xi} + \frac{1}{2}\int d^4x\,\eta(x)\frac{\delta W_k}{\delta\eta(x)} \tag{A9}$$

$$= \xi \frac{\partial \Gamma_K}{\partial \xi} + \frac{1}{2}\int d^4x \, \frac{\delta \Gamma_K}{\delta \psi_c^*(x)} \psi_c^*(x)$$

At this stage specialize to the effective potential, which reduces (A8) to.

(A10)/(8) $$\xi \frac{\partial V}{\partial \xi} - \int d^4x \, \frac{\delta \Gamma(Ox)}{\delta K_j(0)} \Bigg|_{\substack{\phi_{jc}(z)=\phi_j \\ \text{OTHER FIELDS } 0}} \frac{\partial V}{\partial \phi_j} = -\frac{e v_i \phi_{ic} \xi}{\Omega} \cdot \int d^4x \int d^4z \, \frac{\delta \Gamma(Ox)}{\delta \psi_c^*(z)}$$

The mass identity follows by considering $\xi \frac{\partial}{\partial \xi} \frac{\delta^2 \Gamma}{\delta \phi_{1c} \delta \phi_{1c}}$ (and noticing a convolution). This gives, eventually :

$$\left(\xi \frac{\partial}{\partial \xi} + C(\phi,\xi)\frac{\partial}{\partial \phi}\right) \Delta^{-1}_{PHYS}(p^2)\Big|_{\phi=\phi_0} = 2\,\Delta^{-1}_{PHYS} \int d^4r\, e^{ip\cdot r} \int d^4z\, F(z,r)\Big|_{\phi=\phi_0}$$

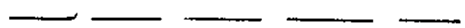
(A11) $$F(z, x-y) = \frac{\delta^2 \Gamma(Ox)}{\delta K_1(z)\, \delta \phi_{1c}(y)}\Bigg|_{\phi_{1c}=\phi}$$

At the potential minimum $\Delta^{-1}{}_{phys}(p^2)$, the inverse propagator for the physical Higgs, is zero, so we arrive at :

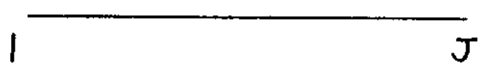
(A12) $$\left(\xi \frac{\partial}{\partial \xi} + C(\phi, \xi)\frac{\partial}{\partial \phi}\right) m^2 \Big|_{\phi=\phi_0} = 0$$

Appendix B

Propagators (For calculating $C^{(1)}$, $V^{(1)}$ and Σ)



$$\frac{i}{k^2 - e^2 \epsilon_{ij} \xi v_i \phi_j}$$

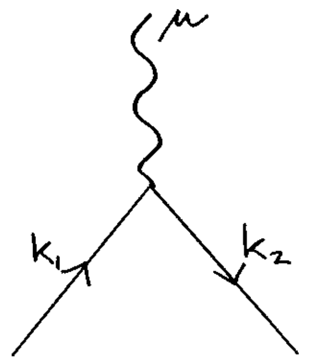


$$\frac{i(k^2 - \xi e^2 \phi^2)(\delta_{IJ} - n_I n_J)}{D_N} + \frac{i\, n_I n_J}{k^2 - m_1^2}$$

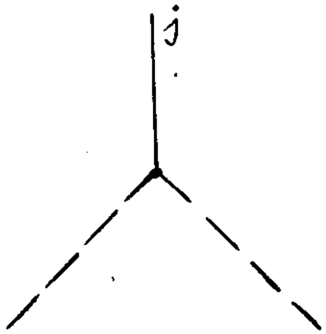
$$-iC\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - iD \frac{k_\mu k_\nu}{k^2}$$

$$C = \frac{1}{k^2 - e^2\phi^2} \qquad D = \frac{\xi(k^2 - m_2^2 - e^2 \xi v^2)}{D_N}$$

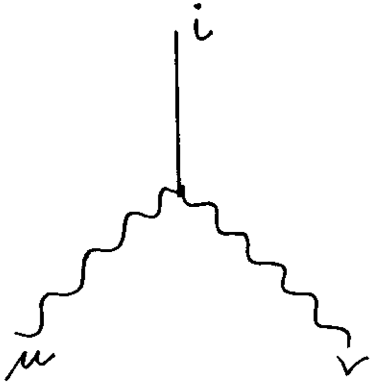
Vertices (Those used in calculation only)



$$-e\,\epsilon_{ij}(k_1+k_2)^{\mu}$$



$$-ie^2\xi V_i\,\epsilon_{ij}$$



$$2ie^2\phi_i\, g_{\mu\nu}$$

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