THE WESS–ZUMINO GAUGE IS A "GOOD" GAUGE

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It is shown, using a recent superfield formulation of Wess–Zumino gauges, that they are "good" gauges i.e. that they are consistent with translational invariance and hence may be used in effective potential calculations. This contrasts with the SUSY covariant gauge and the SUSY R_ξ gauge, which are "bad" gauges.

In a recent preprint [1] Kreuzberger et al. showed that it was possible to implement the Wess–Zumino gauge in a superfield formulation. They used the auxiliary (super)field method of gauge fixing [2] with chiral and antichiral auxiliary superfields B and $\bar{B}$ to give the gauge fixing term.

$$S_{\rm gf} = {\rm tr}\, \frac{1}{8}\int {\rm d}^4x\, {\rm d}^4\theta\, (BKV + \bar{B}\bar{K}V + \alpha\bar{B}B)\ , \qquad (1)$$

V is a vector superfield and K is chosen to be $F^\alpha {\rm D}_\alpha$ with

$$F^\alpha = -\bar{\theta}_{\dot\alpha}(\bar{\sigma}^\mu)^{\alpha\dot\alpha} f_\mu\ . \qquad (2)$$

$\alpha = 0$ imposes the homogeneous gauge conditions

$$\bar{\rm D}^2\bar{K}V = {\rm D}^2KV = 0\ , \qquad (3)$$

which gives rise to the following component field equations:

$$f_\mu v^\mu = 0\ ,$$
$$(f\cdot\partial)C = (f\cdot\partial)(M - {\rm i}N) = (f\cdot\partial)(M + {\rm i}N) = 0$$
$$(+\ \text{fermionic terms})\ . \qquad (4)$$

We have chosen to expand our vector superfield as

$$V = C + {\rm i}\theta\chi - {\rm i}\bar{\theta}\bar{\chi}$$
$$+ \tfrac{1}{2}{\rm i}\theta^2(M + {\rm i}N) - \tfrac{1}{2}{\rm i}\bar{\theta}^2(M - {\rm i}N) - (\theta\sigma^\mu\bar{\theta})v_\mu$$
$$+ {\rm i}\theta^2\bar{\theta}(\bar{\lambda} + \tfrac{1}{2}\bar{\sigma}^\mu\partial_\mu\chi) - {\rm i}\bar{\theta}^2\theta(\lambda + \tfrac{1}{2}\sigma^\mu\partial_\mu\bar{\chi})$$
$$+ \tfrac{1}{2}\theta^2\bar{\theta}^2({\rm d} + \tfrac{1}{2}\nabla^2 C)\ . \qquad (5)$$

Kreuzberger et al. then note that when f_μ is a suitable gauge condition for v_μ, $(f\cdot\partial)$ is the kinetic kernel of the Fadeev–Popov operator, which is invertible, so the conditions (4) are effectively $C = M = N = 0$, which are the Wess–Zumino gauge conditions. In this paper we shall call the $\alpha \neq 0$ case the Wess–Zumino like gauge.

We shall use the gauge-fixing (1) and the superfield techniques used by Piguet and Sibold [3] to investigate the gauge dependence of the effective action and effective potential. The result of these investigations is that the Wess–Zumino like gauges (and hence the Wess–Zumino gauge itself) are "good" gauges in the sense of Fukuda and Kugo [4], in other words, gauges for which it is possible to find translationally invariant minima of the effective action. Another way of stating this is that it is possible to derive the Nielsen identities for Wess–Zumino like gauges [5,6]. It is therefore possible to perform effective potential calculations in the Wess–Zumino like gauges, which contrasts with the case of SUSY covariant gauge-fixing, which, as observed by Miller [7], is not a "good" gauge.

We derive the Nielsen identities by introducing an extra BRS [8] transform on the gauge parameter α

$$\delta\alpha = \epsilon\beta\ , \quad \delta\beta = 0 \qquad (6)$$

(ϵ = BRS parameter, ϵ and β anticommuting). This gives rise to the following extra term in the action which is needed in order to maintain invariance under the extended set of transformations:

$$\frac{1}{16}\int d^4x\, d^4\theta(\beta C_- \bar{B} + \beta \bar{C}_- B) . \quad (7)$$

The other BRS variations are given by

$$\delta V = \epsilon \Big(C_+ - \bar{C}_+ + \sum_{n\geqslant 1} h_n [V, [V, [V, ... [V, C_+ - (-1)^n \bar{C}_+]]]] \Big) = \epsilon Q(V, C_+) ,$$

$$\delta C_+ = -\epsilon C_+ C_+ , \quad \delta \bar{C}_+ = -\epsilon \bar{C}_+ \bar{C}_+ ,$$

$$\delta C_- = \epsilon B , \quad \delta \bar{C}_- = \epsilon \bar{B} , \quad (8)$$

where the coefficients h_n may be derived from

$$\delta \exp(V) = \epsilon \exp(V) C_+ - \epsilon \bar{C}_+ \exp(V) . \quad (9)$$

The rest of the action, apart from (7) is given by

$$S = S_{\rm YM} + \frac{1}{8}\int d^4x\, d^4\theta (BF{\rm D}V + \bar{B}\bar{F}\bar{\rm D}V + \alpha \bar{B}B)$$

$$- \frac{1}{8}\int d^4x\, d^4\theta (C_- F{\rm D} + \bar{C}_- \bar{F}\bar{\rm D}) Q(V, C_+)$$

$$- \int d^4x\, d^4\theta\, \rho Q(V, C_+) - \int d^4x\, d^2\theta\, \sigma C_+ C_+$$

$$- \int d^4x\, d^2\bar{\theta}\, \bar{\sigma} \bar{C}_+ \bar{C}_+ , \quad (10)$$

where ρ and $\sigma, \bar{\sigma}$ are the sources coupled to the BRS variations of the fields and $S_{\rm YM}$ is given by

$$S_{\rm YM} = \frac{1}{32 g^2 k}\int d^4x\, d^2\theta\, [\bar{\rm D}\bar{\rm D} \exp(-gV) {\rm D}^\alpha \exp(gV) \times \bar{\rm D}\bar{\rm D} \exp(-gV) {\rm D}_\alpha \exp(gV)] , \quad (11)$$

with $k\delta^{bd} = f^{abc} f^{acd}$, where f is a structure constant. If we perform the BRS variations on the generating functional and Legendre transform the result we obtain the following Slavnov identity for the effective action Γ.

$$\int \left(\frac{\delta\Gamma}{\delta\rho}\frac{\delta\Gamma}{\delta V} + \frac{\delta\Gamma}{\delta\sigma}\frac{\delta\Gamma}{\delta C_+} + \frac{\delta\Gamma}{\delta\bar{\sigma}}\frac{\delta\Gamma}{\delta\bar{C}_+} + B_{c\ell}\frac{\delta\Gamma}{\delta C_-} + \bar{B}_{c\ell}\frac{\delta\Gamma}{\delta\bar{C}_-} \right) + \beta\frac{\delta\Gamma}{\delta\alpha} = 0 , \quad (12)$$

where the integral sign denotes the measure appropriate to the integrand (full, chiral or anti-chiral) and the subscrpt cℓ denotes a semiclassical field. We can write (12) symbolically as

$$S(\Gamma) + \beta\, \partial\Gamma/\partial\alpha = 0 . \quad (13)$$

If we differentiate eq. (13) with respect to β and then set $\beta = 0$, we find

$$S(-\partial\Gamma/\partial\beta) + \partial\Gamma/\partial\alpha = 0 . \quad (14)$$

For a gauge theory coupled to scalars this equation is the effective action precursor of the Nielsen identity. Just as in the ordinary Yang–Mills case, however, the Nielsen identity will only emerge if the $\int B_{c\ell}\delta\Gamma/\delta C_-$ and $\int \bar{B}_{c\ell}\delta\Gamma/\delta\bar{C}_-$ terms are zero. Carrying out the grassmanian integrations we see that this will be the case if

$$D^2 B_{c\ell} = \bar{D}^2 \bar{B}_{c\ell} = 0 , \quad (15)$$

which, on substituting for $B_{c\ell}$ and $\bar{B}_{c\ell}$ using their equations of motion, becomes

$$D^2 K V_{c\ell} = \bar{D}^2 \bar{K} V_{c\ell} = 0 . \quad (16)$$

This is identical in form to (3). For space–time independent semiclassical scalar fields and with the other fields set equal to zero to obtain the effective potential, (16) is obviously satisfied (see eq. (4)). The Wess–Zumino like gauge is therefore a "good" gauge. The equivalent condition for SUSY covariant gauge fixing

$$D^2 \bar{D}^2 V_{c\ell} = \bar{D}^2 D^2 V_{c\ell} = 0 , \quad (17)$$

gives rise to a component equation of the form [8]

$$d_{c\ell} + \nabla^2 C_{c\ell} = 0 , \quad (18)$$

which is not necessarily satisfied for space–time independent semiclassical fields.

To make these considerations a little more concrete we consider the component expansion for the gauge fixing in (1). We first eliminate B and $\bar{B}$ using their equations of motion (we have set $\beta = 0$) to give

$$S_{\rm gf} = -\frac{1}{8\alpha}\int d^4x\, d^4\theta\, (F{\rm D}V)(\bar{F}\bar{\rm D}V) , \quad (19)$$

which may be written in component form (dropping the fermionic bits) as

$$S_{\rm gf}=\frac{1}{8\alpha}\int d^4x\,[(f_\mu v^\mu)(\bar f_\nu v^\nu) - \tfrac{1}{2}\eta^{\mu\nu}(f_\mu M - i f_\mu N)(\bar f_\nu M + i\bar f_\nu N) + (f_\mu\partial^\mu C)(\bar f_\nu \partial^\nu C)]\ . \qquad (20)$$

If we now choose $f_\mu = 2i\partial_\mu$, we find

$$S_{\rm gf}=-\frac{1}{2\alpha}\int d^4x\,[(\partial_\mu v^\mu)^2+\tfrac{1}{2}M\nabla^2 M+\tfrac{1}{2}N\nabla^2 N + (\nabla^2 C)^2 + (\text{fermionic terms})]\ . \qquad (21)$$

For SUSY covariant gauge-fixing the equivalent is

$$S_{\rm gf}=-\frac{1}{2\alpha}\int d^4x\,[(\partial_\mu v^\mu)^2+(d+\nabla^2 C)^2+M\nabla^2 M + N\nabla^2 N + (\text{fermionic terms})]\ , \qquad (22)$$

where the d term gives rise to the problems with the effective potential.

If we couple our gauge theory to chiral matter superfields

$$\phi_i = A_i + i(\theta\sigma^\mu\bar\theta)\,\partial_\mu A_i + \tfrac{1}{4}\theta^2\bar\theta^2\nabla^2 A_i + \sqrt{2}\theta\psi_i - (i/\sqrt{2})\theta^2(\partial_\mu\psi_i)\sigma^\mu\bar\theta + \theta^2 F_i\ , \qquad (23)$$

the tree-level potential U is given in the Wess–Zumino like gauge by

$$U=\tfrac{1}{2}d^2+\sum \bar F_i F_i\ , \qquad (24)$$

whereas in the SUSY covariant gauge it is given by [8]

$$U=\tfrac{1}{2}[(\alpha-1)/\alpha]\,d^2+\sum \bar F_i F_i\ . \qquad (25)$$

It is also possible to avoid the difficulty of the gauge dependent d term that appears in the SUSY covariant gauge-fixing if one considers the superfield equivalent of an R_ξ gauge [9]

$$S_{\rm gf}=-\frac{1}{8\alpha}\int d^4x\, d^4\theta\,[D^2V^b+\tfrac{1}{2}g\alpha(D^2/\partial^2)(a_i^* T^b{}_{ij}\phi_j)] \times [\bar D^2 V^b + \tfrac{1}{2}g\alpha(\bar D^2/\partial^2)(a_i^* T^b{}_{ij}\phi_j)^*]\ , \qquad (26)$$

where a_i is the vacuum expectation value of ϕ_i and T^b is a generator of the symmetry group. The equivalent of (18) in the constant field limit is now

$$d^b_{\rm c\ell} = -g\alpha a^*_{i\ {\rm c\ell}} T^b{}_{ij} a_{j\ {\rm c\ell}}\ , \qquad (27)$$

but the equation of motion for a constant d field is

$$d^b = -\tfrac{1}{2}g a_i^* T^b{}_{ij} a_j\ , \qquad (28)$$

so (27) cannot be satisfied for general α and the SUSY R_ξ gauge is not a good gauge. However, if one replaces the α in the gauge-fixing by another parameter, say γ

$$S_{\rm gf}=-\frac{1}{8\alpha}\int d^4x\, d^4\theta\,[D^2V^b+\tfrac{1}{2}g\gamma(D^2/\partial^2)(a_i^* T^b{}_{ij}\phi_j)] \times [\bar D^2 V^b + \tfrac{1}{2}g\gamma(\bar D^2/\partial^2)(a_i^* T^b{}_{ij}\phi_j)^*]\ , \qquad (29)$$

one does have a good gauge if $\gamma=\tfrac{1}{2}$, in which the cross terms in the propagators still cancel for $\alpha = 1/2$.

One may check this explicitly by evaluating (29) for constant fields to find

$$S_{\rm gf}=\int d^4x\,[-(1/2\alpha)d^2-(g\gamma/\alpha)(a_i^* T^a{}_{ij}a_j)d^a - (g^2\gamma^2/2\alpha)(a_i^* T^a{}_{ij}a_j)^2]\ , \qquad (30)$$

which gives, using (28).

$$S_{\rm gf}=\int d^4x\,[-(1/2\alpha)d^2+(2\gamma/\alpha)d^2-(2\gamma^2/\alpha)d^2]\ . \qquad (31)$$

If this is to be equal to zero we must have $\gamma=\tfrac{1}{2}$.

The preceding examples should be compared with an 't Hooft type gauge fixing in an ordinary Yang–Mills theory coupled to scalars [6]. There one finds

$$U=(1/2\alpha)(v_i\phi_i)^2+\tfrac{1}{2}\mu^2\phi^2-(\lambda/4!)\phi^4\ , \qquad (32)$$

when one has a gauge fixing of the form

$$B(\partial_\mu A^\mu + v_i\phi_i)+\tfrac{1}{2}\alpha B^2\ . \qquad (33)$$

The problem of the gauge variant term appearing in (32) is eliminated if we demand (c.f. eq. (15)) $B_{\rm c\ell}=0$ which gives

$$v_i\phi_{i\ {\rm c\ell}} = 0\ .$$

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