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Parallel Haskell implementations of the N-body problem

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SUMMARY

This paper provides an assessment of high-level parallel programming models for multi-core programming by implementing two versions of the n-body problem. We compare three different parallel programming models based on parallel Haskell, differing in the ways how potential parallelism is identified and managed. We assess the performance of each implementation, discuss the sequential and parallel tuning steps leading to the final versions, and draw general conclusions on the suitability of high-level parallel programming models for multi-core programming. We achieve speed-ups of up to 7.2 for the all-pairs algorithm and up to 6.5 for the Barnes-Hut algorithm on an 8-core machine.

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KEY WORDS: high-level parallel programming models; functional programming; n-body problem;

1. INTRODUCTION

Modern multi-core architectures make the computational power of parallel hardware available to domain experts, who want to implement compute-intensive applications. However, most established parallel programming technology makes it challenging for non-specialists in parallel programming to exploit this potential. In contrast, high-level parallel programming models simplify the challenging tasks of parallel programming by offering powerful abstractions and by hiding the low level details of synchronisation and coordination from the programmer [30]. Writing parallel programs using these models usually involves only small changes to the sequential algorithm and therefore does not obscure the core computation. This comes, of course, at the expense of additional runtime overhead. Despite being high-level, the programming model should be detailed enough to allow for targeted performance tuning, enabling the domain expert to use algorithmic knowledge to improve the utilisation of multi- and many-cores.

Several high-level language models are built on the purely functional, non-strict programming language Haskell. While all of them are significantly more abstract than explicit, thread-based orchestration as in C+MPI, they vary in their support and flexibility. Completely implicit models of parallelism try to hide all aspects of parallelism from the programmer and rely on compilation and runtime system technology for parallelisation. Semi-explicit models of parallelism only require the programmer to identify and mark worthwhile parallelism. All aspects of synchronisation and communication are handled automatically by the runtime system, the compiler or library code. Explicit models of parallelism expose the notion of independent threads to the application programmer and provide means of explicit synchronisation.

This paper considers three current variants of parallel Haskell including: the semi-explicit GpH [35, 51, 34], which provides basic primitives for introducing parallelism and controlling the order and degree of evaluation; the semi-explicit Eden [32], which identifies parallelism through the notion of processes and is implemented for distributed-memory architectures; and the explicit Par monad [36], which provides a new programming model for explicit, deterministic parallel programming in Haskell.

To compare these models we implement the *n*-body problem which represents an important class of problems in many areas of science including molecular dynamics and astrophysics. It is the core of a real application and representative for a wide range of applications. As a baseline for comparison, we first implement the naive all-pairs algorithm exploiting obvious data-parallelism and tune its parallel performance. Then we implement the more complex but efficient Barnes-Hut algorithm which presents more challenge in getting good parallel performance.

The parallel implementations are based on a proven methodology we have developed for languages with semi-explicit parallelism on the basis of several symbolic computations in the past [31]. Notably, it makes use of a rich set of pre-defined parallel patterns, or skeletons [11], that encode optimised, parallel patterns of computation as higher order functions.

The work in this paper contributes to the wider effort put together by participants in the SICSA Multicore Challenge workshop[*] where parallel implementations of the naive algorithm in a variety of languages were presented. The results varied from language to language. Our aim is to look into implementations using different parallel models in Haskell and evaluate which one is best suited for this particular problem. The comparison takes into account the usability of the models and the required tunings to get the optimal parallel implementation.

The structure of the paper is as follows: Section 2 looks at the *n*-body problem description and the two methods we select to solve the problem. Section 3 covers the three parallel programming models, their implementations, and functions, both low-level and user-level. Sections 4 and 5 look at the sequential and parallel implementations respectively. The performance of each model is discussed in Section 6. We cover related work in the area in Section 7 before concluding in Section 8.

All the Haskell source code and link to paper is available online. [†]

2. THE PROBLEM

The *n*-body problem is the problem of predicting the motion of a system of *N* bodies that interact with each other gravitationally. In astrophysics, the bodies are galaxies or stars, and the movement of the bodies are affected by the gravitational force. The computation proceeds over time steps where the acceleration of each body with respect to the others is calculated and then used to update the velocity and position in each iteration. We look at two commonly used methods of solving the problem: the first method is used mainly for simulation consisting of up to a few thousands number of bodies while the second method is best suited for system of large number of bodies, for example, interactions between molecules in biological systems.

2.1. *All-Pairs Algorithm*

The all-pairs method is the traditional brute-force technique in which the pair-wise accelerations between each body and the remaining are calculated. The body-to-body comparison requires a time complexity of $O(N^2)$ to complete and thus making it convenient for only a small number of bodies. It is relatively simple to implement in an imperative language using a double nested loop with *inplace* update.

In pure Haskell, the absence of *destructive update* requires a different approach for implementation. For instance, the use of recursive function to achieve the same effect as a loop. Alternatively, a nested map function can also be employed to get the same behaviour.

2.2. *Barnes-Hut Algorithm*

The Barnes-Hut algorithm [2] is a widely used approximation method for the *n*-body problem. It is particularly useful for systems consisting of a large number of bodies where an approximate

[*]SICSA Multicore Challenge: *n*-body computation.
http://www.macs.hw.ac.uk/sicsawiki/index.php/Challenge_PhaseII
[†]Source code and paper link:
http://www.macs.hw.ac.uk/~dsg/gph/papers/abstracts/sicsamcc12.html

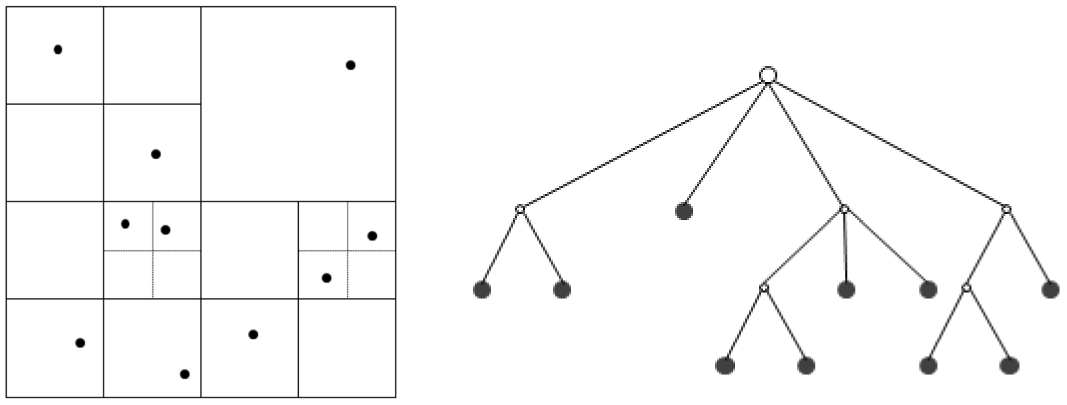

(a) Physical location of points in 2D space (b) Tree representation

Figure 1. 2D Barnes-Hut simulation: Bodies (or points) are contained in a region (bounding box) which is sub-divided recursively into smaller sub-regions.

but efficient solution is more feasible than the naive approach. The Barnes-Hut algorithm proceeds by dividing the region containing the bodies into sub-regions (four for 2D or eight for 3D space) and representing them as nodes in a tree. This division into sub-regions continues recursively until each sub-region contains zero or one body. This process accounts to the tree construction stage of the algorithm, in which the centre of mass and total mass of each region are calculated and stored in the inner nodes. The second stage involves traversing the tree for each body and computing the forces against other bodies directly, or approximating bodies that are too far away, which is determined by threshold, using the region centre of mass. Depending on how dispersed the bodies in the system are, the tree construction phase will usually lead to not-so-well balanced structure which makes achieving good parallel performance difficult. In our implementation, we use random input generation which ensures a fairly balanced tree.

3. TECHNOLOGY

While the problem has been implemented across a wide variety of programming languages covering many paradigms, using a functional approach raises the level of abstraction by making it easier to express the problem, for example, through the use of higher-order functions. As a pure functional language, Haskell is free of many of the inherent problems associated with imperative programming e.g it disallows side effects which leads to *referential transparency*, thus making it easier to parallelise programs.

Haskell supports concurrency by providing a set of primitives to spawn explicit IO threads that are executed in an interleaved fashion [43]. Concurrent execution of threads in this model is non-deterministic and can result in race conditions and deadlocks if proper synchronisation is not implemented. This paper does not deal with concurrency but rather pure parallelism to speed up a program. However, we will shortly see how the Par monad builds on top of concurrent primitives to deliver a new deterministic model for parallel computation. In this section, we cover GpH, the Par monad and Eden as parallel programming models in Haskell.

There are other models for parallel computation in Haskell which we do not use in our implementations. For instance, Data Parallel Haskell (DPH) provides a model of nested data parallelism. An implementation of the Barnes-Hut algorithm using DPH, which covers flattening transformation techniques to improve performance, is given in [25]. This model is particularly well-suited for irregular parallelism. Another example is Cloud Haskell, which is a distributed programming model building on the same principle as the highly successful Erlang programming language.

3.1. Parallelisation Methodology

We follow guidelines established in [31, Sec 3] with some flexibility. The approach we use is as follows:

- **Sequential implementation:** Start with the initial sequential algorithms;
- **Sequential optimisation:** Optimise the sequential algorithms e.g. improve heap consumption by identifying any space leaks and fix them; use tail recursive functions; and add strictness where necessary;
- **Time profile:** To point out “big eaters”, i.e. parts of the programs that are compute-intensive;
- **Top-level parallelisation:** Parallelise top-level data independent operations e.g. map functions representing data-oriented parallelism, and independent tasks representing task-oriented parallelism, using high-level constructs provided in the parallel programming model;
- **Parallel execution and initial results:** Run parallel programs on multi-core machine or cluster to obtain initial results;
- **Parallel tuning:** To achieve better runtimes and speed-ups, use more advanced and explicit features of the model. This step also looks at scalability of the parallel programs by tuning for varying/increasing input sizes.

3.2. GpH: Glasgow parallel Haskell

GpH extends Haskell with two basic primitives to enable semi-explicit parallelism: `par` for parallel composition and `pseq` for sequential composition [51, 34, 35]. The `par` primitive “sparks” its first argument, i.e. it records it to potentially be evaluated in parallel. The `pseq` primitive evaluates its first argument to WHNF (Weak Head Normal Form) before continuing with the evaluation of its second argument and thus enforces sequential ordering. Both primitives return their second argument as the result.

```haskell
-- parallel composition
par :: a -> b -> b
-- sequential composition
pseq :: a -> b -> b
```

For parallel execution, the program needs to be compiled with the `-threaded` option. The runtime option `-Nx` needs to be specified where `x` represents the number of processors.

Sparks are added to a spark pool and are taken up for execution by lightweight Haskell threads which in turn are mapped down to the underlying OS threads. Creating sparks using `par` is cheap, amounting to just a pointer, and thousands of them can be created. Converted sparks represent parallelism extracted from the algorithm, incurring the usual thread creation overhead.

By only using the primitives, it is often possible to specify parallelism in a wrong way, or even obscure the code. *Evaluation strategies* provide further abstraction over this level of programming by separating the coordination and computation concerns. An evaluation strategy is basically a function of type `a -> Eval a` that is executed for coordination effects.

```haskell
data Eval a = Done a

runEval :: Eval a -> a
runEval (Done x) = x

type Strategy a = a -> Eval a

rseq, rpar :: Strategy a
rseq x = x `pseq` Done x
rpar x = x `par` Done x

using :: a -> Strategy a -> a
x `using` strat = runEval (strat x)

-- applying strategy e.g.
somefunc strat = someexpr `using` strat
```

The basic strategies `rpar` and `rseq` are defined directly in terms of their primitives. The `using` function applies a strategy to an expression. Since all parallelism, evaluation order and evaluation degree are specified within the `Eval` monad, an explicit `runEval` is used at the point where it is applied to a concrete expression. Using a monad helps to separate the purely functional aspects of the execution from the behavioural aspects of the execution. It also allows the programmer to use rich sets of libraries and abstractions available for monads in Haskell.

Strategies can be composed just like functions using the `dot` strategy combinator e.g. `(rpar `dot` rdeepseq)` sparks parallel evaluation of its argument and completely evaluates it to normal form. In this example `rdeepseq` is used to specify the evaluation degree. The expressive power of evaluation strategies comes from the ability to compose them, as above, to separate the specification of parallelism from evaluation degree and other parallelism optimisations such as clustering, as we will see later, and the possibility to nest strategies, by providing other strategies as arguments, exploiting higher-order functions in the language.

Skeletons: A number of skeletons are implemented as higher-order functions e.g. parallel map, pipeline and divide&conquer. Parallel map is the most common one which specifies data-oriented parallelism over a list.

```haskell
-- parallel map definition
parMap strat f xs = map f xs `using` parList strat

xs = parMap rdeepseq f [10..25]
```

In the version above, `parMap` exposes the maximal parallelism, creating a spark for each item to be evaluated in parallel. In Section 5 we will discuss several techniques for improving parallel performance by generating fewer, more coarse-grained threads.

3.3. *Par monad*

The Par monad offers a new parallel programming model which is implemented entirely as a Haskell library [36]. Programming in the Par monad looks a lot like programming in Concurrent Haskell but it preserves determinism and is side-effect-free. Par is simply a type declared as a monad. `IVars` are used for communication, an implementation of the I-Structures, a concept from the pH and Id languages [40]. The basic operations use an explicit approach to specify parallel computations. Parallelism is introduced using `fork` which creates a parallel task. Tasks are scheduled using an optimised parallel scheduler among threads. The computation in the monad is extracted using `runPar`. The communication constructs include the following functions:

- `new` to create a new `IVar`.
- `put` to place some value in the `IVar`. It is executed once per `IVar` otherwise an error occurs.
- `get` to retrieve the value from the `IVar`. It is a blocking operation and waits until something is put into the `IVar`.

The derived `spawn` function hides the explicit `put` and `get` operations and therefore ensures that each `IVar` created is only ever put into once. This raises the level of abstraction provided by this model.

```haskell
runPar :: Par a -> a
fork :: Par () -> Par ()
spawn :: NFData a => Par a -> Par (IVar a)

-- communication
data IVar a
new :: Par (IVar a)
put :: NFData a => IVar a -> a -> Par ()
get :: IVar a -> Par a
```

As the library is fairly recent, it has a limited number of higher-level function abstractions. The most obvious being a parallel map implementation, `parMap`. Work on more abstractions is in progress.

3.4. Eden

Eden extends Haskell by providing constructs for parallel *process definition*: abstracting a function that takes an argument and produces a value into a process with input and output that correspond to the function argument and result respectively; and *process instantiation*: evaluating the process in parallel [32].

```haskell
-- process definition
process::(Trans a,Trans b) => (a->b) -> Process a b
-- process instantiation
(#)::(Trans a,Trans b) => Process a b -> a -> b
```

Building on these coordination constructs, a parallel function application operator and eager process creation function are derived. `spawn` is only denotationally specified, ignoring demand control.

```haskell
-- parallel function application
($#)::(Trans a,Trans b) => (a->b) -> a -> b
f $# x = process f # x

-- eager process creation
spawn::(Trans a,Trans b) => [Process a b] -> [a] -> [b]
spawn = zipWith (#)
```

The parallel runtime system distributes the processes to the available processors. Since Eden has been designed for a distributed-memory model, processes communicate messages to each other to provide input and retrieve output. All this synchronisation and coordination is handled implicitly by the runtime system. The programmer does not need to worry about low-level send and receive between parallel processes, and only uses process abstraction or skeletons built on top of these. Eden processes produce output eagerly with the argument to the process being evaluated locally in the parent process before sending. Lists are handled as streams and are sent element-by-element. This can cause significant overhead and techniques to avoid element-wise streaming are used.

EdenSkel: Eden provides a rich set of higher-order functions that abstract common parallel patterns in its skeleton library EdenSkel. For instance, an initial implementation of parallel map uses `spawn` to eagerly instantiate a list of process abstractions. The `parMap` skeleton creates a process for each list element and this often results in far too many processes in comparison to the number of processing elements available.

Farm process: The farm process skeleton adapts the number of processes to the number of available processing elements (given by `noPe`). The input list is grouped into *noPe* sublists and then a process is created for each sublist instead of each individual element. Implementing the farm process, the `parMapFarm` skeleton provides a simpler interface and a familiar name to the programmer who specifies `unshuffle` as the distribution function and `shuffle` as the combination function. `parMapFarm` creates (*noPe*+1) processes in total with *noPe* farm processes and one main process which means that one machine will be allocated two processes. A slight variation of this skeleton, implemented as `parMapFarmMinus`, creates *noPe-1* processes so that each processor gets exactly one process.

Chunking input stream: The farm process reduces the number of processes but does not have any effect on the messages exchanged between the processes. Each element of the list is sent as a single message by default. To improve process communication, the number of messages is reduced using a chunking policy. `parMapFarmChunk` is defined as a new skeleton which decomposes the input list into chunks of a specified size e.g. 1000 then creates the farm processes and distributes the chunks to them. This reduces communication overhead.

```haskell
-- parallel map definition in Eden
parMap f = spawn (repeat (process f))

-- using farm process
```

```
parMapFarm f = shuffle . (parMap (map f)) . (unshuffle noPe)

-- and chunking input stream
parMapFarmChunk f xs = concat (parMapFarm (map f) (chunk size xs))
```

Offline processes: Parallel map implemented using offline processes modifies the communication behaviour where the unevaluated data, when typically smaller than the evaluated result, is sent to be evaluated lazily by the new process instead of being evaluated by the parent process. This reduces the combined effort of the main process having to completely evaluate all input to the farm processes.

```
-- x is strictly reduced and sent to child process
f $# x

-- parameter passing: input serialised and sent to remote PE
(\() -> f x) $# ()
```

3.5. Summary

In summary, all three models provide high-level constructs which can be used to get initial parallelism from the sequential algorithm. Both GpH and Eden represent semi-explicit parallelism, without an explicit notion of threads. Both implementations delegate the coordination of the parallelism to a sophisticated runtime system, in the case of Eden, based on a distributed-memory model, and in the case of GpH, based on a (virtual) shared-memory model. Under closer examination, Eden can be considered slightly more explicit since process instantiation mandates the generation of parallelism, whereas all parallelism in GpH is advisory and may be discarded by the runtime system. In contrast, Par monad is explicit in creating parallel threads, using `spawn`, and it is up to the programmer to coordinate the parallel threads, using `IVars` on a physical shared-memory system.

Skeletons: To further raise the level of abstraction, all three languages provide skeleton libraries. The most advanced of these libraries is the EdenSkel library. It uses internal primitives of the Eden implementation to realise sophisticated topology skeletons, such as rings and tori, as well as optimised, high-level algorithmic skeletons for common patterns such as data parallelism, divide-and-conquer etc. Based on a long history of developed and comparative applications to a range of symbolic applications [30], it represents the best tuned skeleton library of these.

The abstraction provided for GpH are evaluation strategies, which have proven to be very flexible and easily composable in describing patterns of parallelism [35]. They provide the least intrusive way of specifying parallelism, achieving the clean separation between coordination and computation.

Par monad is the youngest of these systems, and thus, it comes with only a small set of skeletons, most notably ones for data-parallelism. Being the most explicit of these languages, it provides the highest level of control to the skeleton programmer. This is reflected in a carefully tuned, work-inlining scheduler, that aims to minimise the parallelism overhead in massively parallel programs.

Runtime system: The most significant difference between these three models is on the runtime system level. GpH implements parallelism on top of a (virtual) shared-memory model. It is implemented very efficiently on physical shared-memory systems, via the GHC-SMP runtime system, which is part of the main release of GHC. It is also implemented on distributed-memory architectures, via the GHC-GUM runtime system and explicit message passing. Eden has been designed for distributed-memory architectures from the start but can also be used on shared-memory machines by exploiting an optimised implementation of MPI or custom shared-memory implementation of internal communication. This generality of design, however, bears the danger of runtime overhead due to duplication of data in different memory locations. The GHC-Eden runtime system is the only stable release of distributed-memory parallelism for Haskell at the moment. The implementation of Par monad supports only physical shared-memory systems. In contrast to GpH and Eden, it does not require any runtime system extension and is implemented entirely as a library. One advantage of this design is the decoupling from any internal changes to the GHC compile chain or runtime system. As of the time of writing, the latest official GHC-based releases are 7.4 for GHC-SMP and 6.12.3 for GHC-Eden. A release candidate for GHC-Eden 7.4 and an unstable version of GHC-GUM 6.12.3 exist.

Because of the different advantages for the individual systems, a combination of some or all of them is desirable in particular for heterogeneous, hierarchical networks of multi-cores. The development of such a merged system is currently being pursued by several research groups.

4. SEQUENTIAL IMPLEMENTATION

In this section, we discuss the sequential implementation of the two algorithms. Starting with a naive initial implementation, we perform several sequential tuning steps to optimise both the all-pairs and the Barnes-Hut algorithm. We consider the problem in three dimensional space. The runtimes reported exclude the generation of input and show the main part of the program that performs the core computation.

4.1. All-Pairs

The all-pairs program proceeds by comparing each body with the rest in order to calculate the accelerations induced by the other bodies. The accumulated accelerations are deducted from the body's initial velocity and the position of the body is updated to reflect its movement. Two algebraic data types are defined to represent a body and the acceleration.

Data types definition

```haskell
data Body   -- body type def
  = Body {  x::Double,   y::Double,   z::Double,
           vx::Double,  vy::Double,  vz::Double,
            m::Double }

data Accel  -- acceleration type def
  = Accel { ax::Double, ay::Double, az::Double }
```

The main part of the program is implemented using two map functions, simulating a double nested loop. The top-level map function applies the composite function `(updatePos . updateVel)` to each body in the list `bs`. The main computation happens in the `updateVel` function, which has another map function to calculate the accelerations against all the bodies. The fold function deducts the accelerations which gives the updated velocity. The code below shows the computation for one iteration.

All-Pairs main iteration

```haskell
doSteps::Int->[Body]->[Body]
doSteps 0 bs = bs
doSteps s bs = doSteps (s-1) new_bs
  where
    new_bs = map (updatePos . updateVel) bs

    updatePos (Body x y z vx vy vz m) = Body (x+timeStep*vx) (y+timeStep*vy) (z
        +timeStep*vz) vx vy vz m

    updateVel b = foldl deductChange b (map (accel b) bs)

    deductChange (Body x y z vx vy vz m) (Accel ax ay az) = Body x y z (vx-ax)
        (vy-ay) (vz-az) m

    accel (Body ix iy iz ivx ivy ivz imass) (Body jx jy jz jvx jvy jvz jmass) =
         Accel (dx*jmass*mag) (dy*jm*mag) (dz*jm*mag)
      where
        mag = timeStep / (dSquared * distance)
        distance = sqrt (dSquared)
        dSquared = dx*dx + dy*dy + dz*dz + eps
        dx = ix - jx
        dy = iy - jy
        dz = iz - jz
```

4.2. Barnes-Hut

The Barnes-Hut implementation is based on the 2D version of the algorithm discussed in [6] and [42]. The program consists of two main steps which are broken down into smaller functions. In addition to the `Body` and `Accel` types used in the all-pairs version, three new data types are defined to represent:

1. `BHTree` the Barnes-Hut tree structure;
2. `Bbox` the bounding box representing a region in 3D space, and
3. `Centroid` the centroid of a region.

The `BHTree` data type implements a rose tree where each node can have an arbitrary number of sub-trees. In our case of modelling a 3D space, this will not be more than 8 children per node, thus an oct-tree. The node of the tree consists of the size of a region, the centre of mass, total mass and sub-trees.

Additional types for Barnes-Hut

```
data BHTree
  = BHT { size::Double, centerx::Double, centery::Double,
       centerz::Double, totalmass::Double, subTrees::[BHTree] }

data Bbox
  = Bbox { minx::Double, miny::Double, minz::Double,
           maxx::Double, maxy::Double, maxz::Double }

data Centroid
  = Centroid { cx::Double, cy::Double, cz::Double, cm::Double }
```

The algorithm proceeds in two main phases:

tree construction first an oct-tree is constructed from the list of bodies (`buildTree`);

force calculation then all forces between bodies are calculated to update the velocities (`updateVel`) and positions (`updatePos`).

Tree construction: This phase constructs an oct-tree from the list of bodies, shown in the `buildTree` function below. The bounding box representing the lower and upper coordinates of the region containing all the points is found (`findBounds`) and the size of the region is calculated. The centre of mass (`cx`, `cy`, `cz`) and total mass (`cm`) are calculated and stored at the root node of the tree to represent the whole space.

The centre of mass (R) and total mass (M) of a list of bodies (m) are given by:

$$M = \sum_{i=1}^{n} m_i \quad \text{where } n \text{ is number of bodies} \tag{1}$$

$$R = \frac{1}{M} \sum_{i=1}^{n} m_i \times r_i \tag{2}$$

The bounding box is used to subdivide the bodies into 8 smaller regions (`splitPoints`) and then the centre of mass and total mass of the bodies contained in each region are computed in the same way and stored in the children nodes. The process continues until a region has no body in it — `buildTree` is essentially a recursive function. The actual bodies are not stored in the tree structure as in some implementation as the centre and total mass are calculated in the tree construction phase.

Barnes-Hut tree building

```haskell
doSteps 0 bs = bs
doSteps s bs = doSteps (s-1) new_bs
  where
    bbox = findBounds bs
    tree = buildTree (bbox,bs)
    new_bs = map (updatePos . updateVel) bs

-- build the Barnes-Hut tree
buildTree :: (Bbox,[Body])->BHTree
buildTree (bb,bs) = BHT size cx cy cz cm subTrees
  where
    subTrees =  if bs <= 1 then []
                else map buildTree (splitPoints bb bs)
    Centroid cx cy cz cm = calcCentroid bs
    size = calcBoxSize bs

findBounds::[Body]->Bbox
-- split bodies into subregions
splitPoints::Bbox->[Body]->[(Bbox,[Body])]
-- calculate the centroid of points
calcCentroid::[Body]->Centroid
-- size of the region
calcBoxSize::Bbox->Double
```

Force calculation: In this phase, the acceleration due to each body is computed by traversing the tree, shown in the `calcAccel` function below.

The traversal along any path is stopped as soon as a node is too far away to make a significant contribution to the overall force (`isFar`). This is determined by dividing the size s of the region by the distance d between the body and the node centre of mass coordinates. If the ratio $\frac{s}{d}$ is less than a certain threshold t where $0 < t < 1$ then the centroid is used as approximation as the point is far from the region. Setting t to zero degenerates to a brute force version while increasing the value improves performance at the expense of losing some precision.

The `accel` function here differs from the one in the all-pairs version in that instead of calculating the acceleration between two *bodies*, it uses the *centroid* of a region and a body. The `updateVel` function deducts the net acceleration due to each body.

Barnes-Hut acceleration calculation

```haskell
updatePos (Body x y z vx vy vz m) = ... -- same as allpairs

updateVel b@(Body x y z vx vy vz m) = Body x y z (vx-ax) (vy-ay) (vz-az) m
  where
    Accel ax ay az = calcAccel b tree

calcAccel::Body->BHTree->Accel
calcAccel b tree@(BHT _ _ _ _ subtrees)
    | null subtrees      = accel tree b
    | isFar tree b       = accel tree b
    | otherwise = foldl addAccel (Accel 0 0 0) (map (calcAccel b) subtrees)
      where
        addAccel (Accel ax1 ay1 az1) (Accel ax2 ay2 az2) = Accel (ax1+ax2) (ay1
            +ay2) (az1+az2)

accel::BHTree->Body->Accel
isFar::BHTree->Body->Bool
```

4.3. *Sequential tuning*

A number of sequential optimisation techniques are used for improving the runtime, heap usage and fixing issues like stack overflow.

Optimisations: The following general optimisations apply to both algorithms:

Reducing stack consumption: The naive implementation of the algorithm suffers from an excessive stack consumption if huge number of bodies are used. Space profiling helps to understand the memory usage of each algorithm and to find any space leak, which led to a stackoverflow in the initial implementation. To fix the problem and improve general performance of the sequential algorithm, the following steps are taken:

- *Tail recursion:*
  Two general, well known techniques for reducing stack consumption in functional languages are to make the function tail recursive and to use accumulating parameters.
- *Strictness:*
  More specifically to a lazily evaluated language such as Haskell, strictness annotations can be added, where delaying evaluation is not necessary thus avoid unnecessary thunking of computations. This is achieved in a number of ways e.g. using the `pseq` primitive, the strict function application (`$!`) or strictness annotation (`!`) from the BangPatterns extension.
- *Types Definition:*
  Initially, type synonyms are used to represent position, velocity and mass as triple tuple, e.g. `type Pos = (Double,Double,Double)`. Through the use of more advanced data types provided in Haskell e.g. with strict data fields, space leaks can be avoided and this improves performance considerably with high input sizes as we will see shortly.

Reducing heap consumption: While the GHC compiler performs numerous automatic optimisations, more opportunities can be exposed by specific code changes, in particular in fusing function applications where necessary. This removes any intermediate data structures that could potentially decrease performance.

```haskell
-- A trivial example
map updatePos (map updateVel bs)

-- rewritten using function composition
map (updatePos . updateVel) bs
```

The composed version using a single `map` is easier to read and also does not depend on compiler optimisation. The use of `foldr` in conjunction with list comprehension (`foldr/build`) also eliminates the intermediate lists produced by build and consumed by foldr. This is used in the Barnes-Hut tuning.

Quantifying sequential tuning: Table I shows the results of each step of tuning the sequential all-pairs and Barnes-Hut algorithms.

All-Pairs

- *Version 1:* the initial version of the all-pairs program uses type synonyms/tuples to represent position, velocity, mass and acceleration. Type synonyms are not new data types but are used mainly for code clarity. For example, it is easy to read that a position consists of 3 doubles, thus representing it using a tuple.

```haskell
type Pos   = (Double,Double,Double)
type Vel   = (Double,Double,Double)
type Mass  = Double
type Accel = (Double,Double,Double)
```

- *Version 2:* change type synonyms to data types. This causes the initial runtime to go up by 60%. Using type synonym is usually more efficient as it incurs one less indirection than data type. However, data types are more powerful and can be used with further optimisation as we will see in the following versions. Data type necessitates deriving appropriate typeclasses e.g. `Eq` if we need to be able to compare them.

```haskell
data Pos   = Pos Double Double Double
data Vel   = Vel Double Double Double
```

Table I. Sequential tuning

(a) All-Pairs (5000 bodies)

Version	Runtime (s)	Max Resi (KB)	Heap Alloc (%)
allpairs1	39.47	1976	+0.0
allpairs2	63.24	3533	+30.9
allpairs3	2.54	1273	-77.9
allpairs4	2.15	726	-28.9
allpairs5	2.14	726	-0.0
allpairs-final	1.94	69	-0.0

(b) Barnes-Hut (80000 bodies)

Version	Runtime (s)	Max Resi (KB)	Heap Alloc (%)
bh1	41.90	28	+0.0%
bh-final	33.37	27	-88.6%

```haskell
data Mass   = Mass Double
data Accel = Accel Double Double Double
```

- *Version 3:* add strictness to avoid unnecessary thunking of computation. For example, return type of the `accel` function below is `Accel`. By default the data fields of `Accel` are evaluated lazily, explaining why Version 2 takes up a lot of memory space. By making them strict, they are computed eagerly. The clearer way to achieve this is by using the (!) strictness annotation instead of the most explicit `pseq`. As the results show, this step accounts for the main reduction in heap and time by 78% and 96% respectively.

```haskell
-- data fields evaluated lazily
accel bodyi bodyj = Accel (dx*jm*mag) (dy*jm*mag) (dz*jm*mag)
-- add strictness annotation (!)
accel bodyi bodyj = Accel ax ay az
  where
    !ax = (dx*jm*mag)
    !ay = (dy*jm*mag)
    !az = (dz*jm*mag)
```

- *Version 4:* Strict data fields. Making the data fields strict removes the need for the previous strictness annotation added inside the function. Additionally, the use of the `UNPACK` pragma indicates to the compiler to unpack the contents of the constructor field into the constructor itself, removing a level of indirection.

```haskell
data Pos = Pos {-# UNPACK #-} !Double {-# UNPACK #-} !Double {-# UNPACK #-}
      !Double
```

- *Version 5:* use appropriate higher order functions from the standard libraries e.g. use `foldl'` instead of `foldl` for its strictness properties.
- *Final version:* use single `Body` data type. This removes the need to deal with many different data types, makes the program more compact, and as a result, reduces the runtime by 10%. The maximum residency sees an important decrease.

```haskell
data Body
      = Body
        { x   :: {-# UNPACK #-} !Double   -- pos of x
        , y   :: {-# UNPACK #-} !Double   -- pos of y
        , z   :: {-# UNPACK #-} !Double   -- pos of z
        , vx  :: {-# UNPACK #-} !Double   -- vel of x
        , vy  :: {-# UNPACK #-} !Double   -- vel of y
        , vz  :: {-# UNPACK #-} !Double   -- vel of z
        , m   :: {-# UNPACK #-} !Double } -- mass
```

		allpairs1		allpairs-final	
Optimisation		Runtime (s)	Max Resi (KB)	Runtime (s)	Max Resi (KB)
`-O0`	(disable optimisations)	48.26	1667000	18.58	71
`-O1`	(standard optimisations)	3.76	1094	0.31	69
`-O2`	(full optimisations)	3.72	1117	0.31	69

Table II. Effect of compiler optimisation (2000 bodies)

Barnes-Hut In addition to the optimisations applied to all-pairs, the main tuning for Barnes-Hut sequential performance is the use of `foldr/build`.

- *Version 1:* The first Barnes-Hut version includes all all-pairs optimisations detailed earlier. This gives a good initial runtime.
- *Version 2:* Use `foldr/build` in the `calcAccel` function which eliminates intermediate lists produced by `build` and consumed by `foldr`.

```haskell
-- before
foldl' addAccel (Accel 0 0 0) [calcAccel st b | st <- subtrees]
-- after
foldr addAccel (Accel 0 0 0) [calcAccel st b | st <- subtrees]
```

Compiler optimisation: The sequential runtimes up to now are based on fully optimised code and on an input size of 5000 bodies for the all-pairs program. GHC's automatic optimisation already manages to improve time and heap performance significantly. Table II shows that without enabling compiler optimisation, the runtime may be very high, and actually too high that we use 2000 bodies here to show the difference in optimisation. The table shows that GHC's aggressive optimisation machinery manages to automatically improve performance of the final version by a factor of 13.0, relative to the unoptimised version, but does not find many more optimisation sources when going to full optimisation. Algorithmic optimisation, represented by all the sequential tuning steps as discussed in this section, gives a performance gain of a factor of 12.0 (combined with full optimisations). Both taken together account for a sequential speed-up of 155.7.

Baseline comparison: We use the *n*-body benchmark from the language shootout website [‡] as baseline for comparing with our sequential all-pairs program. We are particularly interested with the shootout Haskell version, which implements the same all-pairs algorithms, as well as the Fortran, which is the fastest, and C versions. Using 50 million iterations and 5 bodies, the Haskell version is slower by a factor of 1.8 and 1.2 compared to the Fortran and C version respectively.

We adapt the shootout Haskell version to use the same input as we use for our all-pairs program, i.e. 16000 bodies and 1 iteration, so that we have a head-to-head comparison between these two Haskell implementations. We note that our all-pairs program runs at 20.14s compared to 12.55s for the shootout, thus slower by a factor of 1.6. However, it is worth pointing out that the shootout version, though implementing the same algorithm, is highly optimised by expert programmers in Haskell and the GHC compiler, and makes use of inplace update operations organised through monadic code and unsafe operations like `unsafePerformIO`. The disadvantage of this approach is that it introduces sequentialisation in the code as part of the optimisation and therefore loses a lot of potential for parallelism.

As a baseline comparison, we hereby observe a sequential overhead of a factor of 2.9 against the fastest available all-pairs program on the language shootout. By remaining faithful to a purely functional programming model, our implementation provides opportunities for parallelism, that do not exist in the lower-level implementations and have to be refactored in a time-consuming and error prone parallelisation methodology. By exploiting high-level parallelism, we can compensate for the

[‡]The Computer Language Benchmarks Game [21] `http://shootout.alioth.debian.org/`

sequential overhead, using a fairly small number of processors, and achieve high scalability of our code, through the usage of more massively parallel hardware. In particular, the final parallel program will not be tied to one particular class of architectures, nor to a certain number of processors.

5. PARALLEL IMPLEMENTATION

By using the high-level constructs and functions implementing common skeletons in each programming model, the sequential algorithm is expected to require little changes to get initial parallel versions of both algorithms. We present results from the initial implementations and show, through a set of parallel performance tuning steps, how runtimes and speedups are improved. The experimental setup for these results are given in Section 6 where we also evaluate the performance of all models.

Time profiling: Identifying the main source of parallelism is the first step in the parallel implementation. Time profiling points out the "big eaters," that is, functions that take the largest percentage of the total time. Listing 1 shows the time and allocation profiling for both algorithms.

Listing 1: Time and Allocation Profiling Report

```
                                          individual      inherited
COST CENTRE        MODULE    entries   %time  %alloc    %time  %alloc

-- All-Pairs (2000 bodies, 1 iteration)

doSteps             Main           1    0.0     0.0    100.0    99.8
 updatePos          Main        2000    0.0     0.0      0.0     0.0
 updateVel          Main        2000    3.1     1.9     99.9    99.8
  accel             Main     4000000   23.5    37.2     94.5    93.3
  deductChange      Main     4000000    2.4     4.5      2.4     4.5


-- Barnes-Hut (8000 bodies, 1 iteration)

doSteps             Main           1    0.0     0.0     99.9    99.8
 findBounds         Main           1    0.0     0.0      0.0     0.0
 buildTree          Main       11804    0.0     0.0      0.2     0.2
  splitPoints       Main        3804    0.0     0.0      0.1     0.2
  calcCentroid      Main       11804    0.0     0.0      0.0     0.0
 updatePos          Main        8000    0.0     0.0      0.0     0.1
 updateVel          Main        8000    0.0     0.0     99.7    99.5
  calcAccel         Main    14893157    4.8     7.1     99.7    99.5
   accel            Main    12193706   12.9    16.1     65.1    63.3
   isFar            Main    10247211    7.1     9.2     29.7    29.1
```

In both algorithms, the top-level `doSteps` function performs the iterations and inherits the largest percentage of time. The main source of parallelism arises from the update velocity function `updateVel` which is used as the function argument of a map operation in both all-pairs and Barnes-Hut. It accounts for almost 100% of the inherited overall time. As it is used in a map, it presents data-oriented parallelism.

```
new_bs = map (updatePos . updateVel) bs
```

While the tree construction phase can normally be done in parallel, the time profile indicates that `buildTree` accounts for less than 1% of the time in the Barnes-Hut algorithm. Parallelising it may not cause any significant improvement but could, on the other hand, create overheads. However, with the same number of bodies as used for all-pairs, the time percentage spent in `buildTree` reaches approximately 12%. This is explained by lesser computation involved in the acceleration calculation phase and thus better distribution of the time between the two phases. Also, depending on the distance threshold used to determine when to consider a body far enough, the time in `updateVel` can varies significantly. For instance, if the distance threshold is high (closer to 1), the traversal is very fast, as a result of little computation involved. This is not very good for parallelism as the cost of creating parallelism may be higher than the actual computation. Smaller thresholds for example 0.1 runs slower, while 0 degenerates to pair-wise comparison. Ideally, we use 0.25 which gives a reasonable approximation, accuracy and speed.

Both the all-pairs and Barnes-Hut algorithms are structured such that the function to update the velocity and position of each body can be applied to the list of all bodies in a map operation, thus exposing data-oriented parallelism. All of the three models provide at least a basic parallel map for data-parallel operations.

5.1. GpH

All-Pairs: Initial parallelism is obtained by replacing `map` with `parMap` such that each body velocity is computed in parallel.

```
new_bs = parMap rdeepseq (updatePos . updateVel) bs

-- equivalent to
new_bs = map (updatePos . updateVel) bs `using` parList rdeepseq
```

The composition in the `map` function argument can be turned into a pipeline using the parallel `.||` combinator that arranges for a parallel evaluation of both functions being combined. In this case, the result of `updateVel` is evaluated in parallel with the application of the first function. However, given that the `updatePos` function does negligible computation as opposed to `updateVel`, this is not a useful source of parallelism and therefore not considered any further. It demonstrates, however, that this programming model makes it easy to compose different sources of parallelism, and to prototype alternative parallelisations, without having to restructure the existing code in a fundamental way.

The performance results from this naive version are disappointing. In the best case, we observe a speed-up of 1.4 on 4 processors and a slow-down on 8 processors. The reason for this poor performance is considerable overhead associated with generating a thread for every list element, potentially 16000 in total. While the generation of sparks is cheap – it amounts to adding a pointer to a queue – the generation of a thread requires the allocation and initialisation of a thread state object (TSO), which among other data contains the stack used by the thread. In this case, the computation performed by one thread, namely updating the velocity and position of one body, is too small in comparison with the overhead for TSO initialisation and for scheduling the available threads. The following statistics summarises the execution on 2 cores. In total, 16000 sparks are created, one for each list element, and of these 8192 are converted into threads. This lower number is due to the limited size of the spark pool, which is 4k by default. Since the nature of the parallelism is data-parallel, no work can be subsumed by a sibling-thread, and thus lazy task creation is not effective in automatically increasing thread granularities.

Listing 2: Global statistics of a parallel run on 2 cores

```
./allpairs 16000 1 +RTS -N2 -s
56026.00329381344
54897.906546913
time taken: 16.76s
  31,145,652,016 bytes allocated nonein the heap
      27,366,360 bytes copied during GC
       2,999,520 bytes nonemaximum residency (5 sample(s))
         517,760 bytes nonemaximum slop
              10 MB total memory nonein use (0 MB lost due to fragmentation)

  Generation 0: 44953 collections, 44952 parallel,  2.70s,  1.22s elapsed
  Generation 1:     5 collections,     5 parallel,  0.05s,  0.03s elapsed

  Parallel GC work balance: 1.12 (3256443 / 2904329, ideal 2)
```

```
                         MUT time  (elapsed)         GC time   (elapsed)
  Task  0 (worker) :      4.90s    ( 15.63s)         1.51s     (  0.82s)
  Task  1 (worker) :      6.21s    ( 15.63s)         0.52s     (  0.08s)
  Task  2 (bound)  :      9.96s    ( 15.63s)         0.72s     (  0.36s)
  Task  3 (worker) :      0.00s    ( 15.63s)         0.00s     (  0.00s)

  SPARKS: 16000 (8192 converted, 0 pruned)

  INIT   time     0.00s  (  0.01s elapsed)
  MUT    time    21.07s  ( 15.63s elapsed)
  GC     time     2.75s  (  1.25s elapsed)
  EXIT   time     0.00s  (  0.00s elapsed)
  Total  time    23.83s  ( 16.89s elapsed)

  %GC time        11.6%  (7.4% elapsed)

  Alloc rate     1,477,931,568 bytes per MUT second

  Productivity   88.4% noneof total user, 124.8% noneof total elapsed
```

To tune parallel performance, we control the number of sparks created by grouping elements into larger units, known as chunks. Instead of creating a spark for each element in the list, the list is broken down into chunks and a spark is created for each chunk, thus significantly reducing the thread creation overhead. The number of chunks is determined by the number of available processors. Having too few chunks may result in some processors not getting enough work while too many chunks create excessive overhead.

As often in data-parallel programs, a careful balance between low thread management overhead and massive parallelism is crucial. There is no dynamic parallelism here, i.e. all parallelism is generated at the beginning. Thus a low, fixed number of chunks is likely to be the best choice for performance. Each processor does not necessarily get the same number of chunks. Using more chunks retains more flexibility for the runtime system, because a faster or more lightly loaded processor can pick-up new work after having finished its initial work allocation.

We now consider three ways to introduce chunking (or clustering) into the algorithm (the language-level differences between these approaches are discussed in more detail in [35]).

Explicit Chunking: The most obvious way of performing chunking, is to explicitly apply functions performing chunking before and de-chunking after the data-parallel core of the application (see below). Explicit chunking is also used in the Par monad version, and its performance discussed in Section 5.2.

```haskell
s = 1000 -- chunk size
new_bs = concat (map (map (updatePos . updateVel)) (chunk s bs) `using` parList
     rdeepseq)
```

Used directly in the application code, this technique obfuscates the computational core of the application, and introduces an intermediate data structure that is only needed to increase thread granularity.

Strategic Chunking: Another skeleton-based approach to introduce chunking is to modify the definition of the strategy and to encode chunking additionally to the specification of parallelism inside this skeleton. Thus, we change `parList` to `parListChunk`, which takes an additional argument, specifying the chunk size. The `parListChunk` strategy applies the given strategy, in this case `rdeepseq`, to each chunk. This achieves a clean separation of computation and coordination, leaving the core code unchanged, and hiding the intermediate data structure in a custom strategy. However, this strategy is now fixed to one parallel pattern and one way of chunking.

```haskell
s = (length bs) `quot` (numCapabilities * 4) -- 4 chunks/PE
new_bs = map (updatePos . updateVel) bs `using` parListChunk s rdeepseq
```

Table X (a) in Appendix 10 shows the runtime and speed-up results for different number of chunks per processor using `parListChunk` strategy. While generating exactly one chunk per processor

	nochunk		parListChunk		evalCluster	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	20.04	1.00	20.06	1.00	20.02	1.00
1	20.64	0.97	22.10	0.91	19.71	1.02
2	16.76	1.20	11.33	1.77	10.93	1.83
4	13.89	1.44	5.97	3.36	5.83	3.43
8	15.64	1.28	3.29	6.10	3.28	6.10

Table III. *GpH* runtime and speed-up (All-Pairs, 16k bodies, 1 iteration)

might intuitively seem to be the best choice, it is also the least flexible one, because it deprives the runtime system from distributing parallelism in the case where one processor suffers from a high external load. Therefore, a small number of chunks greater than one is usually a good choice. In this case, the right balance is to have approximately four chunks per processor.

Implicit Clustering: A more compositional way to introduce chunking is to delegate it to an instance of a new `Cluster` class, with functions for performing clustering and declustering (or flattening). We can use available abstractions of performing an operation on each element of a cluster (`lift`) and of flattening the resulting data structure (`decluster`). Thus, to define an instance of this class the programmer only needs to define `cluster` in such a way that the specified proof obligation is fulfilled e.g. an instance for lists as given below requires us only to define `cluster`.

```haskell
class (Traversable c, Monoid a) => Cluster a c where
  cluster    :: Int -> a -> c a
  decluster  :: c a -> a
  lift       :: (a -> b) -> c a -> c b

  lift = fmap          -- c is a Functor, via Traversable
  decluster = fold     -- c is Foldable,  via Traversable
 -- we require: decluster . cluster n == id

instance Cluster [a] [] where
  cluster = chunk
```

Based on this class definition, we can then separately define an `evalCluster` strategy, which uses these functions before and after applying its argument strategy to each cluster, thus separating the definition of parallelism from any form of clustering.

```haskell
evalCluster::Cluster c => Int->Strategy a->Strategy a
evalCluster n s x = return (decluster (cluster n x `using` cs))
  where cs = evalTraversable s :: Strategy c
```

Using this approach, we can add clustering to the basic data-parallel strategy, without changing the original strategy at all. We simply replace `evalList (rpar `dot` rdeepseq)`, which is the definition of `parList`, by `evalCluster s (rpar `dot` rdeepseq)`. In short, the compositionality of this style of programming allows to specify a parallel strategy combined with a clustering strategy. This provides more flexibility in aggregating collections in ways that cannot be expressed using only strategies.

In summary, the code below shows the use cases for all three clustering techniques:

```haskell
-- explicit clustering
concat (map (map f) (chunk s bs) `using` parList rdeepseq)
-- strategic clustering
map f xs `using` parListChunk s rdeepseq
-- combining parallel and clustering strategies
map f xs `using` evalCluster s (rpar `dot` rdeepseq)
```

Table III summarises the parallel results of using no chunk and chunking through `parListChunk` and `evalCluster`, both four chunks per PE. Most notably, the implicit `evalCluster` version achieves the same performance as the strategic `parListChunk`. Thus, using this more compositional version, which makes it easy to introduce and modify clustering

	Top-level map only		Parallel buildTree	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	33.31	1.00	33.31	1.00
1	35.77	0.93	35.77	0.93
2	21.50	1.55	21.61	1.54
4	11.00	3.03	10.77	3.09
8	6.77	4.92	6.11	5.45

Table IV. *GpH* runtime and speed-up (Barnes-Hut, 80k bodies, 1 iteration)

strategies separately from specifying parallelism over the data structure, does not incur a significant performance penalty.

Barnes-Hut: Sequential profiling of the Barnes-Hut algorithm identifies the same function – `updateVel` – as the most compute-intensive. As the call count for this function shows, this is due to the iterative use in the top level map. Therefore, the Barnes-Hut algorithm is parallelised in the same, data-parallel way as the all-pairs version. Since an abundance of fine-grained parallelism is also a problem in this version, we use the same form of chunking to tune the parallel performance.

In this version a natural parallel phase is `buildTree`, where sub-trees can be constructed in parallel. But as the profiling report showed earlier, it does not account for a significant percentage of the overall time, and therefore the benefits from parallelising this stage are limited. However, it is cheap to mark the stage as parallel computation, and whether to take the spark for parallel execution is up to the runtime system.

Another generic optimisation that is applied in the `buildTree` function is *thresholding*. By adding an explicit argument to the function that represents the current level of the tree, the generation of parallelism can be restricted to just the top levels. This makes sure there are not too many parallel threads for the tree construction otherwise it would cause overheads with large number of bodies which would require a big tree structure.

As expected, Table IV shows that parallel `buildTree` does not improve performance significantly but it does not cause additional cost either. The main observation though is that the algorithm does not achieve as good speed-up as the all-pairs algorithm. This is expected as all parallel tasks in the all-pairs algorithm have the same amount of computation to perform i.e. the parallelism is regular; whereas the acceleration calculation steps in the Barnes Hut algorithm varies for each body depending on its location. Some bodies require traversing deeper inside the tree to calculate the net acceleration, while for some, it may not require to do so. For instance, quantifying the irregularity of the computation involved in Barnes-Hut, random generation of 80000 bodies gives an unbalanced tree with minimum tree depth 6 and maximum depth 9.

5.2. *Par monad*

All-Pairs: We use the pre-defined parallel map provided in Par monad to add parallelism the same way as we did in GpH. In contrast to GpH, the parallel computation happens in a monad and therefore the result has to be extracted using `runPar`.

```
new_bs = runPar $ parMap (updatePos . updateVel) bs
```

The initial results in Table X (b), without using chunks, already show good performance: a speed-up of 6.17 on 8 cores. The reason for this efficient behaviour is the work-inlining scheduler, which distributes the potential parallel tasks to a number of implicitly created threads and then executes the task within the existing thread. This dramatically reduces the thread creation overhead, at the expense of less flexibility in how to distribute the tasks in the first place. This model is well suited for homogeneous, multi-core architectures and no explicit chunking is needed to improve parallel performance. However, as shown in Table V the use of chunking reduces the maximum residency by 50% from 4822MB to 2417MB for parallel run on 8 cores.

Table X (b) also shows that the number of chunks causes negligible change to the runtime and speed-up. However, to maintain low memory residency and to facilitate scalability beyond the number of cores available for these measurements, a chunking policy is preferred.

Par monad, however, does not come with a pre-defined parallel map function with chunking. So, we use explicit chunking, as discussed above. In the following code extract, *s* is the chunk size and is used in the same way as in GpH to produce an appropriate number of chunks to match the number of cores.

```
new_bs = parMapChunk (updatePos . updateVel) s bs

parMapChunk f n xs = concat ( runPar $ parMap (map f) (chunk n xs) )
```

Barnes-Hut: For the Barnes-Hut algorithm, we note that chunking causes a noticeable improvement in the speed-up from 5.29 to 6.50 on 8 cores (using parallel `buildTree`). This could be due to the fact that a large number of bodies are used in this algorithm and the memory usage is more significant.

The large number of bodies used in Barnes-Hut makes the heap usage significantly higher compared to the all-pairs algorithm. Without chunking, the maximum residency is 83MB and productivity is at 63%. With chunking, residency is 55MB and improved productivity by 10%. This reduced percentage of garbage collection time has an immediate impact on the performance of the parallel program.

As we did for the GpH version, we also try to parallelise the `buildTree` function but the difference is insignificant, as we expected, due to `buildTree` not representing a large part of the overall computation (Table VI).

5.3. Eden

All-Pairs: As with the previous two models, we only need a parallel map implementation to add data-oriented parallelism to the algorithm. Eden offers several skeletal approaches and, in particular, has several implementations of parallel map as described earlier. The default `parMap` implementation creates a process for each list element causing far too much overheads in terms of number of processes instantiated and messages communicated between them (16001 and 64000 respectively as shown in Appendix Table XI (c)). Observing number of processes and communications is motivation for picking a different strategy, and with it numbers drop significantly and speed-up improves.

The farm process skeleton creates the same number of processes as the number of available processing elements. But the message overheads remain. Each list element is communicated as a single message which generates 32048 messages in total. This represents a high number but still the performance is considerably improved compared to the naive parallel map and good speed-up is noted. This indicates that process creation overheads is far more important than the number of messages.

Doing parallel tuning to further reduce message overheads, we use chunking to break the stream into chunks of size 1000 items which are then sent as one message, thus enabling the process to do more computation at one time rather than having to send and receive messages in between. The chunking reduces the total number of messages communicated in the `parMapFarm` version (farm process) from 32048 to just 80 messages. As a result of this, the runtime and speed-up is improved as shown in Table VII.

Par. run on 8 cores	nochunk	chunking
copied during GC (MB)	31527	16171
max residency (MB)	4822	2417

Table V. *Par monad* GC and max. residency (All-Pairs, 16k bodies, 1 iteration)

	Top-level map only		Parallel buildTree	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	33.39	1.00	33.65	1.00
1	34.49	0.97	33.96	0.99
2	17.72	1.88	17.79	1.89
4	9.21	3.63	8.97	3.75
8	5.91	5.65	5.18	6.50

Table VI. *Par monad* runtime and speed-up (Barnes-Hut, 80k bodies, 1 iteration)

The offline farm process, where process input is evaluated by the child process instead of the parent process, causes a small performance improvement compared to the farm process. Sending process input to child processes to be evaluated is intended to reduce the combined time the parent process spent in reducing all inputs. However, in our algorithm, the inputs to farm processes is a `Body` type with strict fields. So there is no much reduction happening after sending it to the child processes.

The all-pairs measurements using all skeletons are given in Appendix Tables XI (a) and (b), with (c) summarising the overheads of each skeleton.

Barnes-Hut: Though the Eden all-pairs implementation has given the best performance so far compared to the other two models, the performance for the Barnes-Hut algorithm using Eden is not as good as the other models. The speed-up is roughly the same on 1 to 4 cores but then there is no further speed-up upto 8 cores. The best speed-up achieved is using offline process with chunking as given in Table VIII. This is partially due to the high maximum residency caused by all PEs combined due to the large number of bodies used. Furthermore, this indicates that spark-oriented parallelism in GpH and Par monad parallel tasks deal better with dynamic and irregular parallelism. We compare the performance of all implementations in more detail in Section 6.

	farm processes `parMapFarm`		with stream chunking `parMapFarmChunk`	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	22.13	1.00	22.13	1.00
1	23.67	0.93	22.91	0.97
2	11.91	1.86	11.57	1.91
4	6.08	3.64	5.82	3.80
8	3.41	6.49	3.09	7.16

Table VII. *Eden* runtime and speed-up (All-Pairs, 16k bodies, 1 iteration)

	Top-level map only		Parallel buildTree	
	`parMapOfflineFarmChunk`			
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	35.34	1.00	35.32	1.00
1	33.11	1.07	33.89	1.04
2	18.41	1.92	18.73	1.89
4	10.62	3.33	11.24	3.14
8	10.37	3.41	10.71	3.30

Table VIII. *Eden* runtime and speed-up (Barnes-Hut, 80k bodies, 1 iteration)

6. PERFORMANCE EVALUATION

Experimental Setup: The machine used for measurements has the following hardware specifications: an Intel Xeon CPU E5410 2.33 GHz processor with 8 cores, 8GB RAM and 12MB L2 cache. The machine runs a 64-bit CentOS 5.8 Linux distribution with kernel version 2.6.18-308.11.1.el5. The GHC compiler version 7.0.1 is used with the `parallel` and `monad-par` packages for GpH and Par monad respectively. The Eden extension to the GHC compiler version 6.12.3 (GHC-6.12.3-Eden) is used for Eden. At the time of writing, both the GHC and GHC-Eden compilers have newer but not yet stable versions. Any comparison between GpH/Par monad and Eden is therefore on the basis of speed-up.

The input size for all-pairs measurements is 16000 bodies, while for Barnes-Hut, being a more efficient algorithm and able to cope with high number of bodies, we use 80000 bodies. The input sizes ensure the runtimes of one iteration are within a minute for both algorithms. Larger input sizes are used for doing scale-up tests and smaller sizes in the sequential optimisation phase. Measurements for the challenge input specification of 1024 bodies and 20 iterations are given at the end of the section, for comparison with other systems. The tables and graphs show *absolute* speed-ups.

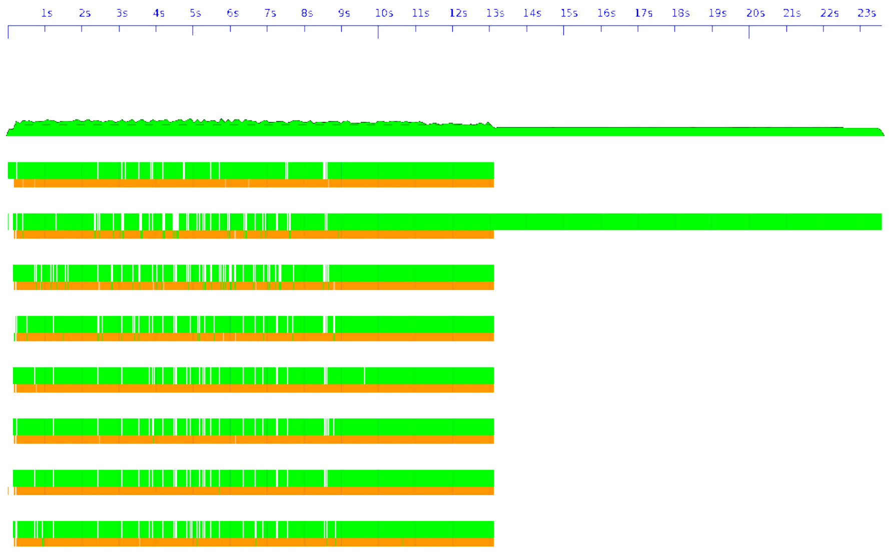
Apart from the performance obtained from using the different models for the parallel implementations, other factors are equally important. For example, each model is built around different concepts and the underlying implementation is fairly technical. However, exposing high-level functions with simple interfaces to the casual programmer is an important part of any programming model. All of them do provide similar interfaces familiar to the programmer, for example, parallel map.

Tuning: While an initial parallel version was easily produced with only a one-line program change, GpH required some parallel performance tuning, in particular by using chunks to generate a bounded number of threads of suitable granularity. Selecting a good chunk size required a series of experiments, establishing four threads per processor to be the best balance between massive parallelism and coarse thread granularity. The more irregular nature of the parallelism in the Barnes-Hut version, compared to the naive all-pairs version, diminishes the achieved speed-up, but also demonstrates that the runtime system effectively manages and distributes the available parallelism, without requiring further application code changes from the programmer.

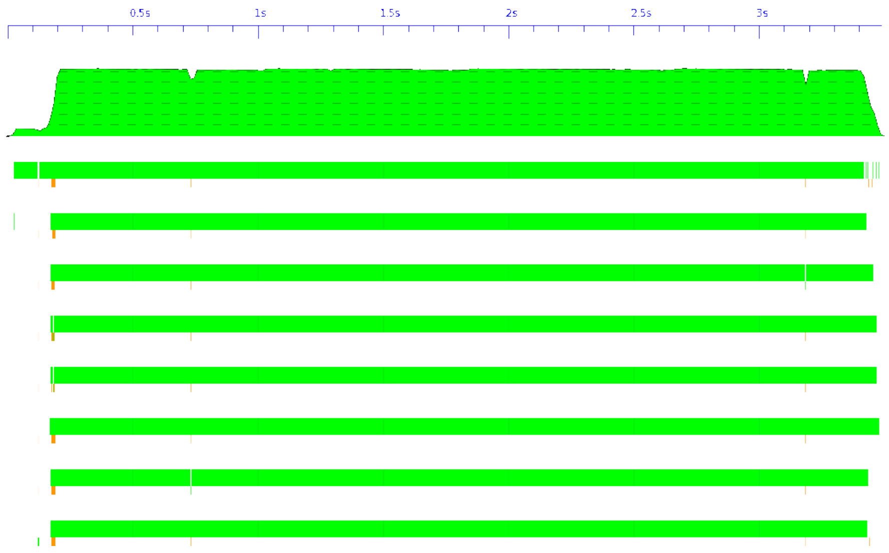
The Par monad version uses a highly tuned, data-parallel map skeleton, and thus can efficiently handle a large number of parallel tasks with an initial version, eliminating the need for explicit chunking. However, chunking does improve the maximum residency and therefore the scalability of the application.

Eden provides the richest set of skeletons available to implement both versions of the algorithm. In this model, parallelisation amounts to selecting the most suitable skeleton for the main worker function. Ample literature on the advantages and disadvantages of different skeletons helps in making the best decision for a specific application and architecture. For fine-tuning the parallel performance, however, an understanding of the process creation and message passing is required to minimise the amount of communication in this distributed-memory model.

For any high-level language model, good tool support is crucial in order to understand the concrete dynamic behaviour of a particular algorithm and to tune its performance. Threadscope helps to visualise the work distribution among the available processors for GpH and Par monad. Figure 2 shows the work distribution of running the all-pairs GpH program on 8 cores before and after parallel tuning using chunking. The first horizontal bar on the graph shows the overall activity, measured in terms of the total number of active processors at each time in the execution. The remaining bars shows the activity for each of the processors, with green representing “running” and red representing “idle”. Additionally, the time spent in garbage collection is indicated in orange. The number of sparks is given in the runtime statistics for GpH. For Par monad, however, the exact number of threads created is not given.



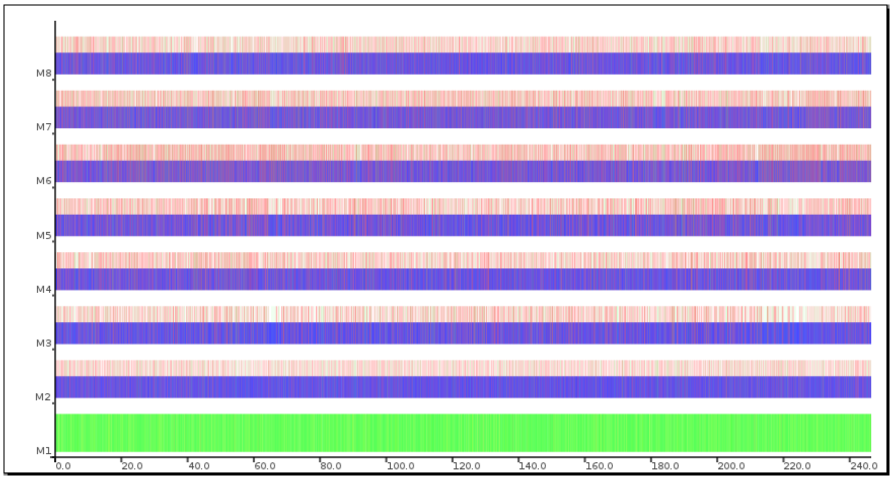
(a) nochunk



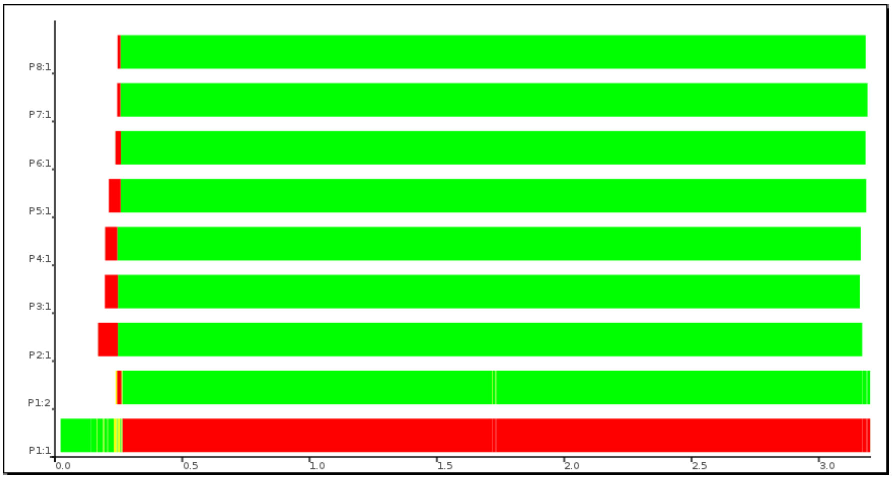
(b) 4chunksPerPE

Figure 2. *Threadscope* work distribution (Par. run on 8 cores)

The Eden Trace Viewer (EdenTV) gives useful information about processes, their placement, conversations and messages between processes. Figure 3 compares the use of the naive parallel map (`parMap`) against the farm process implementation using stream chunking (`parMapFarmChunk`). It shows the overheads of too many processes, and consequently messages, being generated in the former. The overheads are eliminated in the tuned version. Each line represents the activity on one processor with green representing "running" and blue representing "waiting for data". While the first trace shows frequent changes between running and waiting states, reflecting the element-by-element transfer of the input data from the master to the workers, the second trace shows much better utilisation as uninterrupted activity once the entire block of input data has been received by a worker. The master remains idle, while the workers produce their results. A related set of skeletons allows a dual usage of the master process as worker in such a case, and can be used to improve performance further.



(a) parMap

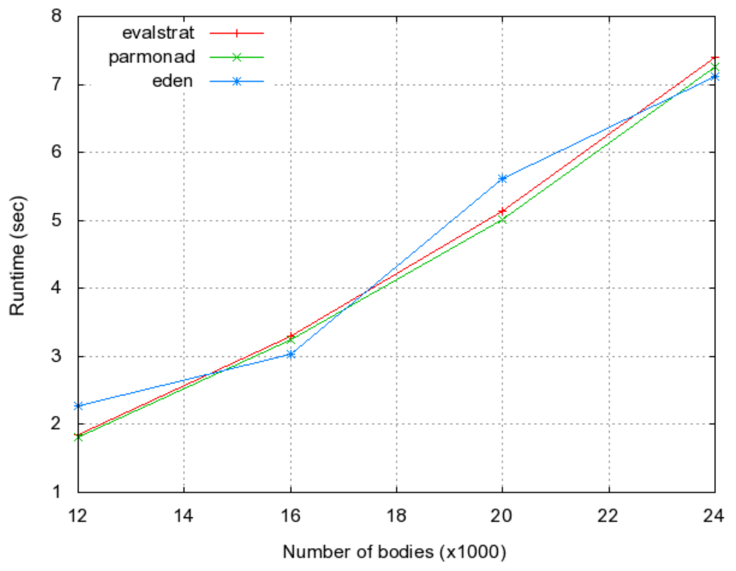


(b) parMapFarmChunk

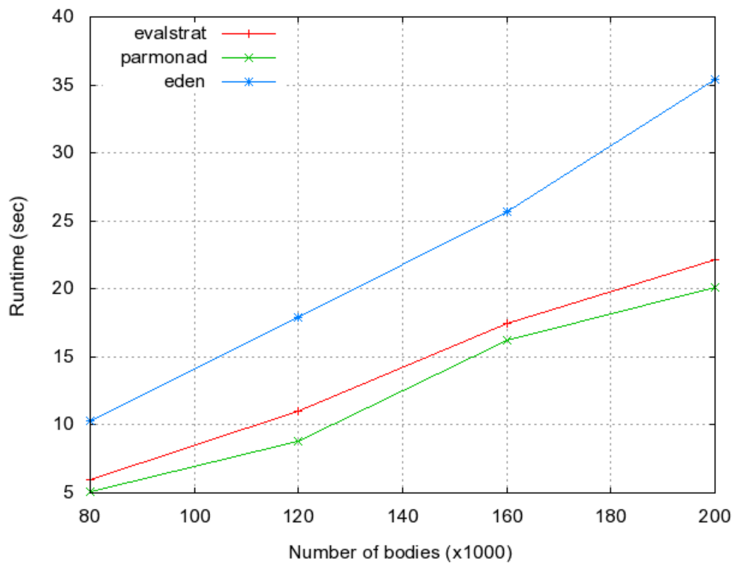
Figure 3. *EdenTV* using different map skeleton (Par. run on 8 PEs)

Scale-up: The optimisations carried out in the sequential tuning phase is crucial in ensuring that both the all-pairs and Barnes-Hut programs can be executed on a large number of bodies by maintaining low heap consumption. Figure 4 assesses the scalability of the two algorithms by plotting the runtimes against increasing input sizes on 8 cores: 12k-24k for all-pairs and 80k-200k for Barnes-Hut. The graph indicates almost linear scale-up within this window by all models. The Barnes-Hut program using Eden has higher runtimes, reflecting a significantly higher memory consumption, which comes from Eden's distributed-memory model, which duplicates data more often than needed on a shared-memory architecture. Figure 5 is a 3D scale-up plot showing the number of cores on one axis. As expected, Barnes-Hut is also able to cope with very large numbers of bodies e.g. 1 million, but not shown here.

Speed-up: The head-to-head comparison of speed-ups for the all-pairs versions in Figure 6(a) show that, despite a higher variability, the Eden implementation performs best, even though it was designed for distributed-memory architectures. This indicates that message passing can work well on shared-memory architectures. Using a highly tuned skeleton that avoids synchronisation bottlenecks on high-latency, distributed-memory systems, is beneficial even on a single multi-core. The support for light-weight parallelism in all three runtime systems helps to reduce the overhead



(a) All-Pairs

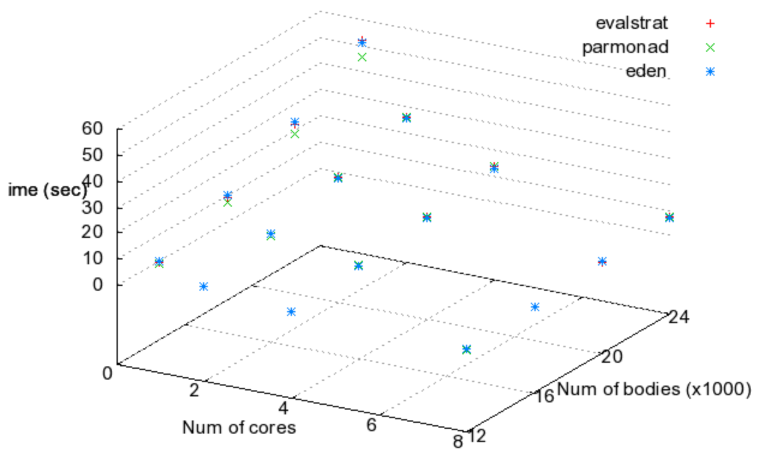


(b) Barnes-Hut

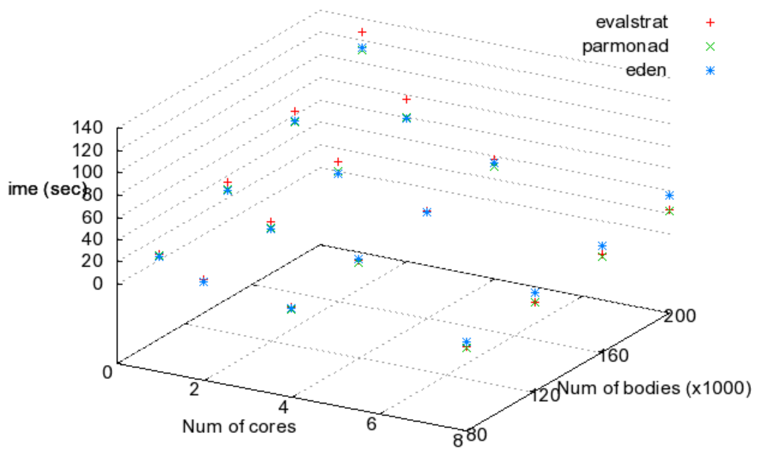
Figure 4. *Scale-up* graphs (Par. run on 8 cores)

that has to be paid for exposing parallelism. The GpH version is potentially more flexible and adaptive, through its dynamic, spark-based load distribution policy. This is beneficial in particular in heterogeneous environments, with dynamically changing external load. On an otherwise idle machine as used for these measurements, however, these benefits cannot be capitalised on, while the overhead still has to be paid for. The Par monad version performs very well with an initial, unoptimised version, but does not exceed the performance of the other systems in its final version. In this case, the overhead of encoding scheduling and other dynamic machinery in the application, rather than the runtime system, is higher compared to the other two systems.

The speed-up results for the Barnes-Hut algorithm in Figure 6(b) show a significantly different picture. The dynamic behaviour of the Barnes-Hut algorithm differs from that of the all-pairs version, in that the parallel threads vary significantly in their granularities. The amount of work



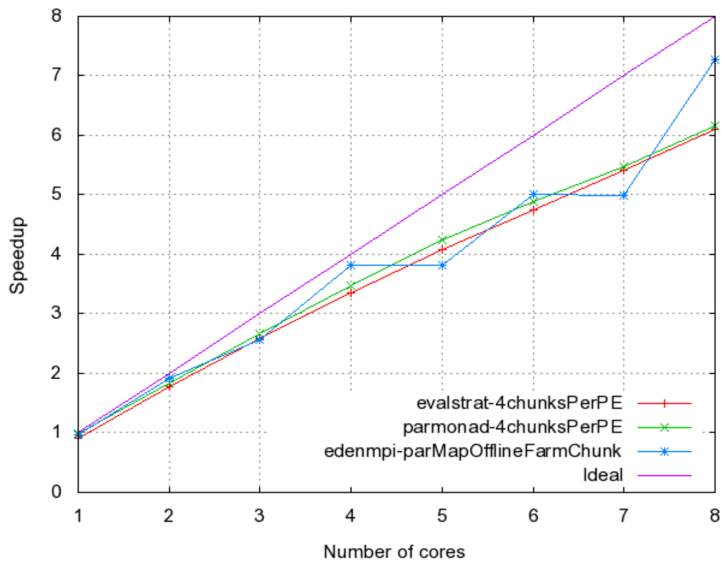
(a) All-Pairs



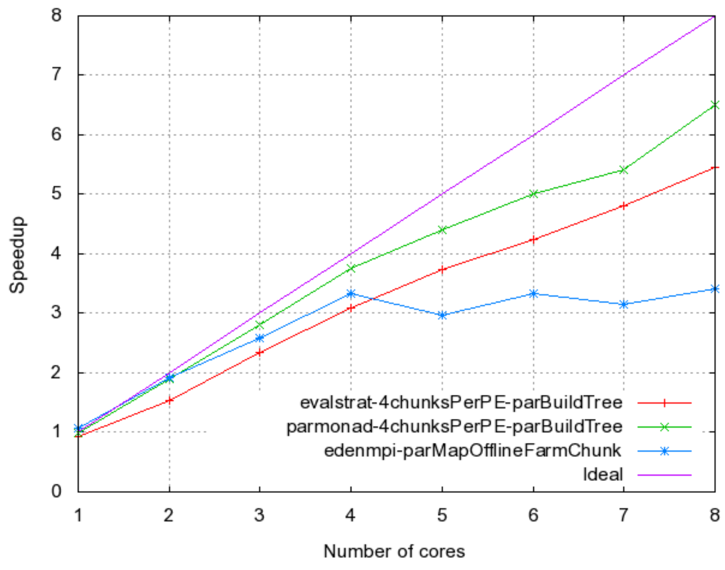
(b) Barnes-Hut

Figure 5. *3D Scale-up* graphs (Par. runs on 1 to 8 cores)

is significantly higher when calculating the impact of a densely populated cube in the oct-tree representation. In contrast, the parallelism in the all-pairs version is more regular, with parallel tasks taking approximately the same amount of time to execute on different processors. The irregular parallelism in the Barnes-Hut version is more challenging to manage efficiently. The underlying runtime system of GpH and the application-level implementation of scheduling for Par monad, are designed to be very flexible and dynamic in their management of parallelism, in particular allowing for cheap transfer of potential parallelism. Considering the more challenging nature of the parallelism, GpH and Par monad achieve good speed-ups. The Eden version, however, suffered most severely from the irregular parallelism. This case shows the limitations of a purely skeleton-based approach, that relies on the existence of a wide range of skeletons for many different architectures. Since Eden is not limited to such a pure skeleton-based approach, but is a skeleton implementation



(a) All-Pairs



(b) Barnes-Hut

Figure 6. *Speed-up* graphs of both algorithms on up to 8 cores

language in its own right, further optimisation should be possible, by fine tuning an existing skeleton for this application. We have not explored a cluster configuration of the Eden execution in sufficient detail to report any solid results on it here.

MultiCore Challenge input specification: Finally, Table IX shows the speed-up results for the tuned versions of all-pairs and Barnes-Hut using the MultiCore Challenge input specification of 1024 bodies and 20 iterations. As expected, the speed-ups are slightly lower for the smaller input set and for an execution which requires synchronisation between the iterations. Still, the speed-ups of 5.23 for GpH and 5.63 for Par monad for the Barnes-Hut version are remarkable, for less than a dozen lines of code changes, and no structural changes to the original Haskell implementation. In particular, we surpass the calculated sequential overhead of a factor 2.9 compared to Fortran on a moderate multi-core architecture and deliver superior, scalable performance with a high-level language model.

7. RELATED WORK

The recent proliferation of parallel hardware poses a significant challenge to the way how these machines are programmed. Traditionally, the coordination of the parallel execution is controlled in every detail to achieve near-optimal performance. This is only feasible in a setting, where a lot of person-effort can be spent on the parallelisation of a single application. Thus, this traditional view of parallelism, *supercomputing parallelism*, represents a niche market in the overall domain of computer science. With multi-core hardware now dominating the architecture landscape, a new view of parallelism, *desktop parallelism*, is of increasing importance. Here the goal is to achieve some speed-up from a general-purpose, compute-intensive application, that has been developed and is maintained by domain experts, rather than experts in parallel programming. The target compute platform is a multi-/many-core machine or a cluster thereof.

Importantly from a programming model point of view, most traditional models for parallel programming are prohibitively expensive in terms of person-effort for this new target community. Therefore, a general trend is to find higher levels of abstractions that can be used to simplify the challenging process of parallelising a complex application. The most successful development in this direction is the use of patterns of parallel computation [38], so called algorithmic skeletons [11], and to instantiate these patterns thereby getting parallelism "for free." The most prominent pattern using this approach is Google's MapReduce pattern [14]. Several different systems provide implementations of this pattern, in particular tuned for large, local clusters: the Google MapReduce system, Apache's Hadoop system, and the Twister system. Already in September 2007 Dean et al. [15] report circa 6500 applications being parallelised using this pattern, and typically obtaining excellent speed-ups on a highly-optimised, cluster based implementation. Naturally, a restriction of this approach is that the application must fit one of the available patterns. However, very active research on skeletons has exposed a very rich set of such patterns, with several systems providing state-of-the-art implementations on different kinds of parallel hardware. More recently, the trend is to generalise the systems beyond just one pattern, and to support skeleton description languages. The next major release of Hadoop, with the working title Yarn [19], provides a high-level, data-flow like language for defining new skeletons. Microsoft's new Dryad system [24] takes a similar approach, extending its Daytona system [13] for iterative MapReduce patterns. Many systems for skeleton-based parallelism have been developed, and the most notable of these systems are eSkel [3], Müsli [10], Skandium [29] and P^{3}L [1]. A good, up-to-date survey on skeletons is given in [23].

The area of declarative programming languages provides a particularly rich source for parallelism, stemming from the referentially transparent nature of the evaluation in a (pure) language [52]. This property guarantees that any order of evaluating an expression will deliver the same result. In particular, independent parts of the program can be evaluated in parallel. A straightforward implementation of this approach would yield an abundance of parallelism, with too fine grained threads to be efficient. Therefore most modern parallel functional languages take an approach of

	All-Pairs				Barnes-Hut			
	GpH		Par monad		GpH		Par monad	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	1.67	1.00	1.70	1.00	1.36	1.00	1.35	1.00
1	1.71	0.98	1.66	1.02	1.40	0.97	1.38	0.98
2	0.94	1.78	0.93	1.83	0.78	1.74	0.72	1.88
4	0.51	3.27	0.52	3.27	0.44	3.09	0.42	3.21
8	0.30	5.57	0.30	5.67	0.26	5.23	0.24	5.63

Table IX. *GpH* and *Par monad* runtime and speed-up (1024 bodies, 20 iterations)

semi-explicit parallelism: they provide a means of identifying those sources of parallelism that are likely to be useful. Still, most of the synchronisation, communication and thread management is hidden from the programmer and done automatically in a sophisticated runtime system.

An influential, early system for parallel functional programming was Mul-T [28] using Lisp. It introduced the concept of futures as handles for a data-structure, that might be evaluated in parallel and on which other threads should synchronise. Importantly for performance, this system introduced lazy task creation [39] as a technique, where one task can automatically subsume the computation of another task, thus increasing the granularity of the parallelism. Both, the language- and the system-level contributions have been picked up in recent implementations of parallel functional languages.

One prominent example of this approach is the Manticore system [18] using a parallel variant of ML that includes futures and constructs for data parallelism. It allows to specify parallelism on several levels in a large-scale applications, typically using explicit synchronisation and coordination on the top level [45] and combining it with implicit, automatically managed, fine-grained threads on lower levels [17].

Another ML extension is Polyml [37], which also supports futures, light-weight threads and implicit scheduling in its implementation. Reflecting its main usage as an implementation language for automated theorem provers, such as Isabelle, it has been used to parallelise its core operations.

Another parallel extension of Haskell is Data Parallel Haskell [25], which has evolved out of the Nepal language [6]. It provides language, compiler and runtime system support for data-parallel operations, in particular parallel list comprehensions. Several program transformations, e.g. data-structure flattening, are performed in the compiler to optimise the parallel code. Its design is heavily based on the Blelloch's NESL language [4], a nested, data-parallel language with an explicit cost model to predict parallel performance.

SAC [46] is another functional, data-parallel language. Its syntax is based on C, but its semantics is based on single assignment, and therefore referentially transparent. It mainly targets numerical applications and achieves excellent speed-ups on the NAS benchmark suite.

Microsoft's Accellerator [47] is another prominent data-parallel system. It supports high-level, language-independent program development, and generates parallel code to be executed on GPGPUs or on multi-cores with vector-processing extensions.

Several experimental languages explored the use of high-level, parallelism language features in object-oriented languages: Fortress [48], X10 [8] and Chapel [7]. Of these, Chapel is currently best supported, in particular on massively parallel supercomputers. These languages introduce high-level constructs such as virtual shared-memory (X10), structured programming constructs for parallel execution (Chapel), and software transactional memory (Fortress) to avoid a re-design of the software architecture due to specifics of the underlying, parallel architecture [9]. These new parallel languages adopt high-level language concepts, pioneered by parallel functional languages:

- *Static type systems* play a central role in all of these languages. X10 even uses dependent types to express constraints on the location of computations [8]. Fortress, designed as a "secure Fortran," puts an emphasis on the security gained from a static type system [48];
- *Higher-order functions* are successfully used in several parallel libraries, such as Intel's thread library [44] or in Google's map-reduce framework [14];
- *Declarative or rule-based computation*, with its inherently decentralised nature of computation, has been identified as one promising direction for (semi-) automatically exploiting parallelism on multi-core machines [9];
- *Generic programming* offers the potential of code-reuse and is used for example in Chapel [7].

Based on the experiences with these languages, high-level abstractions are now entering main-stream languages for parallel programming. In particular, the concept of partitioned global address spaces (PGAS) enables the programmer to use the abstraction of virtual shared-memory, while providing possibilities for co-locating data on specific nodes and thereby tune the parallel execution. The most prominent languages in this family are Unified Parallel C (UPC) [16] and Co-Array Fortran [41].

Increasingly, these high-level abstractions are also used in main-stream languages to facilitate parallel programming. The latest version of the .NET platform, comes with the Task Parallel

Library [5], which provides a set of parallel patterns, in particular for divide-and-conquer and pipeline parallelism, and some advanced parallel programming constructs such as futures. Several languages implemented on top of .NET, including C# and F#, can make direct use of these library functions to introduce parallelism, without extending the language itself. Intel's Threading Building Blocks [44], also a collection of parallel patterns, has been successfully used on a range of parallel architectures.

Crucial for the feasibility of a high-level language approach to parallelism, is an efficient and flexible implementation of basic resource management policies, such as load distribution and scheduling. All three Haskell variants used in this paper profit from light-weight threads as managed in the runtime system. GpH's runtime system manages work distribution through "sparks", effectively handles to potential parallelism [50]. Work represented by one spark can be subsumed by a running thread, effectively achieving lazy task creation [39]. Par monad's approach builds on the existing mechanisms for thread creation and synchronisation as initially developed for Concurrent Haskell [43]. Eden's runtime system is more prescriptive in the way it manages the parallelism, which allows for the specification of skeletons implementing specific topologies [27].

Several other systems have taken similar design decision in producing a system for dynamic and adaptive management of parallelism. Filaments [33] was an early system focusing on light-weight threads, encouraging an approach of parallelisation that exposes massive amounts of parallelism and deciding at runtime whether or not to exploit specific parallelism, rather than restricting it on application level. The Charm++ system [26] builds on top of C++ and provides asynchronous message-driven orchestration together with an adaptive runtime system. It has been used on numerous, large-scale applications, for example biomolecular simulations from the domain of molecular dynamics. The Cilk system [20] achieved excellent results on physical shared-memory systems, in particular for the FFT application. Its C and C++ language extensions are now supported both by Intel's Cilk Plus compiler and by GCC 4.7. Goldstein's thesis [22] provides a detailed study of different representations of light-weight threads and their impact on parallel performance, e.g. in the context of the TAM system [12].

Other functional languages that are gaining in popularity also provide high-level parallel programming support. We provide a performance and programmability comparison in Haskell, F# and Scala in another paper [49]. Developing several parallel versions, we employ skeleton-based, semi-explicit and explicit approaches to parallelism. We focus on advanced language features for separating computational and coordination aspects of the code and tuning performance.

8. CONCLUSIONS

In this paper we have used a very high level language approach to the challenge of obtaining efficient parallelism from a typical, compute-intensive application: the *n*-body problem. As host language we used Haskell, the de-facto standard, non-strict, purely functional language, that is increasingly used in academia and beyond to achieve a high level of programmer productivity.

We studied three variants of parallel Haskell: evaluation strategies, abstractions built on top of Glasgow parallel Haskell (GpH); Par monad, with an explicit way of controlling threads; and Eden, which provides process abstractions akin to lambda abstractions to define parallel computations. Common to all variants is the design philosophy to minimise the code changes that are needed to achieve parallel execution. Ideally, the specification of the parallel execution is orthogonal, and separate from the specification of the code. Indeed, the initial parallel versions of both algorithms required only one-line code changes. All of these languages build on a sophisticated runtime system that automatically manages the synchronisation, coordination and communication necessary to achieve high parallel performance. The resulting programming model is one of semi-explicit parallel programming for GpH and Eden, where the programmer only has to identify the useful sources of parallelism, but explicit for Par monad, which allows to encode archetypical runtime system functionality as high-level Haskell code. More commonly, however, pre-defined parallel skeletons are used to simplify the parallelisation.

The three variants differ in the way they facilitate tuning of the initial parallel algorithm, though. Being first class objects, evaluation strategies can be easily modified to enable different dynamic behaviour. For example, adding chunking to a data parallel algorithm can be easily done by composing strategies. This modularity is one of its main advantages. However, control of data locality is significantly more difficult, because GpH relies on an implicit, work stealing scheduler to distribute the work. In contrast, in Eden, thread creation is mandatory on process application, and it provides finer control of co-location, by using partial applications. These features provide more opportunities for tuning the parallel program without abandoning the high level of abstraction. Finally, Par monad is the most explicit form of controlling parallelism. Here, threads are explicit entities in the program, that have to be synchronised using `IVars`, which raises all the usual issues about parallel programming. However, by providing parallel patterns of computation, skeletons, these low-level issues can be hidden from the programmer. By implementing runtime system functionality on Haskell level, an expert parallel Haskell programmer can also tailor the application, e.g. by implementing a custom scheduling algorithm.

Another current parallel Haskell extension is Data parallel Haskell (DPH), which implements a model of implicit, nested data-parallelism on top of parallel arrays. For a predecessor of DPH, Nepal, it is shown how a Barnes-Hut algorithm can be implemented in this language [25]. Concrete performance comparisons with this Haskell variant would be interesting future work.

Despite the high-level of abstraction, the performance results show good *speed-ups* on 8 cores for all systems: 5.45 for GpH, 6.50 for Par monad, and 3.62 for Eden, all using the Barnes-Hut algorithm. Most notably, these results were achieved changing only a few lines of code. Introducing top-level data-parallelism changes only one line of the original code. Further optimisation code, e.g. for chunking, adds less than a dozen lines of auxiliary functions.

Our main conclusions from this challenge implementation are:

- All three parallel Haskell variants are able to achieve competitive multi-core speed-ups not only for the simple, regular all-pairs algorithm but also for the more sophisticated, irregular Barnes Hut algorithm.
- The performance of the parallel all-pairs version surpasses the calculated performance of the fastest sequential Fortran version from the language shootout [21] and achieves scalable performance up to the maximum of 8 cores.
- The ease of parallelisation allowed us to develop 6 versions of the challenge, using three different variants of parallel Haskell and implementing both an all-pairs and a Barnes-Hut version.
- Well documented program transformations, in the form of local changes to the sequential program, reduce both heap and stack space consumption considerably and improve sequential performance by a factor of 12.0.
- Established techniques for tuning parallel performance, in particular chunking, were important to tune the GpH and Eden implementations of the algorithms.
- The Par monad version already achieves good parallel performance in its initial version, due to a highly optimised, work-inlining scheduler. In contrast, both the GpH and Eden versions require explicit chunking to achieve the same level of performance, but allow for more flexible tuning of performance.
- Interestingly Eden, which is designed for distributed-memory architectures, performs very well on a shared-memory setup using message passing, in particular for the all-pairs version.

We found this exercise of implementing one agreed challenge application, and comparing the parallelisation methodology, the tool support and the achieved performance results with other implementations, that were presented at the workshops, an extremely valuable experience, gaining insights in the relative advantages of each of the approaches. These results also help to focus our research efforts in developing the underlying systems further. In particular, one main direction of further work is to improve our runtime support for hierarchical, heterogeneous parallel architectures, e.g. clusters of multi-cores, and to integrate the different Haskell variants into one unified language that makes use of these variants on different levels in the hierarchy. Eden, based on a distributed-memory model, is a natural match with clusters, whereas GpH and Par monad are natural matches for physical shared-memory architectures. GpH also supports virtual memory, which can be efficiently exploited on closely connected clusters. For the next challenge application we hope to also cover clusters of multi-cores to address the issue of the scalability of the parallelism in more detail. Eden already provides a suitably platform for such a comparison, and we hope to also have a stable cluster-version of GpH ready to use for this next stage.

9. ACKNOWLEDGMENTS

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REFERENCES

1. B. Bacci, M. Danelutto, S. Orlando, S. Pelagatti, and M. Vanneschi. P^{3}L: a Structured High-level Programming Language and its Structured Support. *Concurrency and Computation: Practice and Experience*, 7(3):225–255, 1995. doi:10.1002/cpe.4330070305.
2. J. Barnes and P. Hut. A hierarchical O(N log N) force-calculation algorithm. *Nature*, 324:446–449, December 1986.
3. A. Benoit, M. Cole, S. Gilmore, and J. Hillston. Flexible Skeletal Programming with eSkel. In *EuroPar'05*, LNCS 3648, pages 761–770. Springer, 2005.
4. G. Blelloch. NESL: A Parallel Programming Language, 2012. http://www.cs.cmu.edu/~scandal/nesl.html.
5. C. Campbell, R. Johnson, A. Miller, and S. Toub. *Parallel Programming with Microsoft .NET — Design Patterns for Decomposition and Coordination on Multicore Architectures*. Microsoft Press, August 2010.
6. Manuel M.T. Chakravarty, Gabriele Keller, Roman Lechtchinsky, and Wolf Pfannenstiel. Nepal – nested data-parallelism in haskell. In *IN EURO-PAR 2001*, pages 524–534. Springer-Verlag, 2001.
7. Bradford L. Chamberlain, David Callahan, and Hans P. Zima. Parallel Programmability and the Chapel Language. *International Journal of High Performance Computing Applications*, 21(3):291–312, August 2007.
8. Philippe Charles, Christian Grothoff, Vijay Saraswat, Christopher Donawa, Allan Kielstra, Kemal Ebcioglu, Christoph von Praun, and Vivek Sarkar. X10: an object-oriented approach to non-uniform cluster computing. In *Proceedings of OOPSLA '05*, pages 519–538, New York, NY, USA, 2005. ACM Press.
9. Andrew Chien. Parallelism Drives Computing. Talk given at Manycore Computing Workshop, 2007 June.
10. P. Ciechanowicz, M. Poldner, and H. Kuchen. The Münster Skeleton Library: Müsli. Technical Report ERCIS Working Paper No. 7, University Münster, 2010.
11. M. Cole. *Algorithmic Skeletons: Structural Management of Parallel Computation*. Pitman/MIT Press, 1989.
12. D.E. Culler, S.C. Goldstein, K.E. Schauser, and T. von Eicken. TAM — A Compiler Controlled Threaded Abstract Machine. *Journal of Parallel and Distributed Computing*, 18:347–370, June 1993.
13. Daytona — Iterative MapReduce on Windows Azure, February 2012.
14. J. Dean and S. Ghemawat. MapReduce: Simplified Data Processing on Large Clusters. In *OSDI '04 — Symp. on Operating System Design and Implementation*, pages 137–150, 2004.
15. J. Dean and S. Ghemawat. Mapreduce: simplified data processing on large clusters. *Commun. ACM*, 51(1):107–113, 2008.
16. Tarek El-Ghazawi, William Carlson, Thomas Sterling, and Katherine Yelick. *UPC: Distributed Shared Memory Programming*. John Wiley and Sons, May 2005. ISBN 0-471-22048-5.
17. M. Fluet, M. Rainey, J. Reppy, and A. Shaw. Implicitly-threaded parallelism in Manticore. *Journal of Functional Programming*, 20(5–6):537–576, 2010.
18. M. Fluet, M. Rainey, J. Reppy, A. Shaw, and Y. Xiao. Manticore: A heterogeneous parallel language. In *DAMP 2007: Workshop on Declarative Aspects of Multicore Programming*, pages 37–44, Nice, France, January 2007. ACM Press.
19. Apache Foundation. Apache hadoop nextgen mapreduce (yarn). web page, February 2012.
20. Matteo Frigo, Charles E. Leiserson, and Keith H. Randall. The implementation of the Cilk-5 multithreaded language. In *PLDI98: Conference on Programming Language Design and Implementation*, pages 212–223, Montreal, Quebec, Canada, June 1998. Proceedings published ACM SIGPLAN Notices, Vol. 33, No. 5, May, 1998.
21. B. Fulgham. The Computer Language Shootout Benchmarks, 2012. http://shootout.alioth.debian.org.
22. Seth Copen Goldstein. *Lazy Threads: Compiler and Runtime Structures for Fine-rained Parallel Programming*. PhD thesis, University of California at Berkeley, 1997.

23. Horacio Gonzalez-Velez and Mario Leyton. A Survey of Algorithmic Skeleton Frameworks: High-Level Structured Parallel Programming Enablers. *Software: Practice and Experience*, 2010.
24. Michael Isard, Mihai Budiu, Yuan Yu, Andrew Birrell, and Dennis Fetterly. Dryad: Distributed Data-Parallel Programs from Sequential Building Blocks. In *European Conference on Computer Systems (EuroSys)*, Lisbon, Portugal, March 2007.
25. Simon Peyton Jones, Roman Leshchinskiy, Gabriele Keller, and Manuel M. T. Chakravarty. Harnessing the Multicores: Nested Data Parallelism in Haskell. In *IARCS Annual Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2008)*, IBFI, Schloss Dagstuhl, 2008.
26. Laxmikant V. Kale and Sanjeev Krishnan. Charm++: a portable concurrent object oriented system based on c++. In *Proceedings of the eighth annual conference on Object-oriented programming systems, languages, and applications*, OOPSLA '93, pages 91–108, New York, NY, USA, 1993. ACM.
27. U. Klusik, R. Loogen, S. Priebe, and F. Rubio. Implementation Skeletons in Eden: Low-effort Parallel Programming. In *IFL'00: International Workshop on the Implementation of Functional Languages*, LNCS 2011. Springer-Verlag, 2000.
28. D.A. Kranz, R.H. Halstead Jr., and E. Mohr. Mul-T: A High-Performance Parallel Lisp. In *PLDI'91 — Programming Languages Design and Implementation*, volume 24(7) of *SIGPLAN Notices*, pages 81–90, Portland, Oregon, June 21–23, 1989.
29. Mario Leyton and Jose M. Piquer. Skandium: Multi-core programming with algorithmic skeletons. In *IEEE Euromicro PDP 2010*, 2010.
30. H.-W. Loidl, F. Rubio, N. Scaife, K. Hammond, S. Horiguchi, U. Klusik, R. Loogen, G. J. Michaelson, R. Peña, S. Priebe, Á J. Rebón, and P. W. Trinder. Comparing parallel functional languages: Programming and performance. *Higher Order Symbol. Comput.*, 16:203–251, September 2003.
31. H-W. Loidl, P.W. Trinder, K. Hammond, S.B. Junaidu, R.G. Morgan, and S.L. Peyton Jones. Engineering Parallel Symbolic Programs in GPH. *Concurrency and Computation: Practice and Experience*, 11:701–752, 1999.
32. Rita Loogen, Yolanda Ortega-mallén, and Ricardo Peña marí. Parallel functional programming in eden. *J. Funct. Program.*, 15:431–475, May 2005.
33. David K. Lowenthal and Vincent W. Freeh Gregory R. Andrews. Using Fine-grain Threads and Run-time Decision Making in Parallel Computing. *Journal of Parallel and Distributed Computing*, 37(1), 1996. Special issue on multithreading for multiprocessors.
34. Simon Marlow, Simon Peyton Jones, and Satnam Singh. Runtime support for multicore haskell. In *ICFP '09: Proceeding of the 14th ACM SIGPLAN International Conference on Functional Programming*, August 2009.
35. Simon Marlow, Patrick Maier, Hans-Wolfgang Loidl, Mustafa K. Aswad, and Phil Trinder. Seq no more: better strategies for parallel haskell. In *Proceedings of the third ACM Haskell symposium on Haskell*, Haskell '10, pages 91–102, New York, NY, USA, 2010. ACM.
36. Simon Marlow, Ryan Newton, and Simon Peyton Jones. A monad for deterministic parallelism. In *Proceedings of the 4th ACM symposium on Haskell*, Haskell '11, pages 71–82, New York, NY, USA, 2011. ACM.
37. David C. J. Matthews and Makarius Wenzel. Efficient Parallel Programming in Poly/ML and Isabelle/ML. In *DAMP10: Declarative Aspects of Multicore Programming*, Madrid, Spain, November 2010.
38. T. G. Mattson, B. A. Sanders, and B. L. Massingill. *Patterns for Parallel Programming*. Addison-Wesley, 2004. ISBN 978-0321228116.
39. E. Mohr, D.A. Kranz, and R.H. Halstead Jr. Lazy Task Creation: A Technique for Increasing the Granularity of Parallel Programs. *IEEE Transactions on Parallel and Distributed Systems*, 2(3):264–280, July 1991.
40. Rishiyur Nikhil and Arvind. *Implicit Parallel Programming in pH*. Morgan Kaufmann, 2001. ISBN 978-1558606449.
41. Robert W. Numrich and John Reid. Co-array fortran for parallel programming. *SIGPLAN Fortran Forum*, 17:1–31, August 1998.
42. Simon Peyton Jones. Harnessing the multicores: Nested data parallelism in haskell. In *Proceedings of the 6th Asian Symposium on Programming Languages and Systems*, APLAS '08, pages 138–138, Berlin, Heidelberg, 2008. Springer-Verlag.
43. S.L. Peyton Jones, A. Gordon, and S. Finne. Concurrent Haskell. In *POPL'96 — Symposium on Principles of Programming Languages*, pages 295–308, St Petersburg, Florida, January 1996. ACM.
44. James Reinders. *Intel Threading Building Blocks: Outfitting C++ for Multi-core Processor Parallelism*. O'Reilly, 2007.
45. John Reppy, Claudio Russo, and Yingqi Xiao. Parallel Concurrent ML. In *International Conference on Functional Programming (ICFP 2009)*, September 2009.
46. Sven-Bodo Scholz. Single Assignment C – Efficient Support for High-level Array Operations in a Functional Setting. *Journal of Functional Programming*, 13(6):1005–1059, 2003.
47. Satnam Singh. Declarative Data-Parallel Programming with the Accellerator System. In *DAMP10: Declarative Aspects of Multicore Programming*, Madrid, Spain, November 2010.
48. Sun. The Fortress Language. Talks and Posters, 2012. http://research.sun.com/projects/plrg.
49. P. Totoo, P. Deligiannis, and H-W. Loidl. Haskell vs. f# vs. scala: A high-level language features and parallelism support comparison. In *FHPC'12 – ACM SIGPLAN Workshop on Functional High-Performance Computing*, Copenhagen, Denmark, Sep 2012.
50. P. Trinder, K. Hammond, J.S. Mattson Jr., A.S Partridge, and S.L. Peyton Jones. GUM: a Portable Parallel Implementation of Haskell. In *PLDI'96 — Programming Languages Design and Implementation*, pages 79–88, Philadelphia, PA, USA, May 1996.
51. P. W. Trinder, K. Hammond, H.-W. Loidl, and S. L. Peyton Jones. Algorithm + Strategy = Parallelism. *J. Funct. Program.*, 8:23–60, January 1998.
52. P.W. Trinder, K. Hammond, and H-W. Loidl. *Encyclopedia of Parallel Computing*, chapter Parallel Functional Languages. Springer Verlag, 2011. ISBN 978-0-387-09844-9.

10. APPENDIX

Tables X (a) and (b) give more detailed all-pairs measurements for GpH and Par monad. They show the effect of using different chunk sizes to adapt to the number of cores. For example, two chunks per PE generates enough chunks such that each core roughly has two chunks.

(a) GpH

	nochunk		1chunk/PE		2chunks/PE		4chunks/PE	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	20.04	1.00	20.05	1.00	20.03	1.00	20.06	1.00
1	20.64	0.97	27.80	0.72	24.07	0.83	22.10	0.91
2	16.76	1.20	12.73	1.58	11.98	1.67	11.33	1.77
4	13.89	1.44	6.42	3.12	6.15	3.26	5.97	3.36
8	15.64	1.28	3.40	5.90	3.35	5.98	3.29	6.10

(b) Par monad

	nochunk		1chunk/PE		2chunks/PE		4chunks/PE	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	20.30	1.00	20.06	1.00	20.03	1.00	20.04	1.00
1	20.48	0.99	20.16	1.00	20.08	1.00	20.19	0.99
2	10.96	1.85	10.91	1.84	10.98	1.82	10.94	1.83
4	5.93	3.42	5.85	3.43	5.82	3.44	5.78	3.47
8	3.29	6.17	3.24	6.19	3.22	6.22	3.25	6.17

Table X. *GpH* and *Par monad* measurements (All-Pairs, 16k bodies, 1 iteration)

Tables XI (a) and (b) shows the use of different skeletons and how they affect runtimes and speed-ups. Table XI (c) shows the actual number of processes, threads, conversations and messages overheads using each skeleton.

(a) Eden map skeletons (1)

	parMap		parMapFarm		parMapFarmMinus	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	22.11	1.00	22.13	1.00	22.12	1.00
1	362.38	0.06	23.67	0.93	23.59	0.94
2	294.99	0.07	11.91	1.86	23.33	0.95
4	259.19	0.09	6.08	3.64	7.73	2.86
8	245.72	0.09	3.41	6.49	3.49	6.34

(b) Eden map skeletons (2)

	parMapFarmChunk		parMapOfflineFarmChunk		workpoolSortedChunk	
no. PE	Runtime (s)	Speedup	Runtime (s)	Speedup	Runtime (s)	Speedup
Seq.	22.13	1.00	22.13	1.00	22.12	1.00
1	22.91	0.97	23.02	0.96	23.06	0.96
2	11.57	1.91	11.53	1.92	11.59	1.91
4	5.82	3.80	5.80	3.82	5.84	3.79
8	3.09	7.16	3.04	7.28	3.11	7.11

(c) Eden skeleton overheads - par. run on 8 cores

	processes	threads	conversations	messages
parMap	16001	32001	64000	64000
parMapFarm	9	17	48	32048
parMapFarmMinus	8	15	42	32042
parMapFarmChunk	9	17	48	80
parMapOfflineFarmChunk	9	17	40	56
workpoolSortedChunk	9	17	48	80

Table XI. *Eden* measurements (All-Pairs, 16k bodies, 1 iteration)