Cellular Automata

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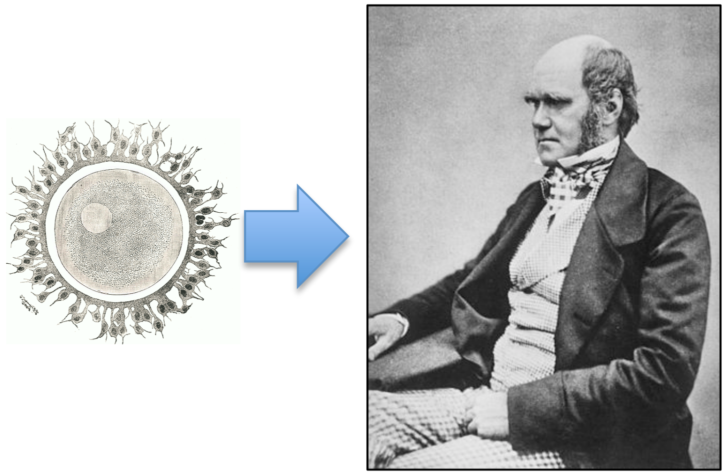
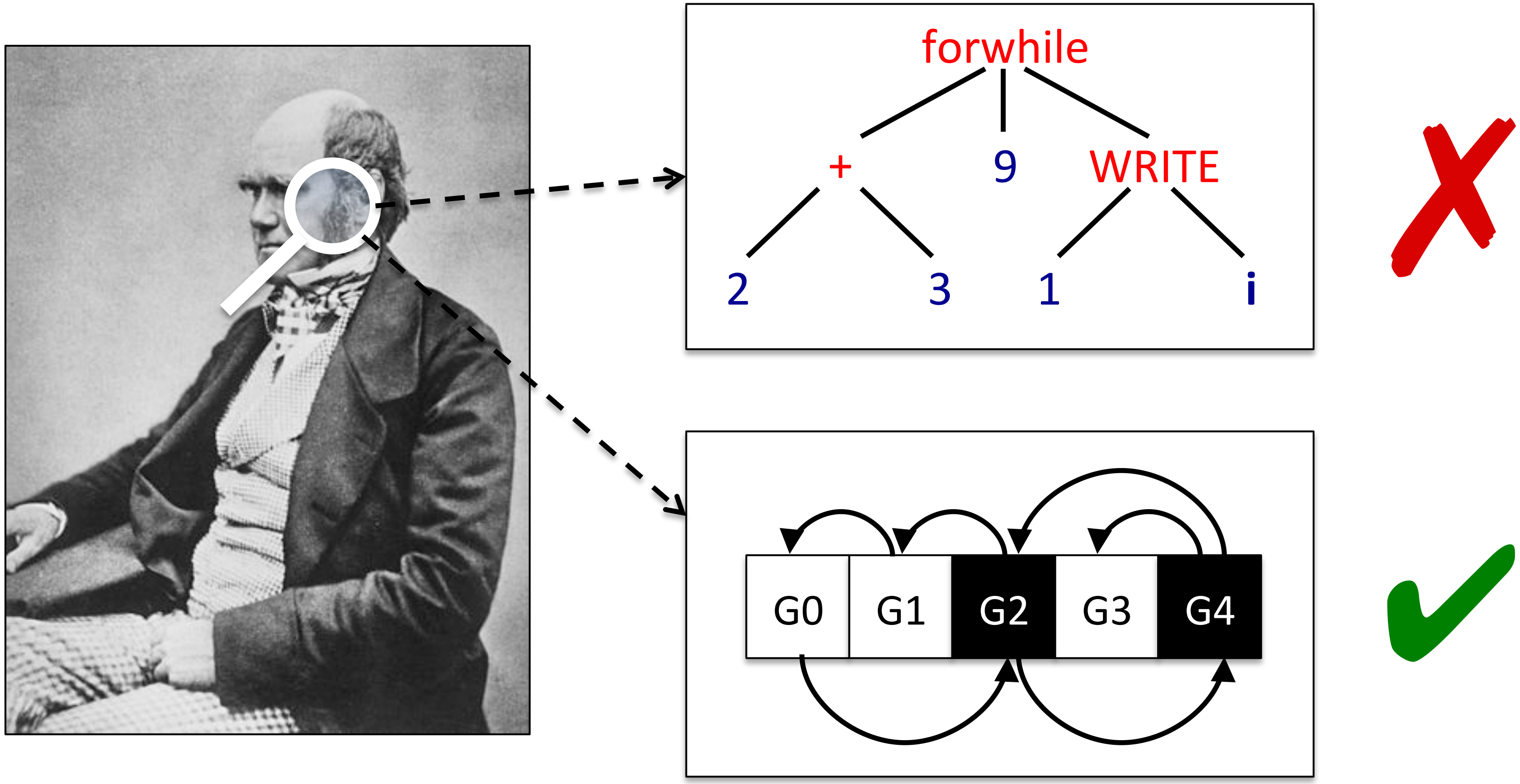
Previous Lectures

- Genetic programming
  - Is about evolving computer programs
  - Mostly conventional GP: tree, graph, linear
  - Mostly conventional issues: memory, syntax

- Developmental GP encodings
  - Programs that build other things
  - e.g. programs, structures
  - Biologically-motivated process
  - The developed programs are still “conventional”



Today’s Lecture



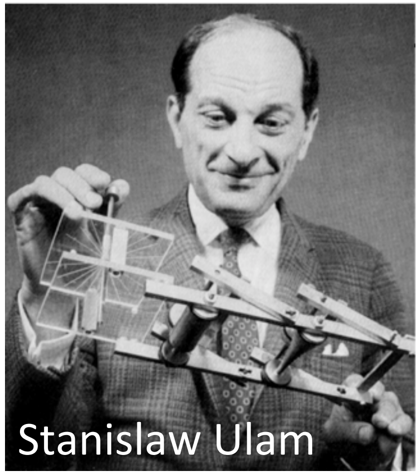
Computation

- Conventional
  - Centralised
  - Top-down
  - Halting
  - Static
  - Exact
  - Fragile
  - Synchronous
- Biological
  - Distributed
  - Bottom-up (emergent)
  - Ongoing
  - Dynamical
  - Inexact
  - Robust
  - Asynchronous

- See Mitchell, "**Biological Computation,**" 2010
  http://www.santafe.edu/media/workingpapers/10-09-021.pdf

Cellular Automata (CA)

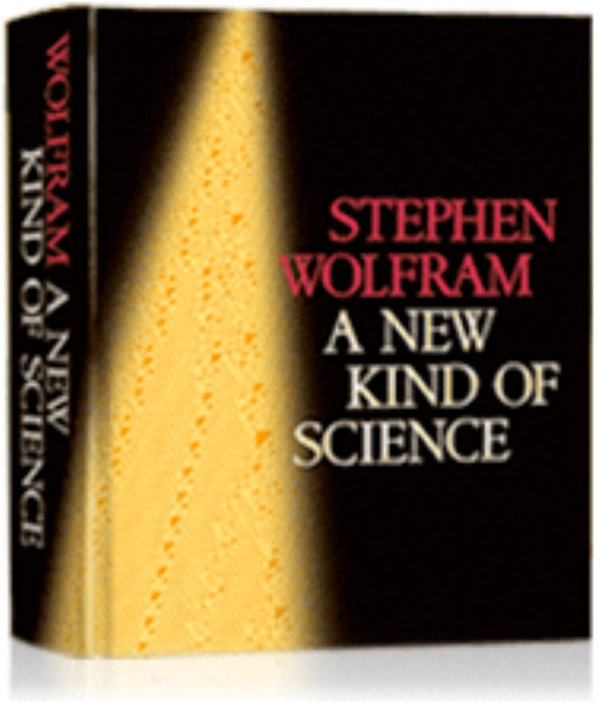
- What is a cellular automaton?
  - A model of distributed computation
    - Of the sort seen in biology
  - A demonstration of "emergence"
    - **complex** behaviour emerges from interactions between **simple** rules
  - Developed by Ulam and von Neumann in the 1940s/50s
  - Popularised by John Conway's work on the 'Game of Life' in the 1970s
  - Significant later work by Stephen Wolfram from the 1980s onwards


Stanislaw Ulam


John von Neumann

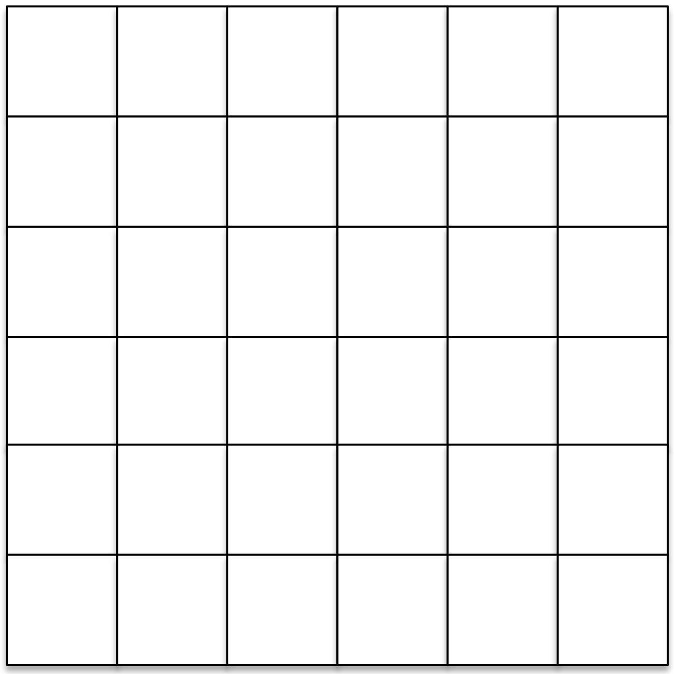
Book – free to read online

- Stephen Wolfram, **A New Kind of Science**, 2002
- https://www.wolframscience.com/nksonline/toc.html



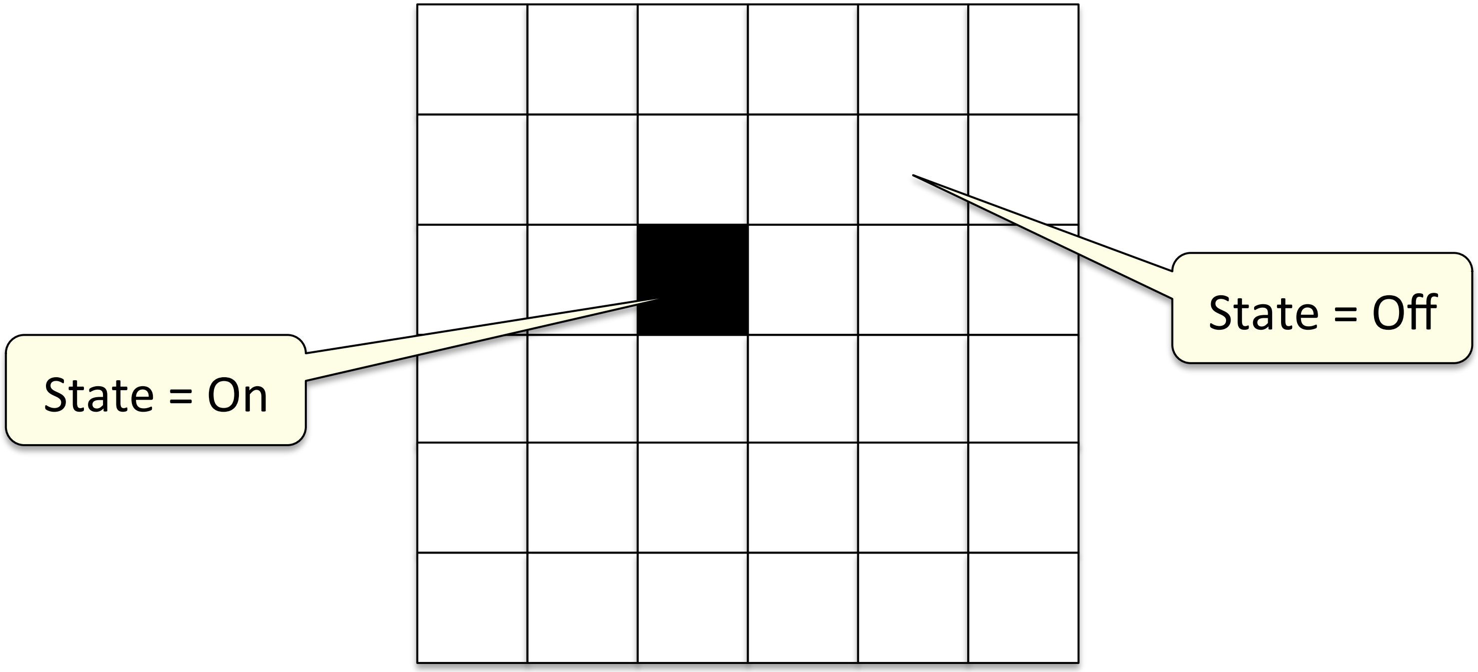
Definition

- Computation takes place on a grid, which may have 1, 2 or more dimensions, e.g. a 2D CA:



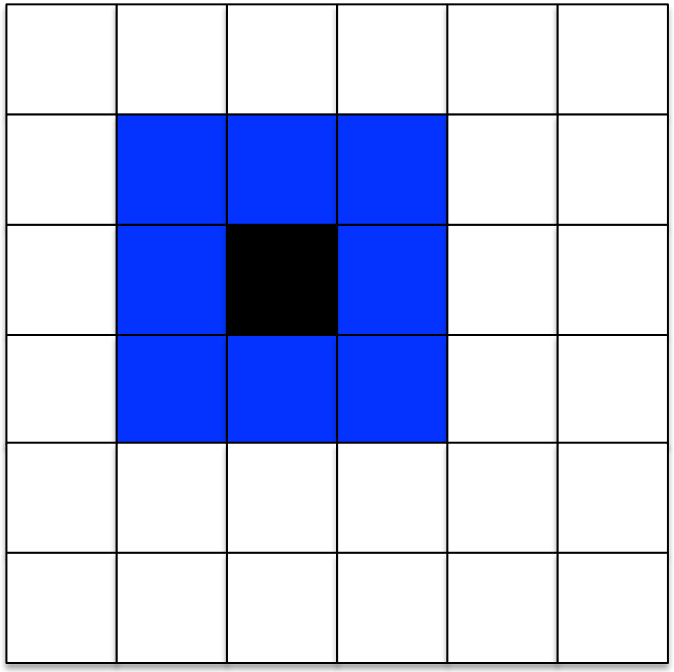
Definition

- At each grid location is a **cell**
  - Which has a **state**
  - In many cases this is binary:



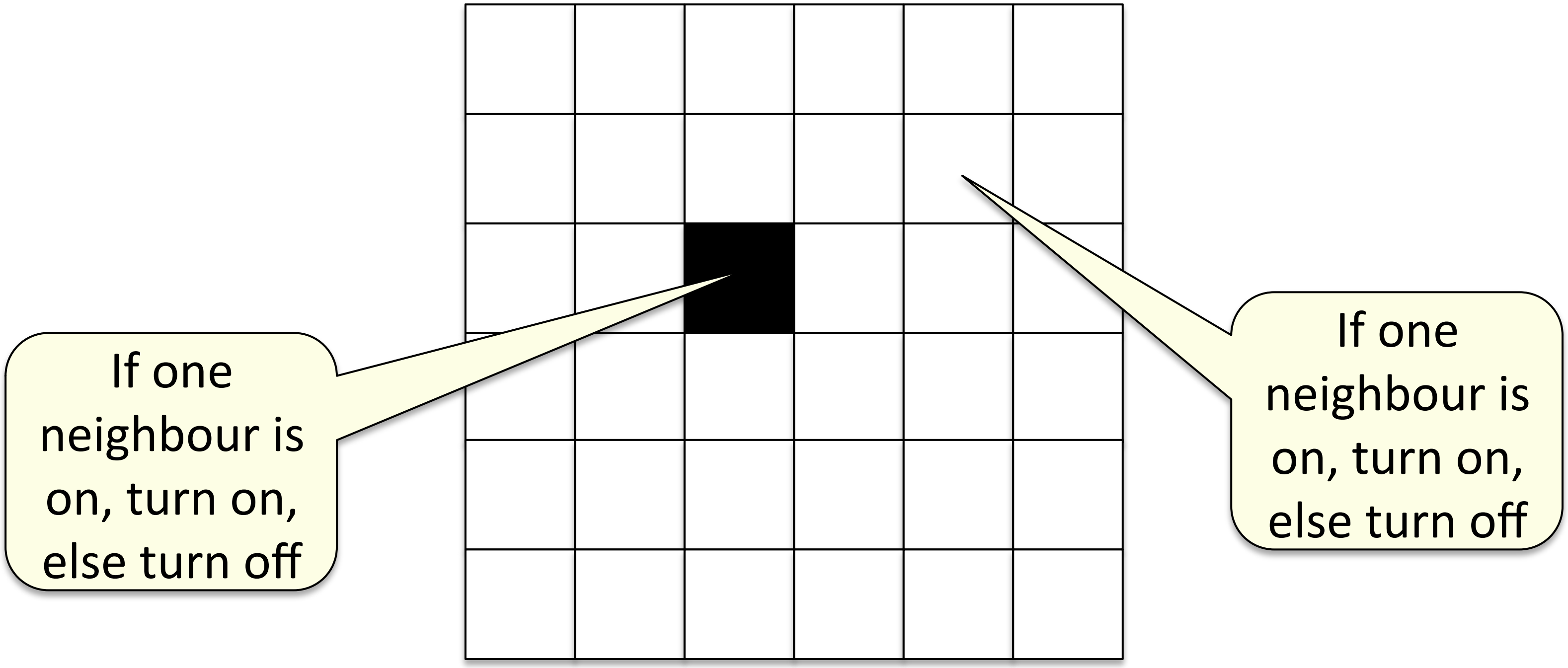
Definition

- Each cell contains an **automaton**
  - Which observes a **neighbourhood** around the cell



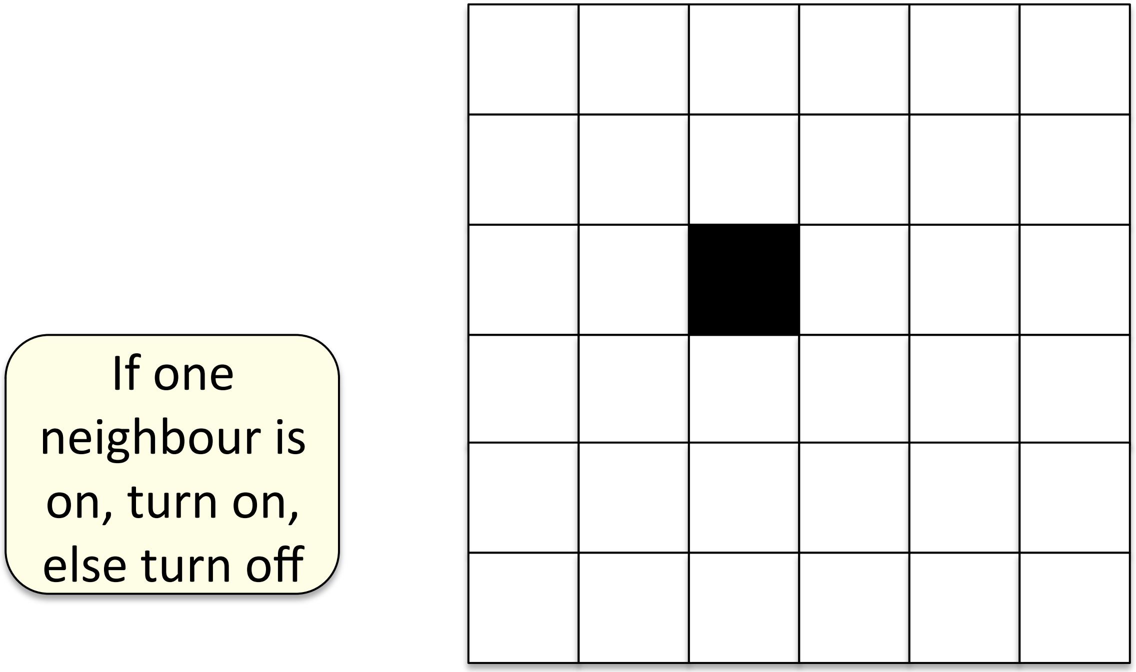
Definition

- Each cell contains an automaton
  - And applies an **update rule** based on this neighbourhood
  - Every automaton uses the same update rule



Definition

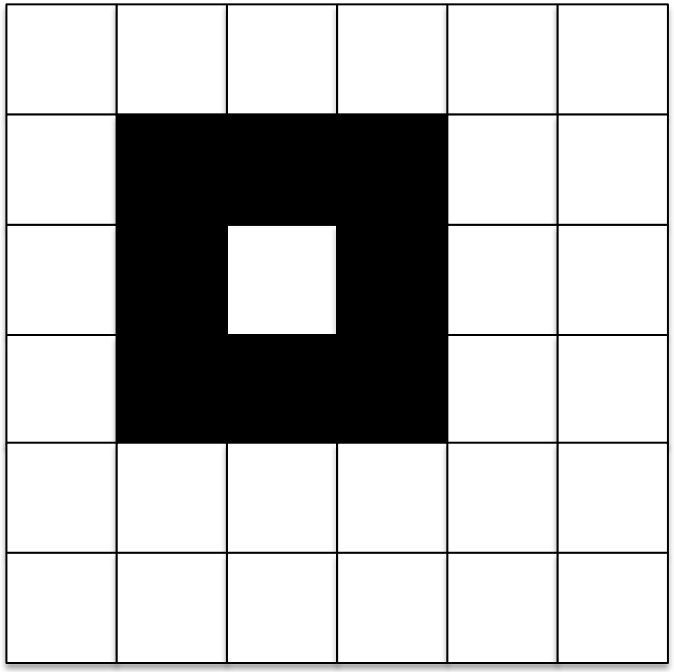
- The CA is run over a number of discrete time steps
  - At each time step, each automaton applies its update rule
  - Time = 0



Definition

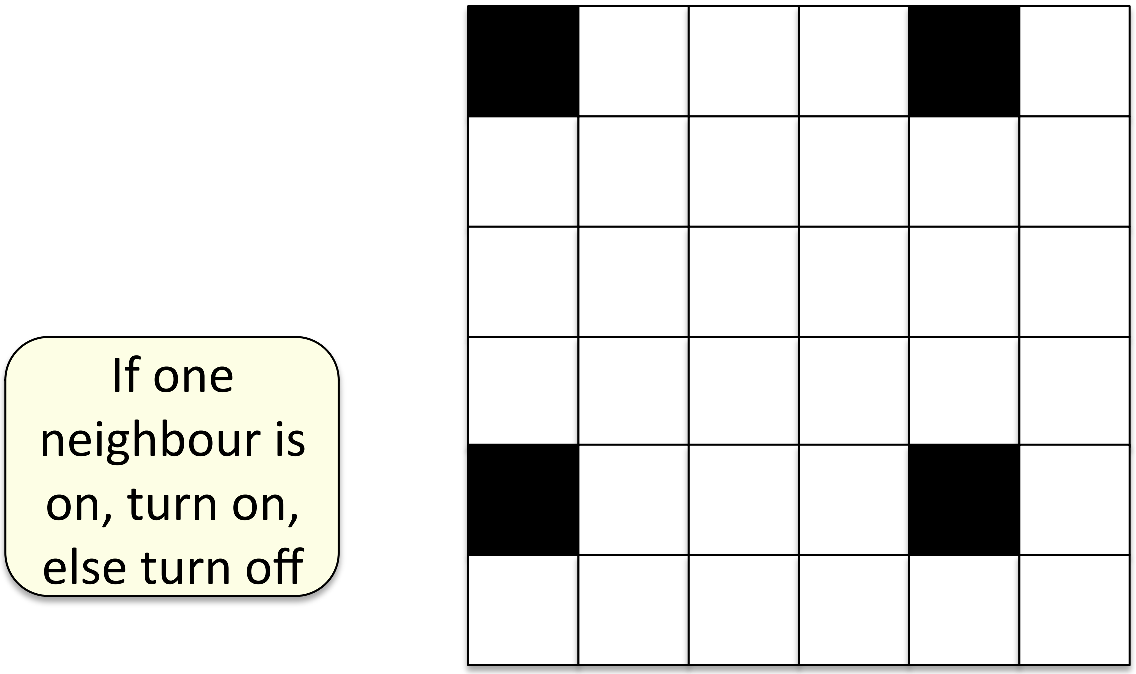
- The CA is run over a number of discrete time steps
  - At each time step, each automaton applies its update rule
  - Time = 1

If one neighbour is on, turn on, else turn off



Definition

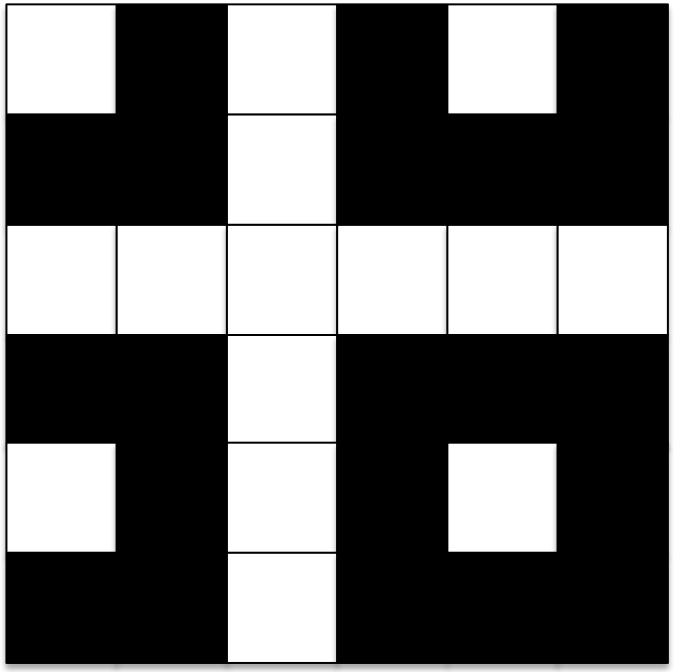
- The CA is run over a number of discrete time steps
  - At each time step, each automaton applies its update rule
  - Time = 2



Definition

- The CA is run over a number of discrete time steps
  - At each time step, each automaton applies its update rule
  - Time = 3

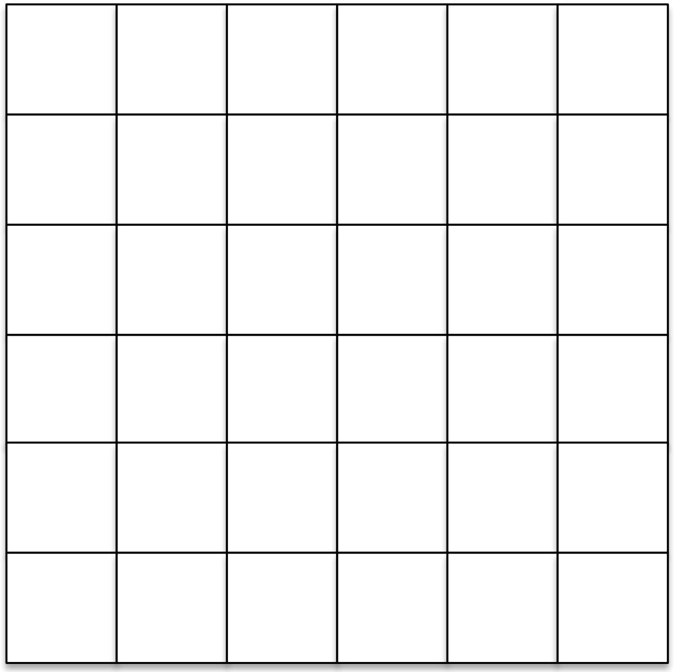
If one neighbour is on, turn on, else turn off



Definition

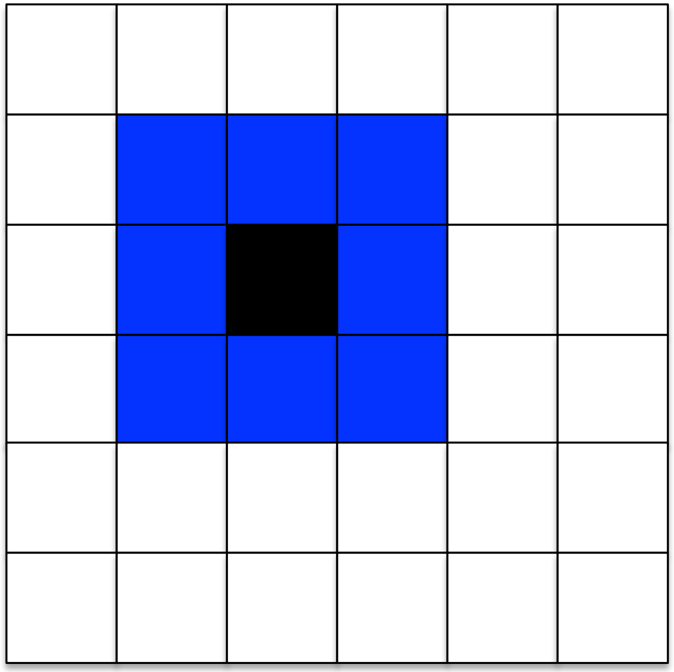
- The CA is run over a number of discrete time steps
  - At each time step, each automaton applies its update rule
  - Time = 4

If one neighbour is on, turn on, else turn off



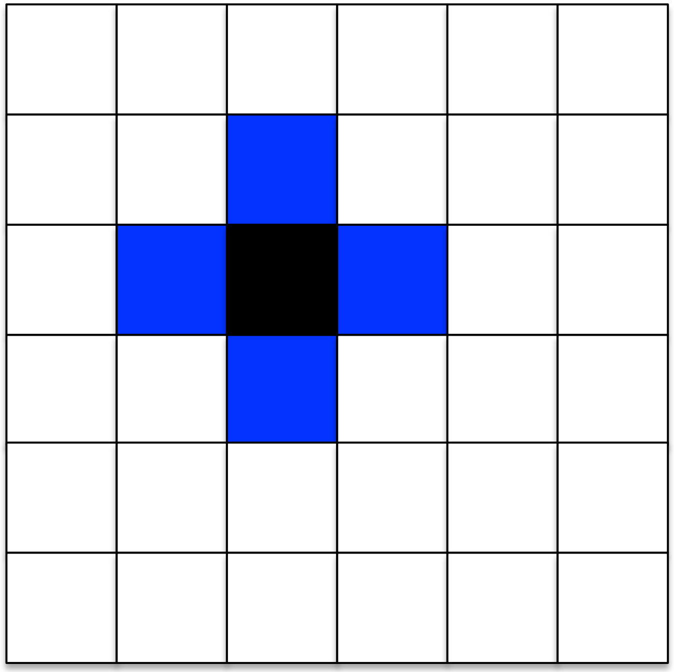
Neighbourhoods

- A number of different neighbourhoods are used in CAs
  - This is called a **Moore** neighbourhood



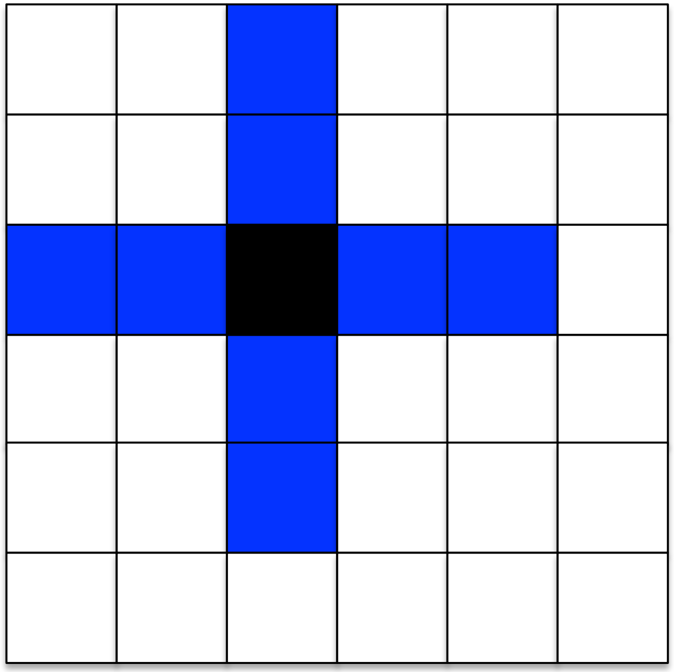
Neighbourhoods

- A number of different neighbourhoods are used in CAs
  - This is called a **von Neumann** neighbourhood



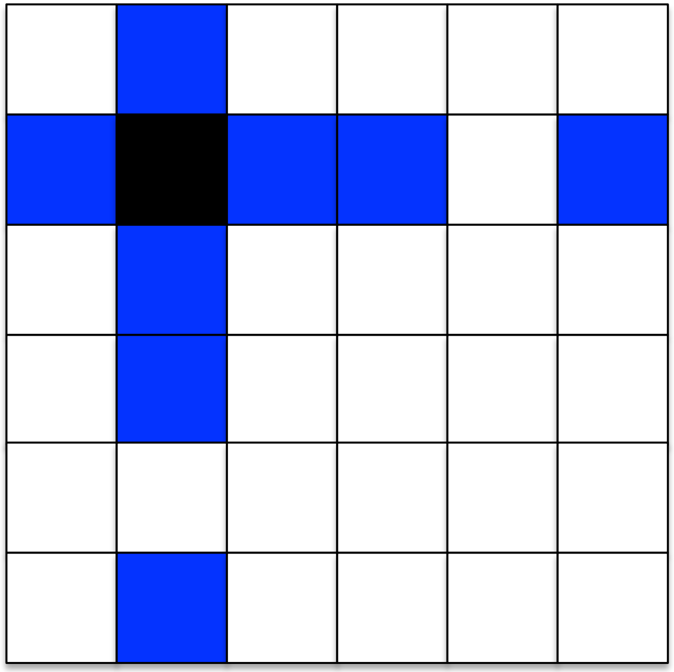
Neighbourhoods

- A number of different neighbourhoods are used in CAs
  - This is called an **extended von Neumann** neighbourhood



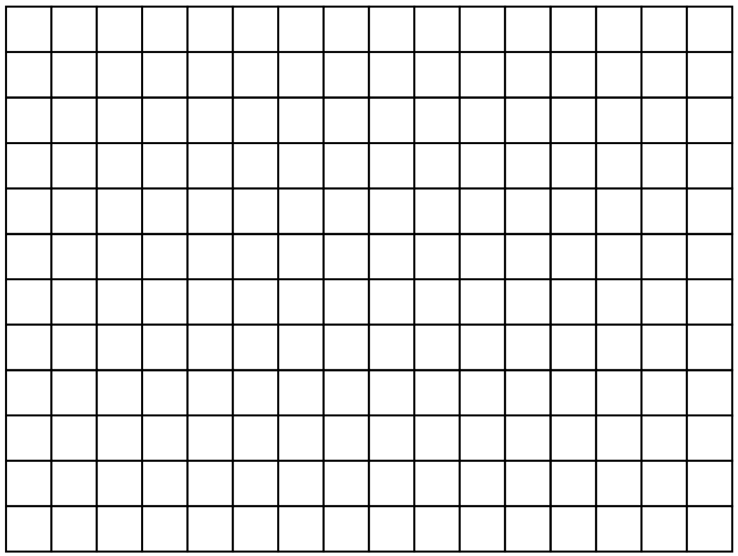
Neighbourhoods

- A number of different neighbourhoods are used in CAs
  - At the edges, **toroidal** neighbourhoods are often used
  - Also known as periodic boundary conditions



Simple Example

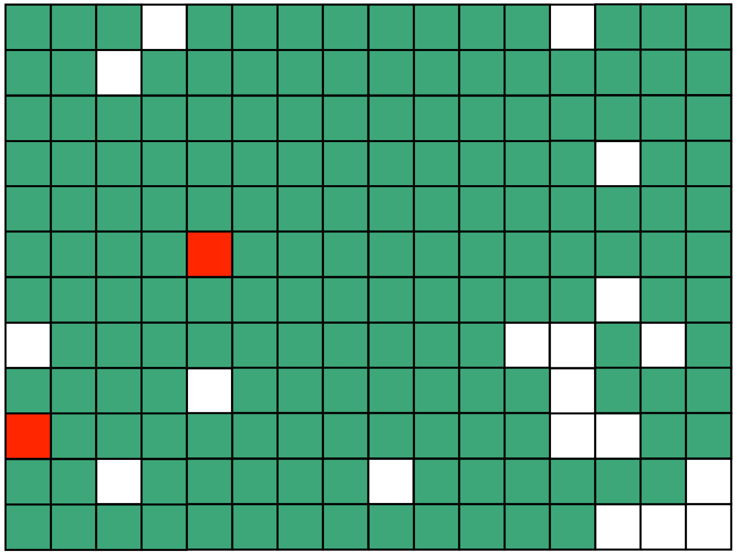
- Modelling how a forest fire spreads*
  - A forest is modelled as a 2D grid of automata
  - A tree may or may not grow in each cell



*Thanks to David Corne for this example

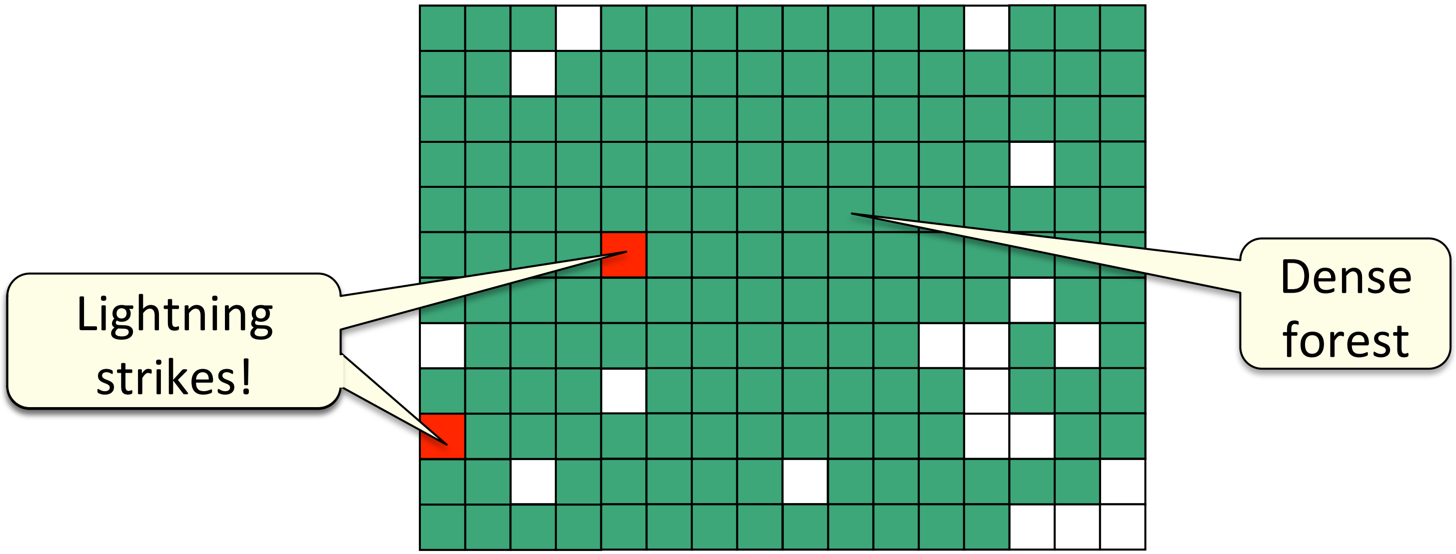
Simple Example

- Each automata has one of three states
  - **Empty** indicating no tree
  - **ok_tree** indicating a healthy tree
  - **fire_tree** a tree that's on fire



Simple Example

- Each automata applied this rule in a Moore neighbourhood
  - If a tree is not on fire, and has *n* neighbours on fire, it catches fire with probabilty *n*/8. If on fire for 3 steps, a tree dies
  - Time = 0



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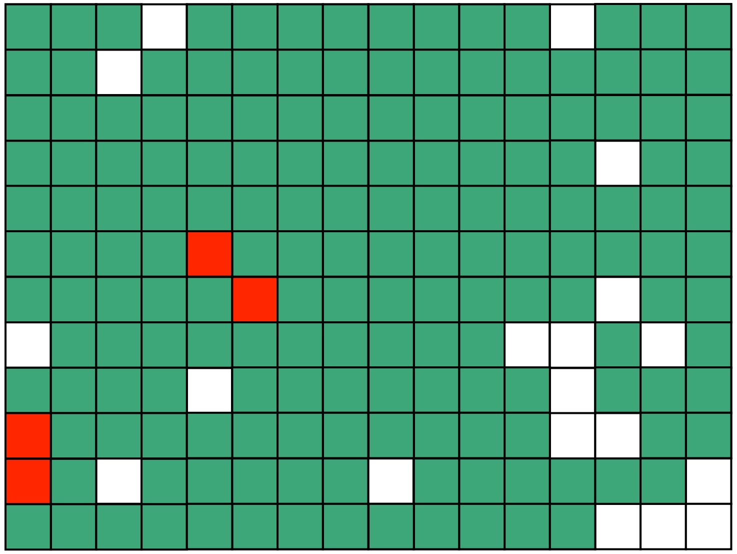
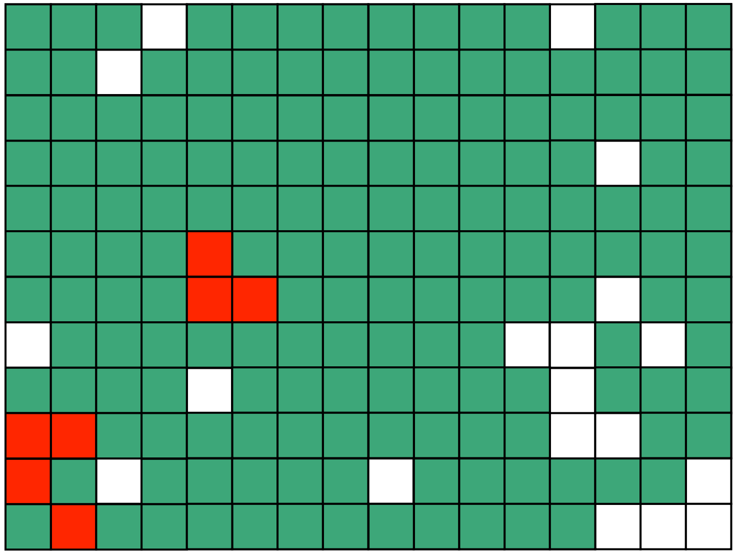


Simple Example

- Each automata applied this rule in a Moore neighbourhood
  - If a tree is not on fire, and has *n* neighbours on fire, it catches fire with probabilty *n*/8. If on fire for 3 steps, a tree dies
  - Time = 2



Simple Example

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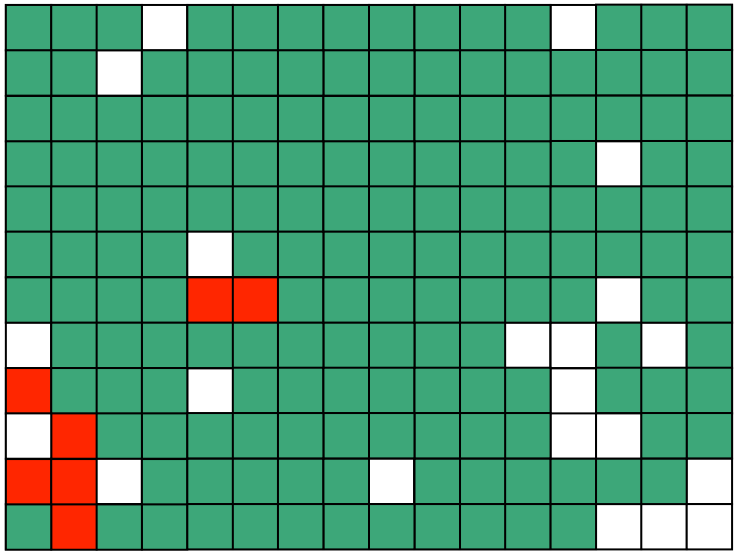

Simple Example

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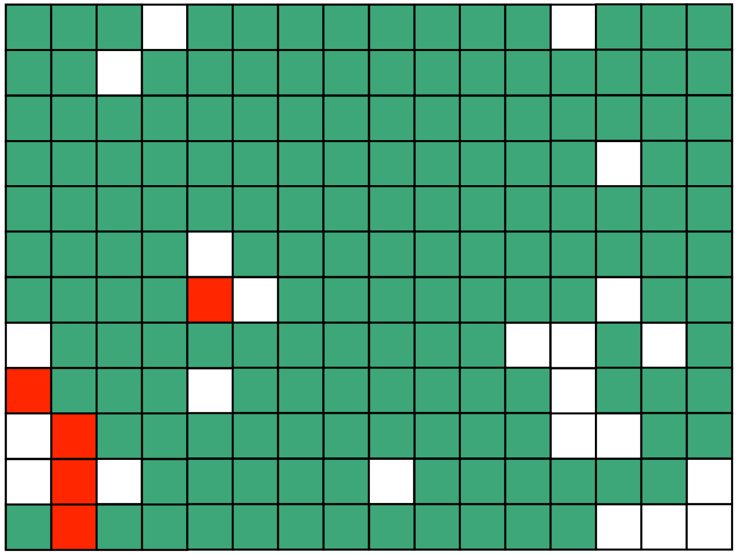
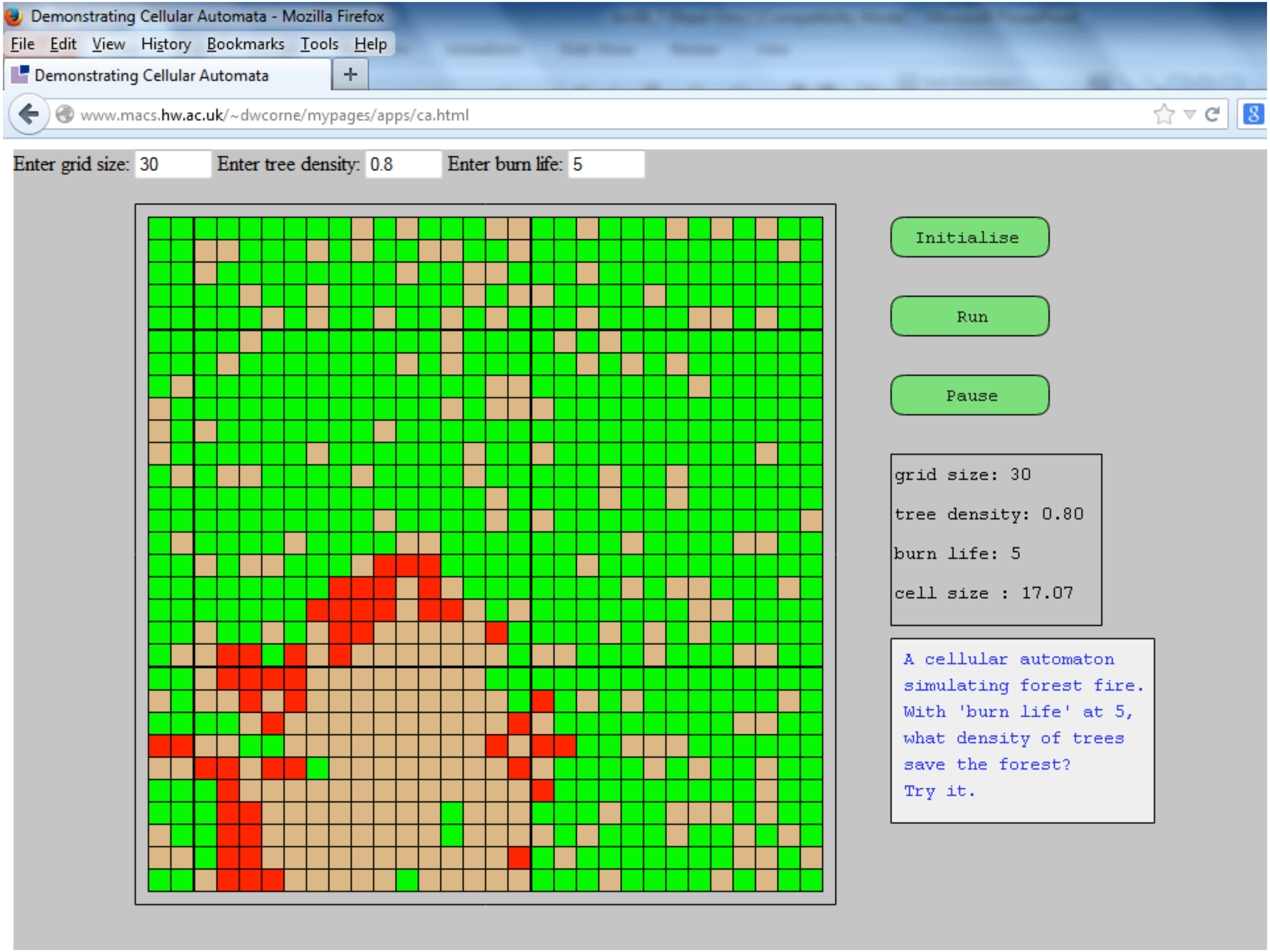
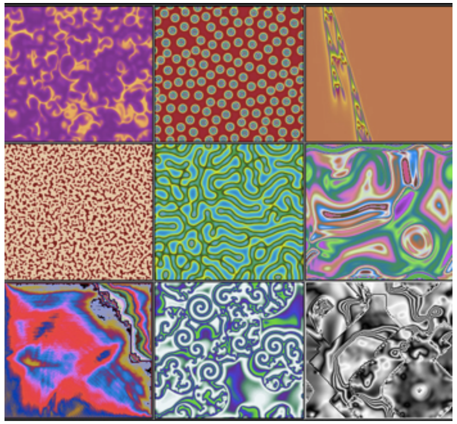
  - Time = 4




Demonstrating Cellular Automata - Mozilla Firefox
File Edit View History Bookmarks Tools Help
Demonstrating Cellular Automata
www.macs.hw.ac.uk/~dwcorne/mypages/apps/ca.html
Enter grid size: 30
Enter tree density: 0.8
Enter burn life: 5
Initialise
Run
Pause
grid size: 30
tree density: 0.80
burn life: 5
cell size : 17.07
A cellular automaton simulating forest fire. With 'burn life' at 5, what density of trees save the forest? Try it.

Cellular Automata (CA)

- What are cellular automata used for?
  - Modelling spatial processes
    - e.g. forest fires, disease spread
  - Modelling physical processes
    - e.g. crystal formation, thermodynamics
  - Modelling biological processes
    - e.g. pattern formation, self-replication
  - Solving computational problems
    - e.g. random number generators, ciphers
  - Parallel processing architectures
    - e.g. systolic arrays, Connection Machine ➔

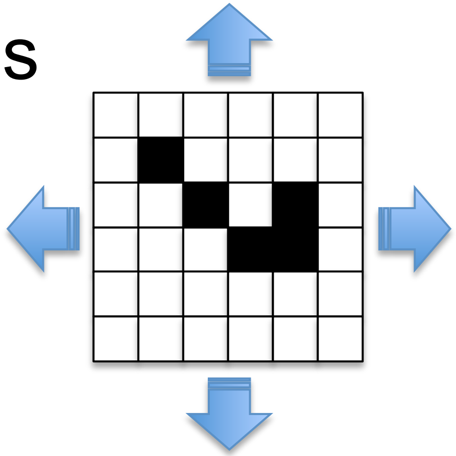


http://www.rudyrucker.com

www.mission-base.com/tamiko

Conway’s Game of Life

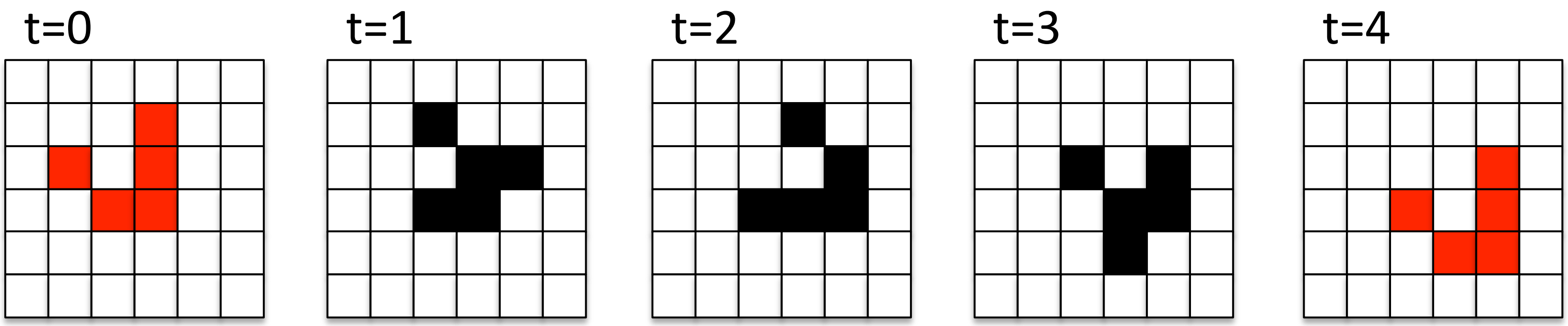
- Developed by John Conway in the 1970s
  - A simple model of self-replication
  - Surprisingly complex behaviour
  - Led to wider interest in CAs

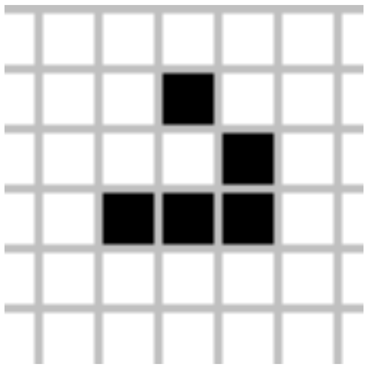


- 2 states (**live**, dead), Moore neighbourhood, 4 rules:
  - A live cell with <2 live neighbours dies (*under-population*)
  - A live cell with 2-3 live neighbours remains alive
  - A live cell with >3 live neighbours dies (*over-crowding*)
  - A dead cell with 3 live neighbours becomes a live cell (*reproduction*)

Spaceships

- Moving elements that emerge from these rules
  - The most famous is the glider:





Spaceships

- Moving elements that emerge from these rules
  - Some more complex examples:

http://en.wikipedia.org/wiki/File:Animated_spaceships.gif (animated)

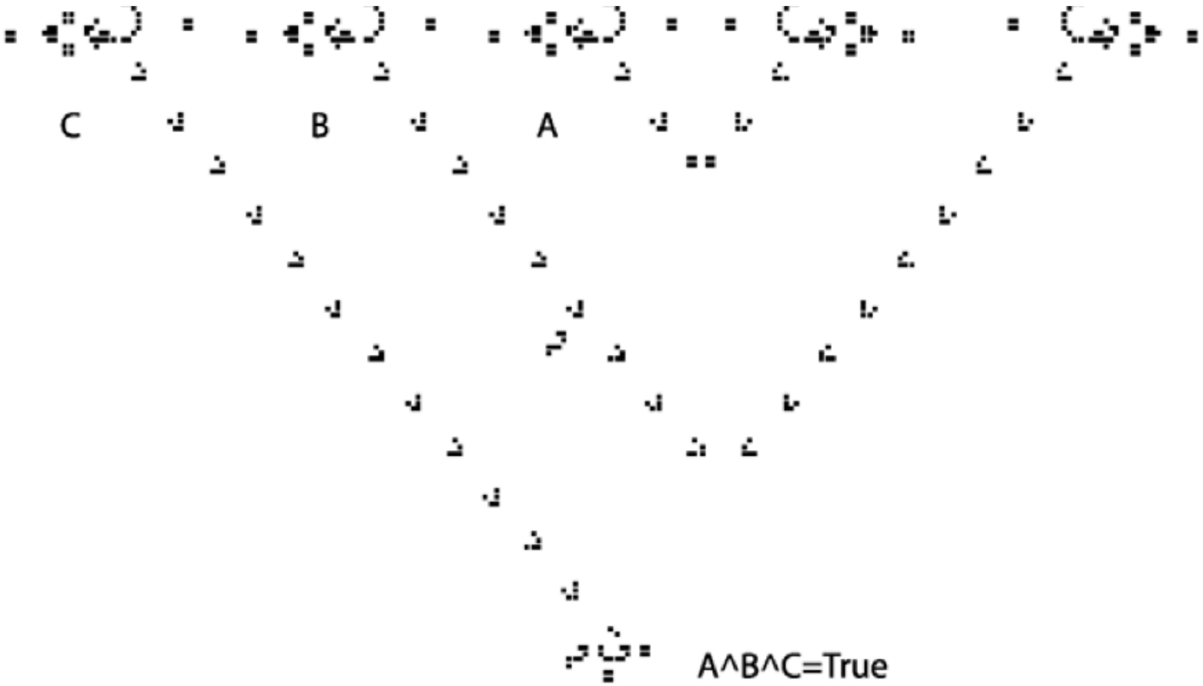
Guns

- Structures that generate streams of spaceships
  - Gosper's glider gun is the smallest known example:

http://en.wikipedia.org/wiki/File:Gospers_glider_gun.gif (animated)

Turing Completeness

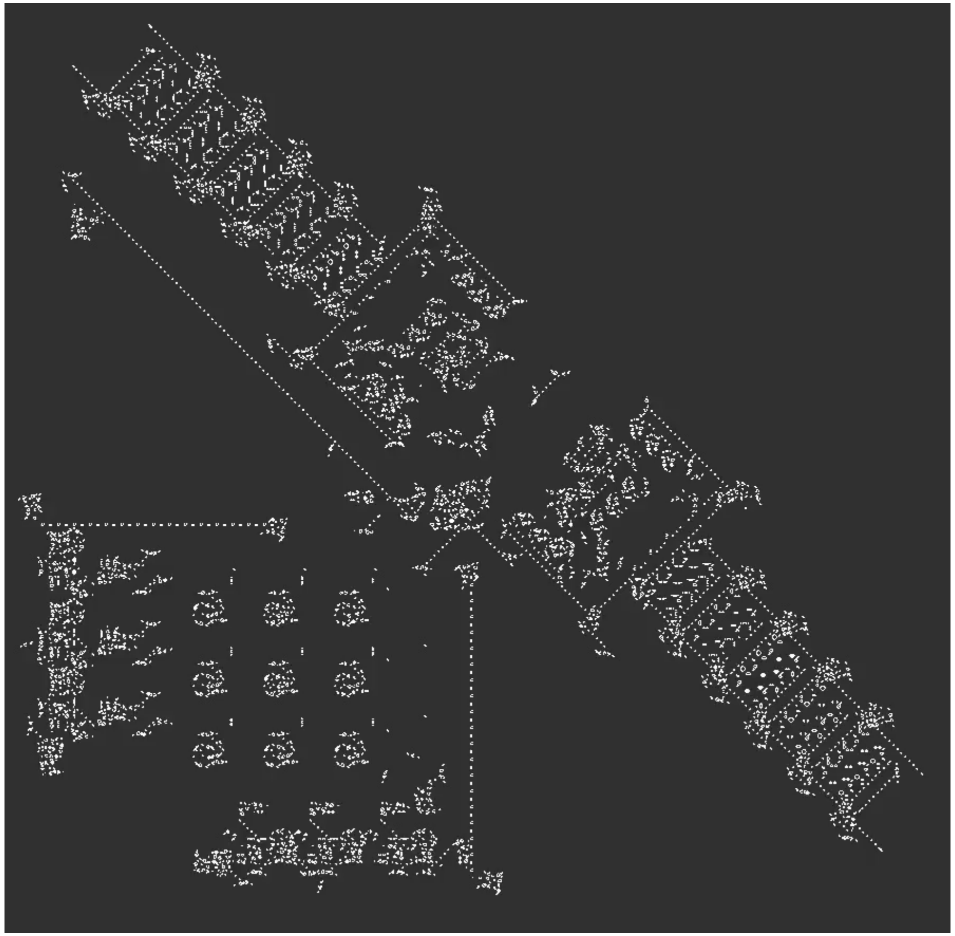
- Game of Life famously shown to be Turing complete
  - i.e. capable of universal computation
  - Proven by implementing logic gates with gliders

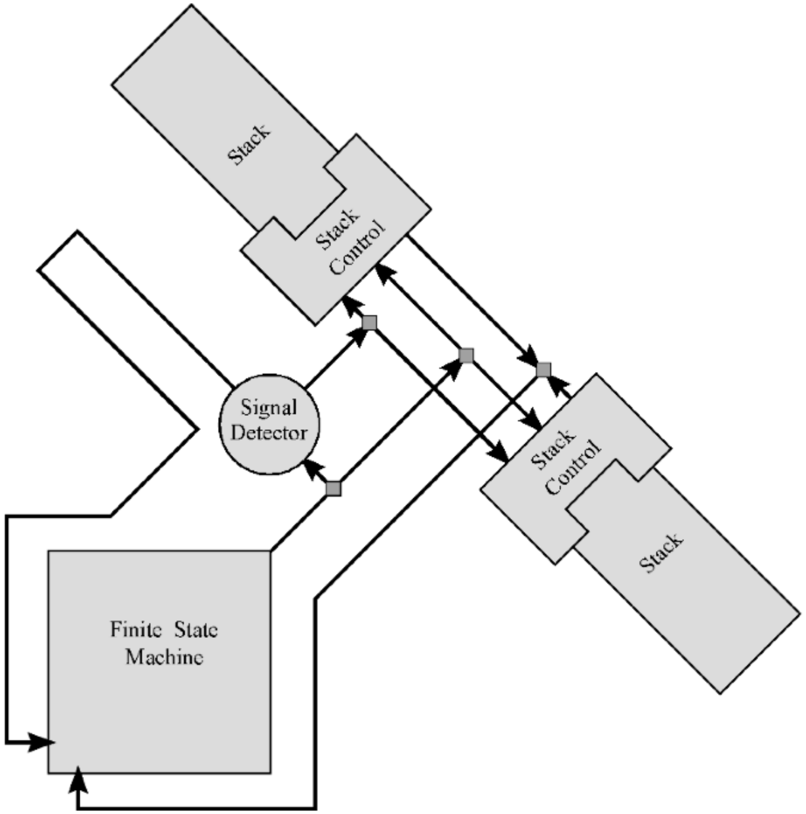


Jean-Philippe Rennard, http://arxiv.org/pdf/cs/0406009.pdf

Turing Machine

- Rendell, A Universal Turing Machine in Conway's Game of Life
  - http://uncomp.uwe.ac.uk/CAAA2011/Program_files/764-772.pdf

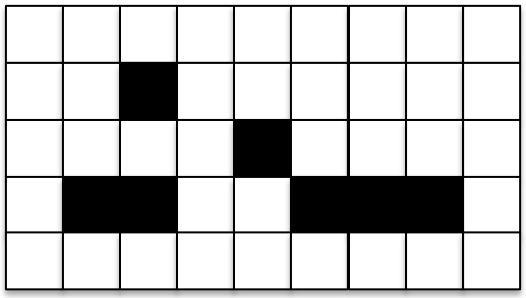




Methuselahs

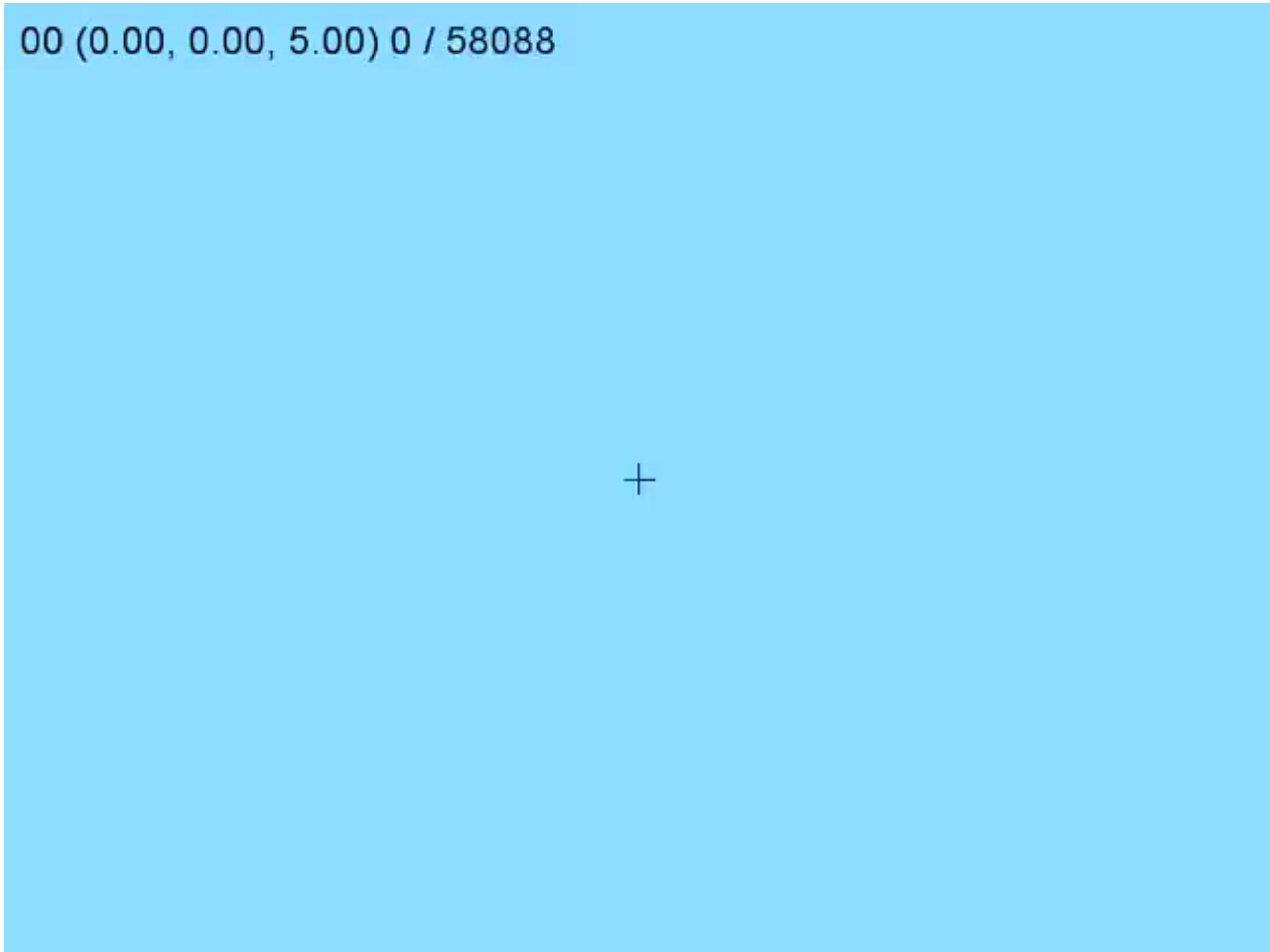
- Patterns that grow and take a long time to stabilise
  - Complexity emerges from simple rule and initial state
  - Can be seen as carrying out a complex computation

- Acorn: size 7, grows to 1057, lasts 5206 time steps
  - Stable pattern consists of 41 blinkers, 4 traffic lights, 34 blocks, 30 beehives, 1 honey farm, 13 gliders, 8 boats, 5 loaves, 3 ships, 2 barges, 2 ponds and 1 mango



http://www.conwaylife.com/wiki/Methuselah

Acorn



https://www.youtube.com/watch?v=U2dB57bwIWQ
http://altsoph.com/projects/conwaytree/

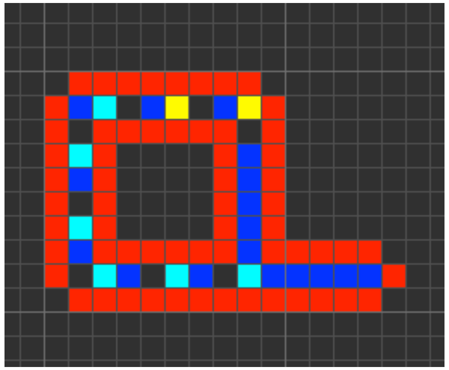
Game of Life

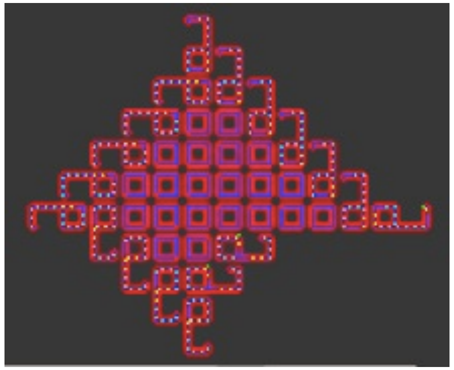
- A computationally interesting cellular automata
  - Simple definition, complex behaviour
  - Unexpected emergent phenomena
  - i.e. spaceships, methuselah
  - Computationally universal

- Lots of Game of Life implementations:
  - http://www.bitstorm.org/gameoflife/ [Java, online]
  - http://golly.sourceforge.net [cross-platform]
  - Do try this at home!

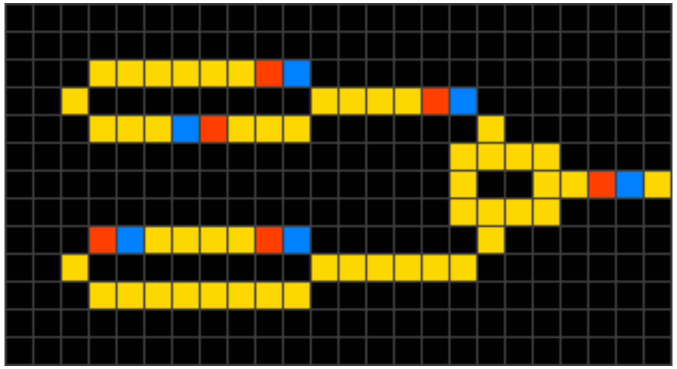
Multi-Valued States

- Various multi-valued state CAs have been studied
  - e.g. Langton's loops model self-replication
    - Uses 8 states:

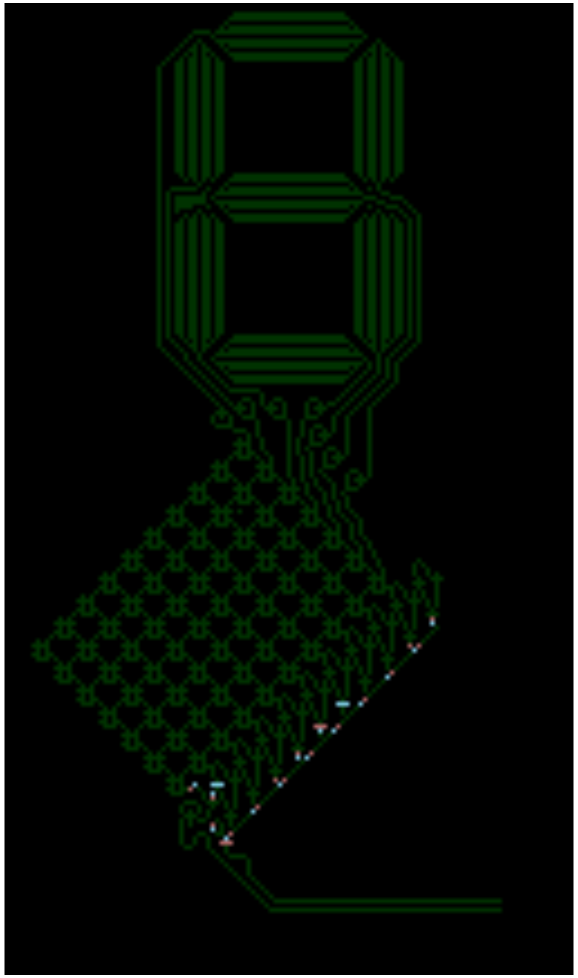




  - e.g. WireWorld models electron flow in circuits
    - Uses 4 states:



WireWorld



http://en.wikipedia.org/wiki/File:Animated_display.gif (animated)

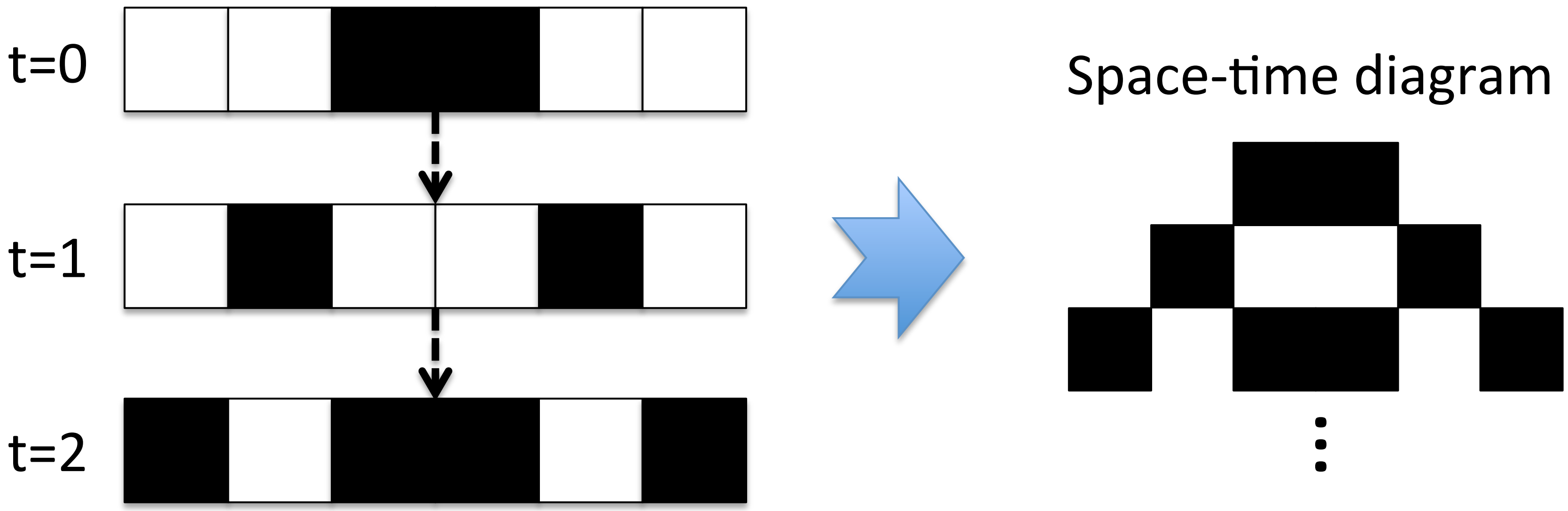
Other Model Extensions

- Probabilistic cellular automata
  - Transitions occur with a certain probability
- Asynchronous cellular automata
  - Updates don't occur at the same time
- Use of non-rectangular grids
  - e.g. irregular Penrose tilings
- Continuous cellular automata
  - Continuous state, functions or spaces

http://en.wikipedia.org/wiki/File:Oscillator.gif

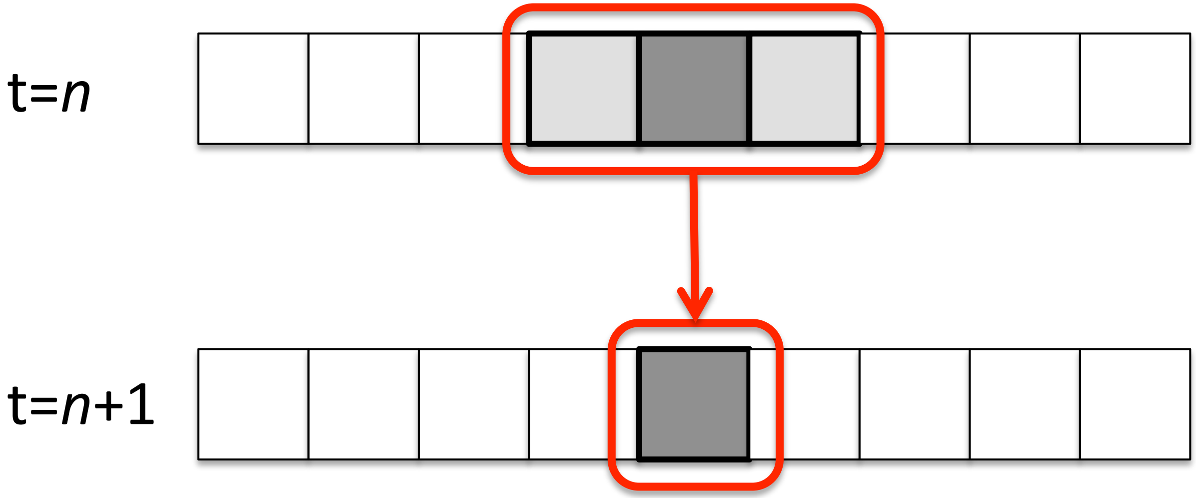
Elementary Cellular Automata

- 1D binary CAs that take place on a single grid row
  - Appear simple, but can be deceptively complex
  - Probably the most studied form of CA
  - Stephen Wolfram's work on these is very well known



Elementary Cellular Automata

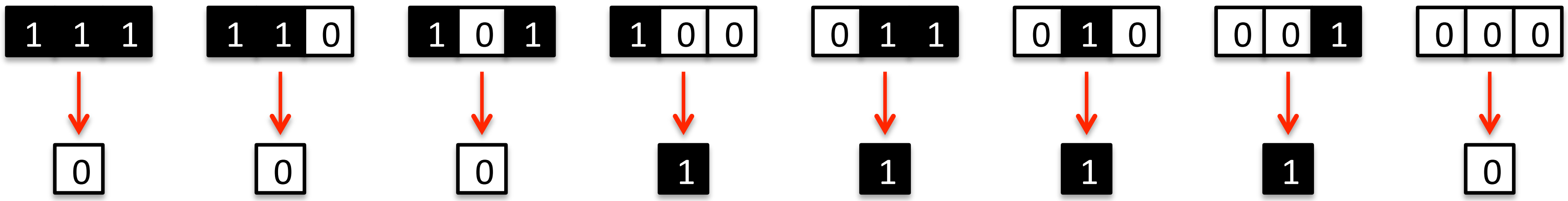
- They are based around a neighbourhood of size 3:



- Hence, it maps 2^3=8 possible patterns to 0 or 1
  - Meaning there are 2^8=256 possible update rules

Elementary Cellular Automata

- An example of a rule:



- This is known as Rule 30 … can you see why?
  - Hint: binary!
  - Every elementary CA has a number between 0 and 255

Rule 30

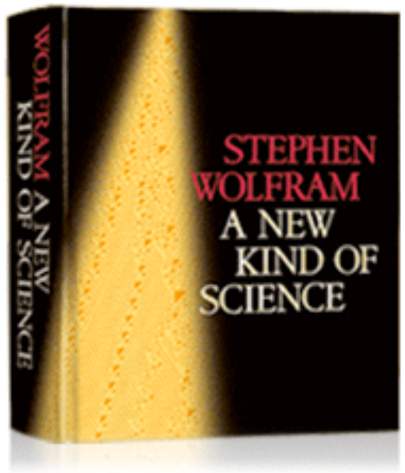
- One of Wolfram's famous rules
  - Leads to complex, aperiodic, chaotic behaviour
  - Space-time diagram resembles the shell of *conus textile*
  - Used as a random number generator in Mathematica

From A New Kind of Science, © Stephen Wolfram, LLC

http://en.wikipedia.org/wiki/File:Textile_cone.JPG

"Simple Programs"

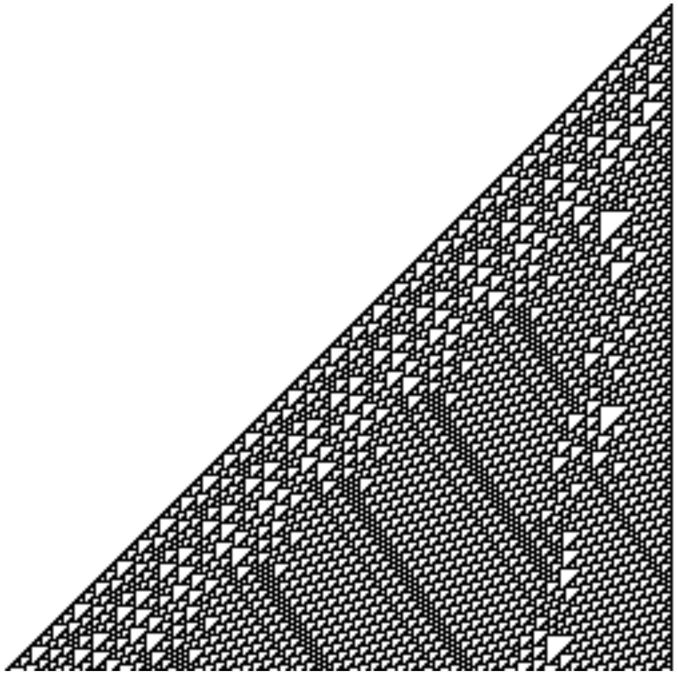
- Stephen Wolfram's book in a nutshell
  - Simple programs (such as CAs) can generate complex behaviours
  - They can generate a lot of the patterns we see in natural systems
  - They might therefore be used as a means for studying natural systems
  - Rather than using top-down models
  - e.g. in physics, chemistry, biology, …



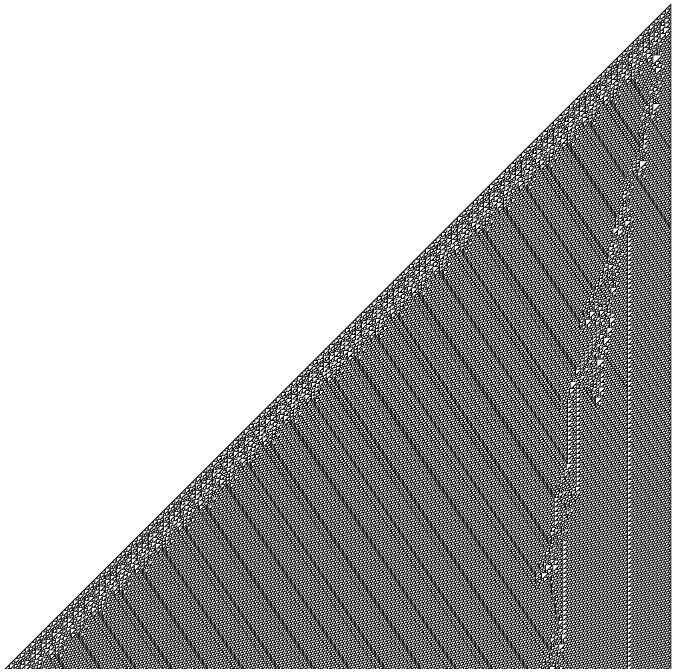
- Often misunderstood as saying the universe is a CA

Rule 110

- This one is known to be Turing complete
  - The simplest known Turing complete system
  - Very simple definition, complex behaviour
  - Behaviour appears to take place on the "edge of chaos"



From A New Kind of Science, © Stephen Wolfram, LLC



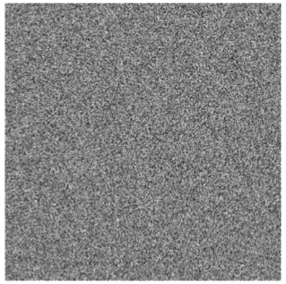
Evolving Cellular Automata

- Rules 30 and 110 were discovered by exhaustively enumerating and simulating the rule space
  - Feasible for these elementary CAs
  - Quickly becomes infeasible for more complex CAs
- Is it possible to use EAs to find useful rules?
  - Yes, this has been done.
- Is this a form of genetic programming?
  - If the aim is to perform computation, then yes!
  - If the aim is modeling, then probably not

Computing with CAs

- How do I compute using a CA?
  - 1) Find a suitable rule
  - 2) Encode the problem instance in the initial state
  - 3) Execute the CA for a certain number of steps
  - 4) Read the result from the final state of the CA

- Lots of cryptography applications
  - Google ‘cellular automata encryption’
  - Lots of different CA models used
  - Have a look!



Things you should know

- Know how to execute a CA, and their uses
- Know how CAs differ from traditional computation
- Know about the Game of Life:
  - Know it is computationally universal, and understand why
  - Know its emergent phenomena: gliders, methuselah
  - *You don't need to remember patterns*
- Know about Elementary CAs:
  - Know what they are
  - Be aware of their computational properties
  - *You don't need to remember individual rules*

Things to try out

- Try out some CAs:
  - Forest fire
    http://www.macs.hw.ac.uk/~dwcorne/mypages/apps/ca.html
  - Elementary cellular automata
    http://www.macs.hw.ac.uk/~dwcorne/mypages/apps/1dca.html
  - Golly (game of life, Langton's loops, WireWorld)
    http://golly.sourceforge.net/

- Read about using CAs for modelling:
  - See papers on course website
  - Urban growth, traffic simulation, flu infection, brain tumour growth, HIV infection

Next Lecture

- How biological cells actually "compute"
  - Gene regulatory networks (GRNs)
  - Computational models of GRNs
  - Boolean networks