

DRAFT Proposal for a Masters Training Package

## **Advanced Mathematics and Statistics for Engineers**

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### **Summary**

We are seeking support to develop a suite of modules in Mathematics and Statistics to provide engineering graduates with transferable, modern mathematical skills at a high level. We are applying for funds to develop the content and delivery process of these modules, to advertise widely and to attract participants from technology-based companies, which form a vital part of the UK economy, particularly in Scotland. We already have substantial support from BAE Systems, who are committed to use these modules as training for employees in Edinburgh on a day-release basis. Any course materials developed will be available for use in whole or part by the various masters courses and MTPs already taught by our engineering departments, and will be open to suitably-qualified PhD students across the University. Once converted to distance mode, the courses will be available throughout the UK.

### **Heriot-Watt expertise**

The **Department of Mathematics** ([www.ma.hw.ac.uk/math.html](http://www.ma.hw.ac.uk/math.html)) is a leading UK and international centre for research in applied mathematics and was rated 5 in RAE 1996. The department's research covers a broad area ranging from industrial mathematical modelling, numerical analysis, mathematical biology through to the theoretical aspects of differential and other systems of equations. The strengths of the department relevant to this proposal are industrial mathematical modelling and computational mathematics.

The **Department of Actuarial Mathematics and Statistics** ([www.ma.hw.ac.uk/ams.html](http://www.ma.hw.ac.uk/ams.html)) plays a leading role in research and teaching for the actuarial profession worldwide, and has recently greatly enhanced its research in statistics and applied probability with the appointment of Professor Gavin Gibson from BIOS and Dr Serguei Foss from Novosibirsk State University. The main areas of interest relevant to this proposal are stochastic modelling, spatial stochastic interaction, communications and queueing networks, optimization and data analysis.

**Interdisciplinary Research** is a major part of the work of both departments. We run a series of interdisciplinary seminars and half-day workshops in which the speakers are from local industry or from the science and engineering departments in Heriot Watt, and this has led to the formation of new partnerships between engineers and mathematicians. Uniquely in mathematics, we have recently completed three TCS projects with oil and gas service companies (both Edinburgh-based SMEs), two of which won national prizes. One was awarded the UK TCS prize for "Best Application of Knowledge and Technology" in 1999 for work on simulation and modelling of the oil welltesting process, while the most recent (Spatial Positioning in Seismic Testing) with Concept Systems won a prize for reaching the final 7 from a total of 196 top-rated projects in 2000.

**Mathematics and Statistics courses for Engineers and Scientists** are a major part of our teaching. Both departments provide service teaching of engineering mathematics across Heriot-Watt, with courses ranging from basic mathematics and statistics in years 1 and 2 up to courses in advanced and vector calculus, linear algebra, and offering optional classes in computer algebra, PDE's, statistics and optimization in later years for BEng and MEng degrees. Unlike many mathematics departments we have maintained most of our science and engineering service teaching by providing an efficient, high-quality service, which we update regularly in consultation with our client departments.

Heriot-Watt University is a leader in the development and provision of **distance learning** and **computer based learning and assessment**, and both departments play a significant part in this work. We have been running a joint access course with Aberdeen University since 1992, using carefully prepared text materials and video conferencing with students from all over the Highlands and Islands. Our statistics courses are taught and used world-wide in training Actuaries. The CALM (Computer Assisted Learning in Mathematics) group founded in 1986 is a world leader. Most of this effort has been in producing materials for engineers and scientists. A major current project is the SCHOLAR programme which is making on-line material covering the new Advanced Higher courses (similar to A-level) available in all Scottish schools. The material is also being adapted for first year university courses in engineering mathematics. The Mathematics Department was also involved in developing the Webtest assesment software (<http://flex-learn.ma.hw.ac.uk>) which we plan to use in this MTP.

## **1. The national need**

Engineers play a key part in the UK economy, and mathematical techniques are a vital part of their toolkit. However, it seems to be generally accepted that UK engineers now learn much less mathematics in their studies at university than do their counterparts in other countries. Moreover, statistical and probability techniques are becoming ever more important in the understanding of complex systems, but are often neglected in undergraduate engineering courses. Many developments in engineering rely on mathematics beyond that normally found in a UK degree course, and ignorance of appropriate techniques slows the process of adopting new ideas and curtails innovation. In response to concerns raised by employers, the Engineering Council and the associated specialised professional engineering Institutions are actively trying to raise the mathematical (and other) skills of professional engineers through the recent overhaul of SARTOR regulations and other means.

In these circumstances there is a need for a strong mathematical component in the lifelong learning process of engineers in this country, which EPSRC has recognised specifically in the present call for MTP proposals.

## **2. Criticality of EPSRC support**

We had already identified BAE, a major employer in Edinburgh and throughout the UK, as being interested in our providing Continuing Professional Development modules for their staff before the current call for MTP proposals. We have come to an agreement with BAE to teach their staff courses in mathematical and statistical techniques for at least the next five years. The modules will be delivered to participants on day-release and will commence in October 2001. We will provide four modules (Phase I of the plan below) and develop more for delivery in later years.

The participation of BAE will generate initial income to cover some of the costs of delivery

of the first set of modules, and we are now soliciting interest from other local companies and organisations to expand student numbers. To go further and establish a sustainable and useful service for engineers throughout UK industry, we need to extend our portfolio of modules and to develop the materials into a flexible format for delivery in distance learning mode as well as the day-release on-campus model we are starting out with. Also, given that courses of this type are not widely available, we have to publicise our services to make companies aware of the potential benefits.

The University is absorbing the costs of development of the content of the first set of modules for delivery on-campus, but we need external funding to have a wider impact, and are therefore seeking support from EPSRC to

- develop the delivery mechanism of the first group of modules (Phase I of the list in the next section) by converting them to materials, both paper based and interactive, which can be studied in distance mode
- develop both the content and distance mode delivery mechanism of the second group of modules (Phase II of the list in the next section)
- market these courses through the University Publicity and Information Office and through visits to and from companies and possible partners in CPD provision.

### **3. Course content and method of delivery**

The aim is to provide high-quality, relevant masters-level courses which can be delivered on-campus or in distance mode. Courses on-campus will be designed to run weekly in half-day blocks for students in central Scotland. Initially, two modules will run on the same day so that students can attend both. Off-campus students will be taught using print and computer-based distance learning materials, supported by e-mail or video-conferencing, and augmented by short, concentrated periods on-campus.

As indicated earlier, we have a great deal of experience in the University and our departments of distance learning and computer based teaching and assessment. We will be using skilled staff in the Heriot-Watt Learning Technology Centre to develop print and web-based materials to support the teaching of the modules. The materials will include web-based simulations, for example to carry out repeated sampling and verify coverage properties of confidence intervals or to validate asymptotic distributions of estimators. The courses will also feature progress monitoring by sophisticated, randomised on-line testing by software developed over the past few years at Heriot-Watt.

The modules are listed below and described in detail in the Appendix. Our philosophy in teaching these modules is to give an outline of the theory behind the techniques and to concentrate on how and when to use them in practice. We aim to provide students with the information that they need to choose appropriate methods for solving mathematical and statistical problems. Particular emphasis will be placed on illustrating all methodology using engineering applications whenever possible, and a number of case studies will be used, each sufficiently complex as to require the use of multiple techniques, to provide a thread linking different topics within and between modules.

A small number of case studies will be used, .

#### **Phase I**

To be delivered on-campus (two per term) from 2001-2002 academic year and converted to distance learning mode as part of the MTP. See the Appendix for detailed course content.

- Introduction to probability and statistical inference
- Optimization (theory and application)
- Computational mathematics I and II

## Phase II

At least four of the following to be developed for on-campus and distance mode as part of the MTP. See the Appendix for detailed course content.

- Mathematics of communications:
- Transforms
- Computational mathematics III: differential equations
- New developments
- Mathematical methods with Maple

**Assessment** will be by a mixture of standard written examination at the end of the module, and practical, project-based assignments (usually with a computational component) submitted as short reports.

**Marketing** the modules has already met with success. We are starting with a baseline demand of at least 7 students per module from BAE alone and have had expressions of interest from many sources. We need EPSRC funding to bring the courses to a wider audience. To facilitate this we are working with the Institute for System Level Integration (ISLI), an organisation set up by a consortium of universities in central Scotland to provide postgraduate education, professional training and research in system level integration and system-on-chip technologies and our Petroleum Engineering department to refine our proposed modules to a form that can be used in their portfolios of postgraduate and CPD courses of direct relevance to engineers working in electronics and petroleum science. We will also work with the Smith Institute Farady Partnership (see letter of support) to promote the courses and develop content.

The **Course Director** is Dr Dugald B Duncan of the Mathematics Department who has wide experience as an undergraduate course director, and in working with engineers in industry, first as an employee of Atomic Energy of Canada Ltd. and then through the TCS and a current EPSRC grant in projects with oil service companies. The course will be monitored and run locally by the Internal Course Committee, which will initially involve everyone associated with this proposal. We will actively seek feedback from the participants and their employers. An External Examiner will be appointed to scrutinize the examination process and standards. The Industrial Advisory Panel will also receive regular reports and will monitor the effectiveness of the delivery and relevance of course content. The **quality assurance** measures will be governed by the University's strict guidelines.

## 4. Cost effectiveness

The same materials will be used for on-campus courses and in distance mode. We plan to use a uniform style, breaking down each module into smaller self-contained units wherever possible, so that new combinations of topics can be built up as the need arises.

The courses will be used in whole or part to augment existing masters degree programmes, including the Flexible Learning Advanced Masters Degree in Energy and the Safety, Risk and Reliability Engineering MTPs taught on campus. For example, parts of the "Introduction to probability and statistical inference" module will be used to provide the theoretical component of the statistics course in the Water Resources Engineering Management and Geotechnical

Engineering MScs. We also plan to use some of the materials in an MSc course for students with a background in mathematics and statistics at a later date.

## **5. Level and nature of employer engagement**

We have set up an Industrial Advisory Board consisting of Professor Steve Beaumont (Director, ISLI), Mr Laurence Ormerod (MD, Edinburgh Petroleum Services Ltd), Professor John Roulston OBE (Director of Technology, BAE Systems Avionics Group) and Professor David Broomhead (now UMIST, formerly of DRA) and have already had extensive and valuable discussions with them, particularly about the Phase I courses.

We have already agreed to provide the four Phase I modules on-campus for the next five years (starting in October 2001) for BAE employees in or near Edinburgh. Their agreed fees over this period will be in the region of £100,000, which will go some way to covering the cost of delivery. This gives us a financial cushion and gives the course credibility with other employers, to ease our task of expanding student numbers.

We have other letters of support and interest, including offers of case studies and “guest lecturer” contributions.

## **6. Relevance of training opportunities**

We will be offering CPD courses at masters level to be taken on-campus or in distance mode. Each module will require the equivalent of 120 hours student effort. The courses will also be used in whole or part in other masters programmes in the university. Further, the university is discussing the establishment of an MSc degree and postgraduate diploma programme in which part time students accumulate a portfolio of masters level course credits in a range of subjects relevant to the full range of their work, including this mathematics MTP, as well as carrying out a suitable research project undertaken partly in the workplace.

Close liaison with employers will ensure the relevance of material to the needs of students on the course. Our Phase I modules have been designed in consultation with BAE Systems; detailed syllabuses for later modules will be the result of further consultation with the Industrial Advisory Board and interested employers. We are also seeking ways to fit our modules into SARTOR “matching section” training.

Our **target entry qualification** is at the level of a graduate engineer from an accredited programme in a UK university.

## **7. Relationship to existing provision**

We aim to provide high-quality courses providing generic mathematical skills at an advanced level for engineers with a “standard” UK engineering undergraduate background. Current provision of such courses in the UK seems to be non-existent. We think that we can fill the need for on-campus CPD in central Scotland, and for distance learning CPD across the UK.

## **8. How should the proposal be judged?**

The success of the MTP will be judged on:

- meeting national and corporate needs
- favourable feedback from companies and course participants
- production of a suite of high quality modules which can be delivered on campus or in distance mode

- External Examiner's confirmation of the rigour of the course materials and examinations
- Industrial Advisory Panel's confirmation of the usefulness of the material and effectiveness of its delivery
- significant increase in our range of clients and number of course participants
- achievement of financial viability and contribution to the finances of the University

## Financial Summary

To be completed very soon - can be ignored for now.

|                                | 2002-3 | 2003-4 | 2004-5 | 2005-6 | 2006-7 |
|--------------------------------|--------|--------|--------|--------|--------|
| Module registration income     | 25500  | 25500  | 25500  | 25500  | 25500  |
| Support for fees for SMEs      | £4500  | £4500  | £4500  | £4500  | £4500  |
| Distance Materials Development |        |        |        |        |        |
| Marketing                      | £3000  | £3000  | £3000  | £3000  | £3000  |

# Appendix

## Modules for 2001-2002

### Introduction to probability and statistical inference

This module will provide students with an introduction to the key concepts in probability theory and the application of these to statistical modelling, parameter estimation and hypothesis testing. At the end of the module, students should be familiar with the fundamental concepts of statistical inference that will underpin subsequent work using statistics.

Topics covered in the course will include:

- Sample spaces and events, conditional probability and Bayes' Theorem, independence of events;
- The concept of discrete and continuous random variables, distribution and expectation;
- Standard distributions including normal, Poisson, Gamma, Chi-square etc.;
- Distributions of particular engineering relevance such as Weibull, k-distribution, Rayleigh;
- Generating functions and their application;
- The Central Limit Theorem and its applications;
- Maximum likelihood and method of moment estimators;
- Pivotal quantities and construction of confidence intervals;
- Hypothesis testing, likelihood ratio tests and the Neyman-Pearson lemma;
- Introduction to Bayesian inference.

### Optimization

This module aims to introduce optimization theory and applications. It ranges from fundamental concepts to a wide range of applications. Matlab will be used for practical demonstrations and exercises.

Topics covered in the course will include:

- Fundamental analytical results on existence, uniqueness and location of maxima and minima; stationarity.
- Unconstrained and constrained optimization.
- Numerical techniques.
- Lagrangian theory - strong and weak (classical), KKT theory, Duality
- Linear programming: theory, algorithms, duality, degeneracy, special problems, e.g. transportation, flows in networks.
- Other network problems, e.g. shortest path, routing problems.
- Branch and bound techniques, integer and mixed integer programming.
- Evolutionary algorithms, genetic programming and genetic algorithms.
- Simulated annealing.
- Tabu search.

## Computational Mathematics I and II

This course is a double module running over two terms which aims to provide a toolkit of modern techniques in numerical computation. A combination of background theory with practical applications using Matlab will be used to get the main ideas across and case studies (e.g. circuit theory) will be used to act as a unifying thread through the material.

Topics covered in the course will include:

- Introduction to the Matlab scientific computing package.
- Linear Algebra - solving dense and sparse linear systems directly
- Linear Algebra - iterative methods for linear systems of equations
- Linear Algebra - finding eigenvalues and the singular value decomposition.
- Solving Nonlinear Algebraic Equations - rootfinding in  $f(x)=0$  and methods for system of equations
- Interpolation and Approximation - polynomials
- Interpolation and Approximation - splines
- Data Fitting and Parameter Estimation
- Numerical Integration - Midpoint, Trapezoidal and Gauss methods
- Numerical Integration - automatic adaptive methods using error estimation
- Introduction to the solution of ODEs - methods, stiff systems, automatic adaptive methods
- Introduction to the solution of PDEs - basic methods, finite elements used in standard packages

## Modules for Further Development

Below are outlines of further modules which we are actively discussing with our Advisory Panel. We expect to go ahead with at least three from this selection.

### Mathematics of Communications

Security (RSA algorithms, etc)

Coding, compression, error correction algorithms, etc

Digital signal processing

Multiplexing

Mathematics of packet-switched networks: routing and switching, buffering, effective bandwidth, dimensioning of networks and components, transmission and acknowledgement protocols, admission and congestion control.

Capacity reservation in networks.

### Mathematical Methods with Maple

We use Maple to develop skills in using modern algebraic manipulation systems as a tool for applying mathematics. The topics covered will include: differentiation, integration, solving linear and nonlinear equations, matrix manipulation, eigenvalues+vectors, Taylor series, solving differential equations, vector calculus.

The course will also serve as a refresher in a wide range of mathematical techniques.



## **Transforms**

The aim of this course is to provide a unifying guide to the theory and uses of transforms in signal processing, data compression, solution of linear PDEs etc.

Theory and applications of discrete and continuous transforms including: Fourier, Laplace, Z, Discrete Fourier and FFTs, Wavelets.

## **New Developments**

The aim of this course is to provide a balanced view of what these new techniques are and how they can be used to solve engineering problems.

An overview of chaos, fractals, dynamical systems, wavelets, genetic algorithms and other “hot topics” in modern mathematics.

## **Computational Mathematics III**

A combination of background theory with practical applications using Matlab and the PDE Toolbox or FEMLAB will be used to get the main ideas across and case studies will be used to act as a unifying thread through the material.

Advanced methods for solving ODE initial and boundary value problems.

Elliptic, Parabolic and Hyperbolic PDEs by finite differences, finite elements and finite volume methods