Two Lane Traffic Simulations using Cellular Automata

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Abstract

We examine a simple two lane cellular automaton based upon the single lane CA introduced by Nagel and Schreckenberg. We point out important parameters defining the shape of the fundamental diagram. Moreover we investigate the importance of stochastic elements with respect to real life traffic.

1 Introduction

In recent times cellular automata based simulations of traffic flow have gained considerable importance. By extending the range of rules from nearest neighbours to a range of 5 grid sites and introducing 6 discrete velocities $0 \ldots 5$ Nagel and Schreckenberg [1] have found a striking resemblance of simulation and realistic traffic behaviour. For $v_{max} = 1$ Schadschneider and Schreckenberg have found an analytic solution [2]. For higher v_{max}, these analytic approaches lead to good approximations for the average behavior [3]. Further analytic results can be found in [4]. Nagel [5] pointed out the strong connections between particle hopping models and fluid-dynamical approaches for traffic flow.

Much less is known about modelling of multi-lane traffic. This statement is not only true for particle hopping models for traffic flow, but for traffic flow theory in general. Queueing models are not truly multilane, but emulate multiple lanes by switching the order of vehicles on one lane whenever a passing would have occurred in reality [7]. Fluid-dynamical models incorporate multi-lane traffic only by parametrization [8, 9], although sometimes based on kinetic theory [10]. Traditional car-following theory (see [11]) by and large never dealt with multi-lane traffic. Modern microscopic traffic simulation models (e.g. [12, 13, 14, 15]) obviously handle multi-lane traffic by necessity. Cremer and coworkers [16, 17] even treat multi-lane traffic in the context of cellular automata models. Yet, all these papers approach the problem by using heuristic rules of human behavior, without checking which of these rules exactly cause which kind of behavior. In validations then (e.g. [15]), it often enough turns out that certain features of the model are not realistic; and because of the heuristic approach it is difficult to decide which rules have to be changed or added in order to correct the problem.

For that reason, a more systematic approach seems justified. Our approach here is to search for *minimal* sets of rules which reproduce certain macroscopic facts. The advantage is that relations between rules and macroscopic behavior can be more easily identified; and as a welcome side-effect one also obtains higher computational speed.

We again choose particle hopping models as starting point for this investigation because their highly discrete nature reduces the number of free parameters even further. It is clear that a similar analysis could be applied to continuous microscopic models, hopefully benefiting from the results obtained in this and following papers.

Nagatani examined a two lane system with completely deterministic rules and $v_{max} = 1$ [18, 19], where cars either move forward or change lanes. A very unrealistic feature of this model are states where blocks of several cars oscillate between lanes without moving forward at all. He corrected this by introducing randomness into the lane changing [20]. Latour has developed the two lane model which served as the basis for the one discussed here [22]. Rickert used a more elaborate rule set for two lane traffic which reproduced the phenomenon of increased flow with an imposed speed limit [23].

2 Single Lane Model

For the convenience of the reader we would like to outline the single lane model. The system consists of a one dimensional grid of L sites with periodic boundary conditions. A site can either be empty, or occupied by a vehicle of velocity zero to v_{max}. The velocity is equivalent to the number of sites that a vehicle advances in one update — provided that there are no obstacles ahead. Vehicles move only in one direction. The index i denotes the number of a vehicle, $x(i)$ its position, $v(i)$ its current velocity, $v_d(i)$ its maximum speed[1], $pred(i)$ the number of the preceding[2] vehicle, $gap(i) := x(pred(i)) - x(i) - 1$ the width of the gap to the predecessor. At the beginning of each time step the rules are applied to all vehicles simultaneously (parallel update, in contrast to sequential updates which yield slightly different results). Then the vehicles are advanced according to their new velocities.

- **IF** $v(i) \neq v_d(i)$ **THEN** $v(i) := v(i) + 1$ (**S1**)
- **IF** $v(i) > gap(i)$ **THEN** $v(i) := gap(i)$ (**S2**)
- **IF** $v(i) > 0$ **AND** $rand < p_d(i)$ **THEN** $v(i) := v(i) - 1$ (**S3**)

S1 represents a linear acceleration until the vehicle has reached its maximum velocity v_d. **S2** ensures that vehicles having predecessors in their way slow down in order not to run into them. In **S3** a random generator is used to decelerate a vehicle with a certain probability modelling erratic driver behaviour. The free–flow average velocity is $v_{max} - p_d$ (for $p_d \neq 1$).

3 A Generic Two Lane Model

The single lane model is not capable of modelling realistic traffic mainly for one reason: a realistic fleet is usually composed of vehicles types having different desired velocities. Introducing such different vehicle types in the single lane model only results in *platooning* with slow vehicles being followed by faster ones and the average velocity reduced to the free–flow velocity of the slowest vehicle [21, 24].

We introduce a two lane model[3] consisting of two parallel single lane models with periodic boundary conditions and four additional rules defining the exchange of vehicles between the lanes. The update step is split into two sub-steps:

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[1] Note that in the original model all vehicles had the same maximum velocity v_{max}. We now allow for different desired velocities $v_d(i)$ to include an inhomogeneous fleet

[2] *A precedes B* in this context means that A is followed by B

[3] Note that the results presented here still concentrate on a single desired velocity only. The effects of different desired velocities will be investigated in future papers.

1. Check the exchange of vehicles between the two lanes according to the new rule set. Vehicles are only moved *sideways*. They do not *advance*. Note that in reality this sub-step regarded by itself seems unfeasible since vehicles are usually incapable of purely transversal motion. Only together with the second sub-step our update rules make physically sense.

   This first sub-step is implemented as strict parallel update with each vehicle making its decision based upon the configuration at the beginning of the time step.

2. Perform independent single lane updates on both lanes according to the single lane update rules. In this second sub-step the resulting configuration of the first sub-step is used.

The most important parameters of the two lane model are as follows:

Symmetry: The rule set defining the lane changing of vehicles can be both symmetric and asymmetric. The symmetric model is interesting for theoretical considerations whereas the the asymmetric model is more realistic.

Stochasticity: The single lane model proved that a strictly deterministic model is not realistic: the model did not show the desired spontaneous formation of jams. In the case of the two lane model the lack of stochasticity in combination with the parallel update results in strange behaviour of slow platoons occupying either lane: since none of the vehicles has reached its maximum velocity and all evaluate the other lane to be better there is collective change sidewise which is usually reversed over and over again until the platoon dissolves or the platoon is passed by other vehicles.

We introduce stochasticity into the two lane rule set to reduce the effective number of lane changes and thus dissolve those platoons. The simulation also revealed that is effect is also important in the asymmetric free–flow case (see 4.4).

Direction of Causality: In the single lane model a vehicle only looks ahead (= downstream = in the direction of vehicle flow) so that causality can only travel upstream (= in the opposite direction of vehicle flow). A reasonable lane changing rule must include a check of sites *upstream* in order not to disturb the traffic of the destination lane. This would result in causality travelling downstream.

A somewhat generic starting point for modeling passing rules is the following: (T1) You look ahead if somebody is in your way. (T2) You look on the other lane if it is any better there. (T3) You look back on the other lane if you would get in somebody else's way.

Technically, we keep using $gap(i)$ for the number of empty sites ahead in the same lane, and we add the definitions of $gap_o(i)$ for the forward gap on the other lane, and $gap_{o,back}$ for the backward gap on the other lane. Note that if there is a vehicle on a neighbouring site both return -1. The generic multi-lane model then reads as follows. A vehicle i changes to the other lane if all of the following conditions are fulfilled:

- $gap(i) < l$ (**T1**),
- $gap_o(i) > l_o$ (**T2**),
- $gap_{o,back}(i) > l_{o,back}$ (**T3**), and
- $rand() < p_{change}$ (**T4**).

l, l_o, and $l_{o,back}$ are the parameters which decide how far you look ahead on your lane, ahead on the other lane, or back on the other lane, respectively. According to the before mentioned characteristics we associate the parameters of our rule set:

characteristic	yes	no
symmetry	**T1** for L→R	no **T1** for L→R
stochasticity	$prob_c < 1$	$prob_c = 1$
backward causality	$l_{o,back} > 0$	$l_{o,back} = 0$

4 A lane changing model

As an example, we start with $l = v + 1$, $l_o = l$, $l_{o,back} = 5 = v_{max}$, and $p_{change} = 1$. That means that both l and l_o are essentially proportional to the velocity, whereas looking back is not: It depends mostly on the expected velocity of other cars, not on one's own.

In the symmetric version of this model, cars remain on their lane as long as they don't "see" anybody else. If they see somebody ahead on their own lane (i.e. $gap < v + 1$), then they check on the other lane if they can switch lanes and do so if possible. Afterwards, if they are satisfied, they remain on this lane until they become dissatisfied again.

In the asymmetric version, cars always try to return to the right lane, independent of their situation on the left lane.

Space-time-plots both of the symmetric and the asymmetric version are shown in Figs. 1 and 2. For these plots, we simulated a system with a length of 12,000 sites of which we plot 400 sites in 400 consecutive time-steps. The density is 0.09 which is slightly above the density of maximum flow (see below). Vehicles go from left to right (spatial axis) and from top to bottom (time axis). Traffic jams appear as solid areas with steep positive inclination whereas free flow areas are light and have a more shallow negative inclination. Each plot is split into two parts: The left part containing the left lane and the right part containing the right lane, respectively.

Note that plot 2 (left lane) gives a good impression of the great number of lane changes through the high frequency of short vehicle life lines appearing and disappearing: These are vehicles that temporarily leave the right lane to avoid an obstacle. They go back to their old lane as soon as the obstacle has been passed. It will be confirmed quantitatively that indeed the rate of lane changes is much higher for the asymmetric model than for the symmetric model.

4.1 Simulation Setup

Before going on, we would like to describe our standard simulations set-up for the following observations. Note that quantitative simulation results were obtained with a much larger system than the qualitative space-time plots. We simulated a system of length

$$L \;=\; 133,333 \; sites \;\approx\; 1000 \; km$$

with closed boundary conditions, i.e. traffic was running in a loop. We started with random initial conditions, i.e. N cars were randomly distributed on both lanes around the complete loop with initial velocity $v_0 = 0$.

Since the system is closed, the average density per lane is now fixed at

$$\langle \rho \rangle_L = \frac{N}{2\,L} \;,$$

where the “2” stands for the number of lanes.

The simulation was then started, 1000 time steps were executed to let the transients die out, and then the data extraction was started. The flow which is found in the fundamental diagrams is both space and time[4] averaged, i.e.

$$\langle j \rangle_{L,T} = \frac{1}{T}\,\frac{1}{L}\,\sum_{i}^{L}\,\sum_{t}^{T/5} v(i, 5t) \;.$$

Values for lane change frequency and ping pong lane change frequency (see below) are obtained by the same averaging procedure except that statistics are gathered every time step, since *by definition* ping pong lane changes occur in subsequent time steps.

We usually used $T = 5000$, and the same procedure was repeated for each density found in the plots. With a resolution of $\Delta\varrho = 0.01$ an average plot took about 22 hours of computation time on a Sparc 10 workstation.

[4] We gather statistics every fifth time-step only, since subsequent time steps are correlated.

4.2 Flow behavior

By comparing these models with each other and with earlier results, we make the following observations (Fig. 3 unless otherwise noted):

(i) Both for the symmetric and the asymmetric version, maximum flow is higher than twice the maximum flow of the single lane model (Fig. 4). Which means that, in spite of the additional disturbances which the lane changing behavior introduces into the traffic flow, the general effects are beneficial, probably by diminishing large deviations from "good" flow patterns.

(ii) Both for the symmetric and the asymmetric version, the combined 2-lane flow reaches a maximum at $\rho_{jmax} \approx 0.08$, which is at or near a sharp bend of the flow curve.

(iii) For the asymmetric model, flow on the left lane keeps increasing slightly for $\rho > \rho_{jmax}$, but this is over-compensated by the decreasing flow on the right lane.

(ii) and (iii) together lead one to the speculation that maximum flow in the asymmetric case here actually is connected to a "critical" flow on the right lane and a "sub-critical" flow on the left lane. Any addition of density beyond here leads to occasional break-downs on the right lane and thus to a much lower flow there. Obviously, such interpretations would have to be clarified by further investigations, and the word "critical" would have to be used with more care, such as is pointed out in [6] for the single lane case.

(iv) For both lanes combined, the curves for symmetric and asymmetric traffic actually look fairly similar. If the above interpretation is right, this means that the overall density of maximum flow is a fairly robust quantity, but one can stabilize one lane at a much higher density if this density is taken from the other lane.

(v) At very low densities in the asymmetric case, flow on the left lane, j_{left}, only slowly builds up. This is to be expected, since at least two cars have to be close to each other to force one of them on the left lane, leading to a mean field solution of $j_{left}(\rho) \sim \rho^2$ for $\rho \approx 0$.

(vi) For $\rho > 0.4$, flows on both lanes in the asymmetric models are fairly similar and similar to the lane flows in the symmetric models.

4.3 Lane changing behavior

To get some further insight into the lane changing dynamics, Fig. 5 shows the frequency of lane changing both for the asymmetric and the symmetric model.

(i) Note that in the asymmetric case there is a sharp bend in the curve, which is not found for the symmetric case. This bend is also near ρ_{jmax}, giving further indication that the dynamics above and below ρ_{jmax} are different.

(ii) For the symmetric case, lane changing occurs with less than half the frequency compared to the asymmetric case.

(iii) In the symmetric case, the lane changing frequency per site for small densities increases approximately quadratically up to rather high densities, whereas the same quantity for the asymmetric model grows approximately linearly already for fairly low densities. suggests that for the symmetric case a mean field description of interaction, $P(change) \propto \rho^2$, would be valid up to comparably high densities. obvious that this does not work. Since the vehicles have a strong tendency to be on the right lane, already a density of 0.04 per lane would be a density of 0.08 if everybody were on the right lane. Yet, $\rho = 0.08$ is known to be already a density of high interaction in single lane traffic. Since this high interaction tends to spread vehicles out [2, 3], each additional vehicle simply adds its own share of lane changes, making the relation roughly linear.

(iv) The maximum number of lane changes occurs at densities much higher than ρ_{jmax}. The lane changing probability *per vehicle*, however, reaches a maximum below the critical density (Fig. 6).

4.4 Ping pong lane changes

An artifact of the so far described algorithm is easily recognizable when one starts with all cars on the same lane, say the right one. Assuming fairly high density, then all cars see somebody in front of them, but nobody on the left lane. In consequence, everybody decides to change to the left lane, so that *all* cars end up on the left lane. Here, they now all decide to change to the right lane again, etc., such that these coordinated lane changes go on for a long time ("cooperative ping-pong effect"). This effect has already been observed by Nagatani for the much simpler two-lane model [18, 19].

One way around this is to randomize the lane changing decision [20]. The decision rules remain the same as above, but even if rules T1 to T3 lead to a yes, it is only accepted with probability p_{change}. With this fourth lane changing rule, patterns like the above are quickly destroyed.

In order to quantify the effects of a different p_{change}, simulations with $p_{change} = 0.5$ were run. The observations can be summarized as follows:

(i) The flow-density curves are only marginally changed (Fig. 7).

(ii) The frequency of lane changes is decreased in general, but, except for $\rho < \rho_{jmax}$ in the asymmetric case, by much less than the factor of two which one would naively expect (Fig. 5). That means that usually there is a *dynamic* reason for the lane change, that is, if it is not done in one time step due $p_{change} < 1$, then it is re-tried in the following time step, etc.

(iii) To better quantify in how far a $p_{change} < 1$ actually changes the pattern of vehicles changing lanes back and forth in consecutive time steps, we also determined the frequency of "ping pong lane changes", where a car makes two lane changes in two consecutive iterations. Obviously, there are left-right-left (lrl) and right-left-right (rlr) ping pong lane changes.

Fig. 8 shows that reducing the probability to change lanes, p_{change}, from 1 to 1/2 has indeed a beneficial effect: The number of ping pong lane changes decreases by about a factor of five.

Yet, for the symmetric case, the frequency of ping pong lane changes is more than an order of magnitude lower in both cases anyway. This indicates that in simulations starting from random initial conditions, the cooperative effect as described further above as cooperative ping-pong effect does not really play a role for the statistical frequency, because this effect should be the same for the symmetric and the asymmetric model. Instead, the cause of the ping pong lane changes in the asymmetric model is as follows: Assume just two cars on the road, with a gap of 5 between them. With respect to velocity, both cars are in the free driving regime, and their velocities will fluctuate between 4 and 5. Now assume that the following car has velocity 5 from the last movement. That means that it looks 6 sites ahead, sees the other car, and changes to the left lane. Then, assume that in the velocity update, the leading car obtains velocity 5 and the following car obtains velocity 4. Then, after the movement step, there is now a gap of 6 between both cars, and in the lane changing step, the follower changes back to the right lane. And this can happen over and over again in the asymmetric model, but will not happen in the symmetric model: Once the following car in the above situation has changed to the left lane, it will remain there until it runs into another car on the left lane.

To investigate this second kind of ping pong lane changes we ran simulations recording whether a ping pong lane change was made at low velocities $0 \leq v \leq 3$ or high velocities $4 \leq v \leq 5$. Fig. 9 shows a very distinct peak for *fast* ping–pong–changes at low densities whereas *slow* ping–pong–changes have a lower peak at higher densities similar to that of the symmetric case.

This gives a strong indication that most lane changes are actually caused by the "tailgating effect" as described above, which is an artifact of the rules. It is, though, to be expected that this behavior does not have a strong influence on the overall dynamics: It mostly happens in the free driving regime; as soon as, for example, another car is nearby in the left lane, it is suppressed by the looking back and forward on the other lane.

4.5 Other Parameter Combinations

We would like to mention two other parameter combinations. They are presented because they generate artifacts which contradict the common sense one would apply to the phenomena of traffic flow.

(i) In the first case the lookahead is reduced to $l = v$ instead of $l = v + 1$. While this change is negligible for vehicles at higher velocities it becomes crucial to a vehicles stopped in a jam: assuming the current velocity to be zero the vehicle looks *zero* sites ahead and decides to remain in the current lane due to the non–fulfilled rule $T1$. This state will persist until the predecessor moves even if the other lane is *completely* free! Fig. 10 shows the impact the reduced look ahead on overall flow: for density $\rho > 0.75$ there is no perceptible flow in the right lane which corresponds to a traffic jam that occupies more or less the whole right lane.

(ii) In the second case the look-back is reduced to $l_{o,back} = 0$. Vehicles no longer check whether their lane changing could have a disadvantageous effect on the other lane which corresponds to a very egoistic driver behaviour. Fig. 11 shows flow density relationships for look-back $l_{o,back} = 5$ and $l_{o,back} = 0$. It is obvious that the decrease in look-back also decreases the maximum flow at critical densities. Moreover $l_{o,back} = 0$ seems to split the curves of the symmetric and asymmetric cases which used to be almost identical for $l_{o,back} = 5$: the lack of look-back is much more disadvantageous for asymmetric than for symmetric rules.

In Figs. 12 and 13 we used $l_{o,back} = 0$ for the symmetric and asymmetric rule sets with one plot per lane. It is clearly visible (compare to Figs. 1 and 2) how $l_{o,back} = 0$ completely disrupts the laminar flow regime. Vehicles change lanes without looking back; and due to the formulation of the model this does not cause accidents, but causes the obstructed vehicles to make sudden stops. Since these stops are caused more or less randomly, the regime becomes much more randomly disturbed than before, somewhat reminiscent of the Asymmetric Stochastic Exclusion Process (see [3, 5]).

As seen before the effect is even more drastic for the asymmetric rule set since the number of lane changes is higher than in the symmetric case. In Fig. 13 with $l_{o,back} = 0$ dynamics are dominated by small traffic jams caused by lane changes, while in Fig. 1 there are still some fairly laminar areas.

5 Discussion

Compared to reality (e.g. [15]), the lane change frequency in the asymmetric models presented here is by about a factor of 10 too high. Using $p_{change} = 0.1$ would correct this number, but is dynamically not a good fix: It would mean that a driver follows a slower car in the average for 10 seconds before he decides to change lanes. Besides, it was shown that about 90% of the lane changes in the asymmetric models here are produced by an artificial "tailgating dance", where a follower changes lanes back and forth when following another car. It remains an open question in how far artifacts like this can be corrected by the current modeling approach or if it will be necessary to, e.g., introduce memory:

If one remembers to just have changed lane from right to left, one will probably stay on that lane for some time before changing back.

Another defect of the models presented in this paper is that the maximum flow regime is most probably represented incorrectly. Both measurements (e.g. [15] or Fig. 3.6 in [25]) or everyday observation show that real traffic shows a "density inversion" long before maximum flow, that is, more cars drive on the left lanes than on the right lanes. This effect is more pronounced for countries with higher speed limits. Let us denote by ρ_{jmax}^{1lane} the density of maximum flow of the *single lane* case. It follows for the real world two lane case that at a certain point the left lane will have a density higher than this density ρ_{jmax}^{1lane} whereas the right lane has a density lower than ρ_{jmax}^{1lane}. When further increasing the overall density, then the flow on the left lane will decrease whereas it still increases on the right lane. It is unclear if the net flow here increases or decreases; but it should become clear that instabilities here are caused by the left lane first. This is in contrast to the models of this paper, where the right lane reaches the critical density first. Work dealing with this problem is currently in progress.

Also the effect of different maximum velocities will be addressed in later papers.

6 Summary and conclusion

We have presented straightforward extensions of the cellular automata approach to traffic flow so that it includes two-lane traffic. The basic scheme introduced here is fairly general, essentially consisting of two rules: Look ahead in your own lane for obstructions, and look in the other lane if there is enough space. The flow-density relations of several realizations of this scheme have been investigated in detail; possible artifacts for certain parameter choices have been pointed out. In general, there seem to be two important lessons to be drawn from our investigations:

- Checking for enough space on the other lane ("look-back") is important if one wants to maintain the dynamics consisting of laminar traffic plus start–stop waves which is so typical for traffic.
- Especially in countries with high speed limits, observations show a density inversion near maximum flow, that is, the density is higher on the left lane than on the right lane. This effect is not reproduced by our models (work in progress).

Yet, in general, it seems that the approach to multi lane traffic using simple discrete models is a useful one for understanding fundamental relations between microscopic rules and macroscopic measurements.

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Figure 1: *symmetric*, $l_{o,back} = 5$, *left + right lanes*

Figure 2: *asymmetric*, $l_{o,back} = 5$, *left + right lanes*

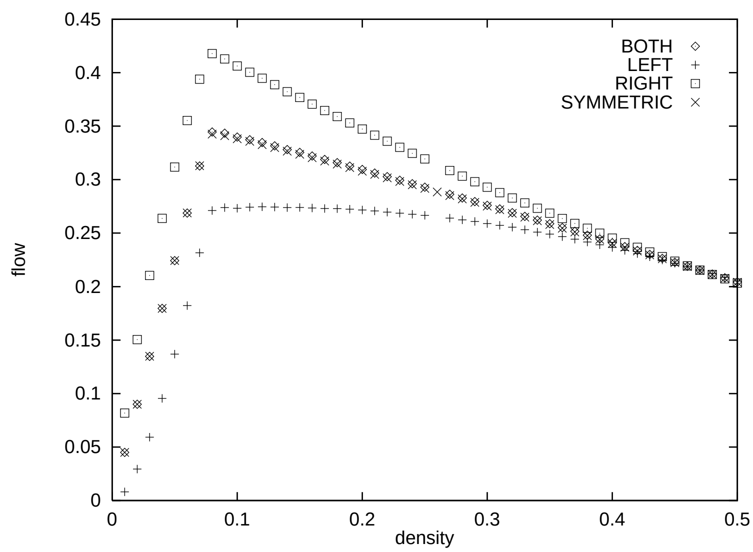

Figure 3: *Flow LookBack* $= 5$, $p_{change} = 1.0$

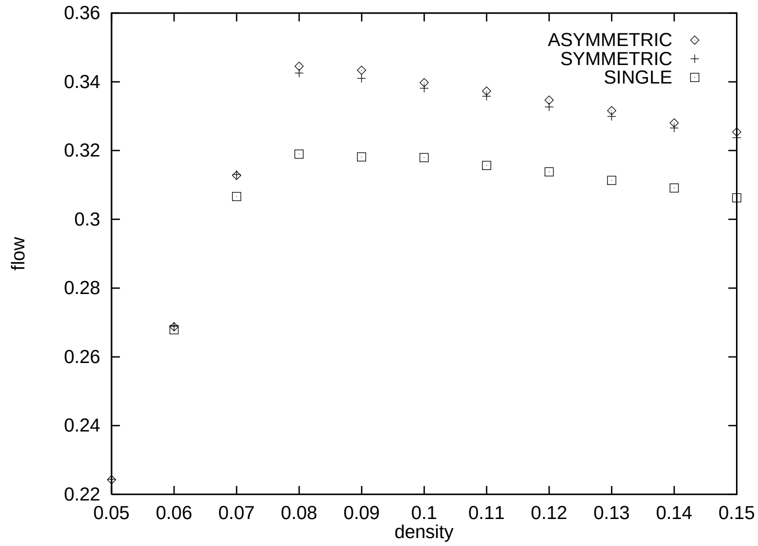

Figure 4: *Comparison Single Lane to Multi Lane*

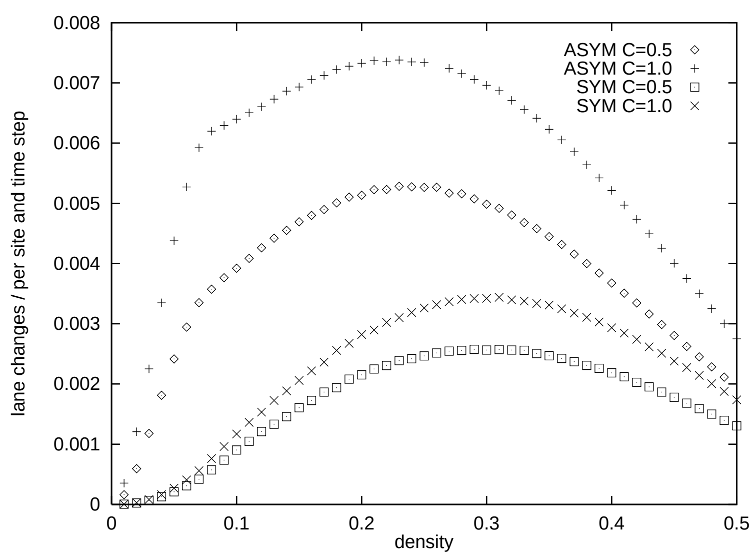

Figure 5: *Simple Lane Changes* $LookBack = 5$

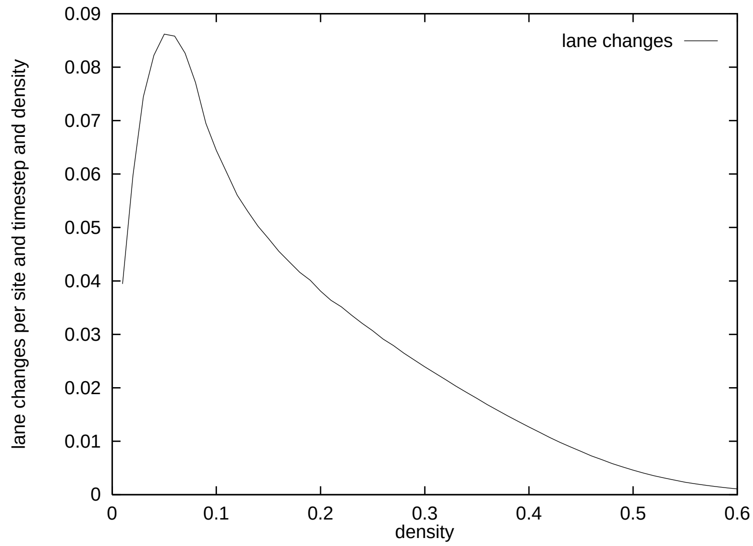

Figure 6: *Lane Changes Per Vehicle* $l = 5$, $p_{change} = 1.0$

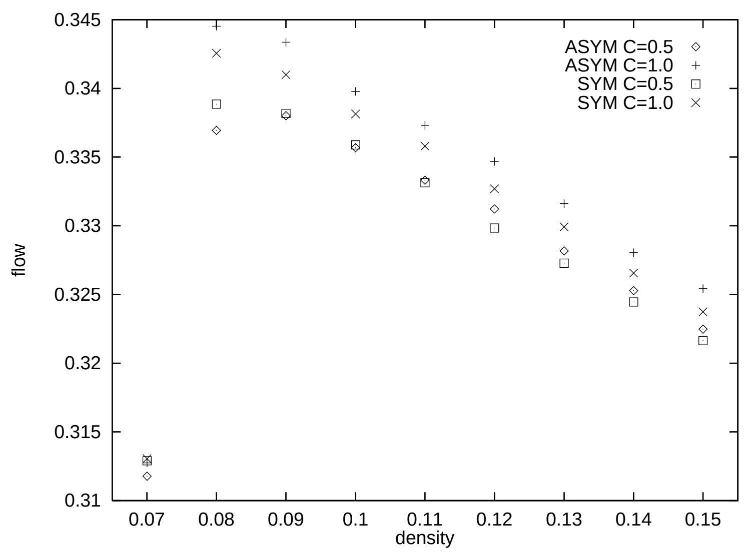

Figure 7: *Flow* $LookBack = 5$, *different* p_{change}

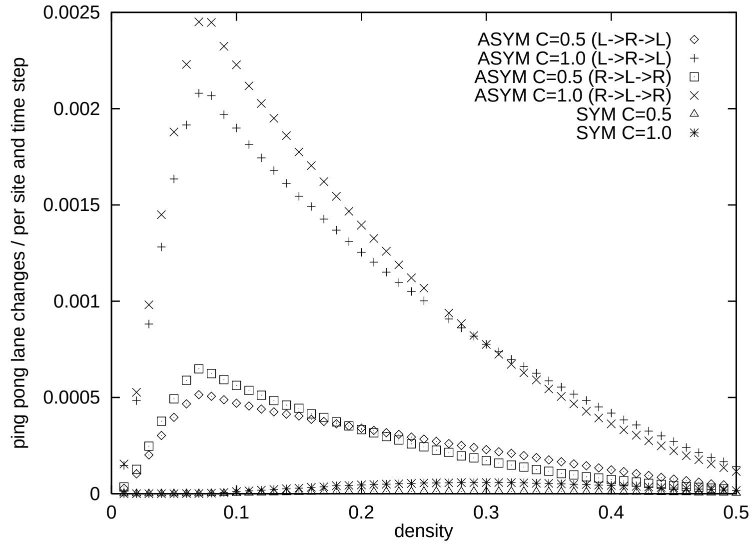

Figure 8: *Ping Pong Lane Changes* $LookBack = 5$*, different* p_{change}

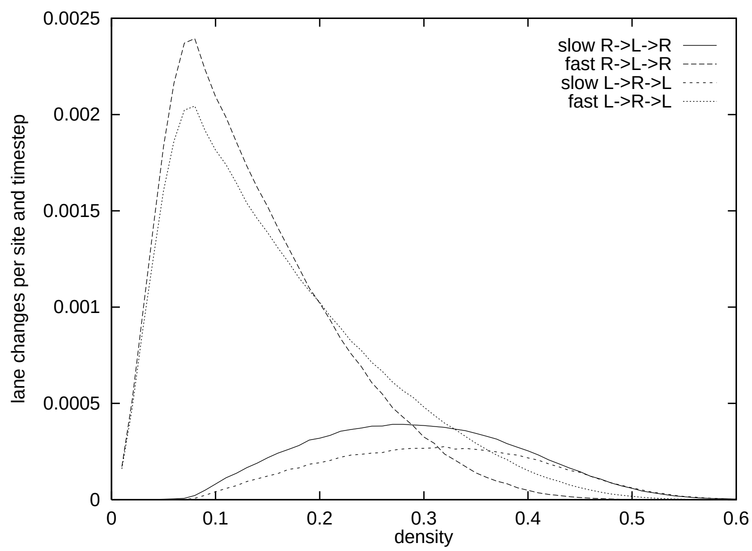

Figure 9: *Slow and Fast Ping Pong Lane Changes*

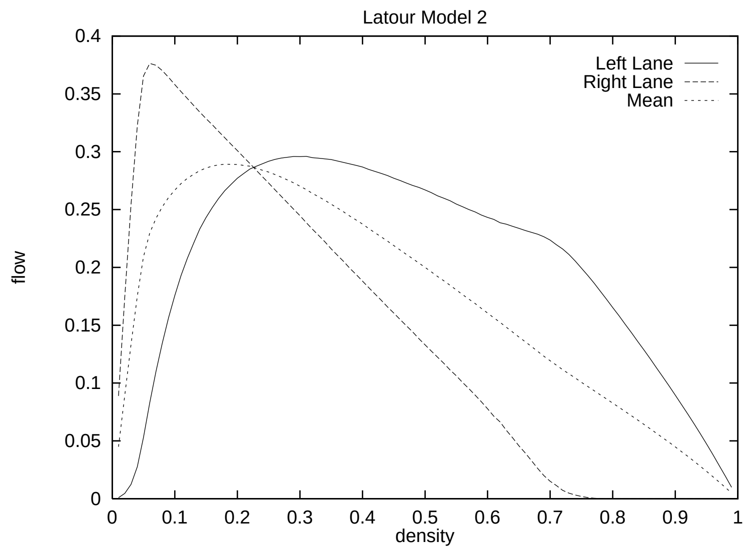

Figure 10: *Flow with* $l = v$

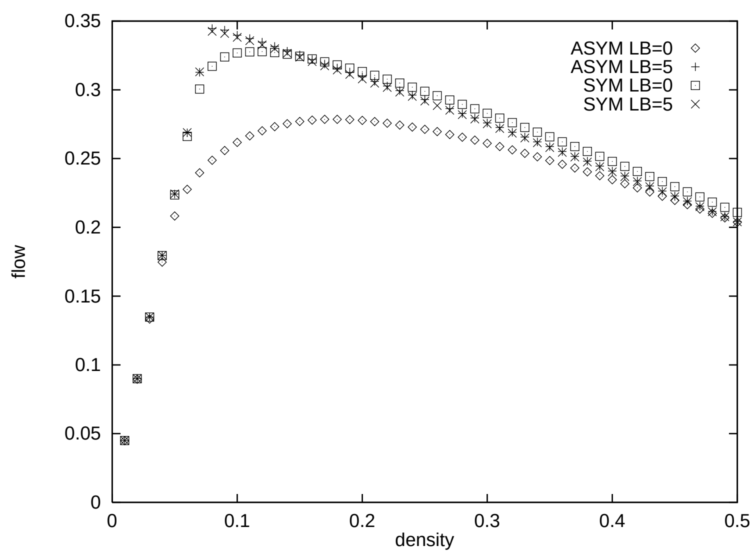

Figure 11: *Flow* $p_{change} = 1.0$, *different LookBack*

Figure 12: *symmetric,* $l_{o,back} = 0$*, left + right lanes*

Figure 13: *asymmetric,* $l_{o,back} = 0$*, left + right lanes*