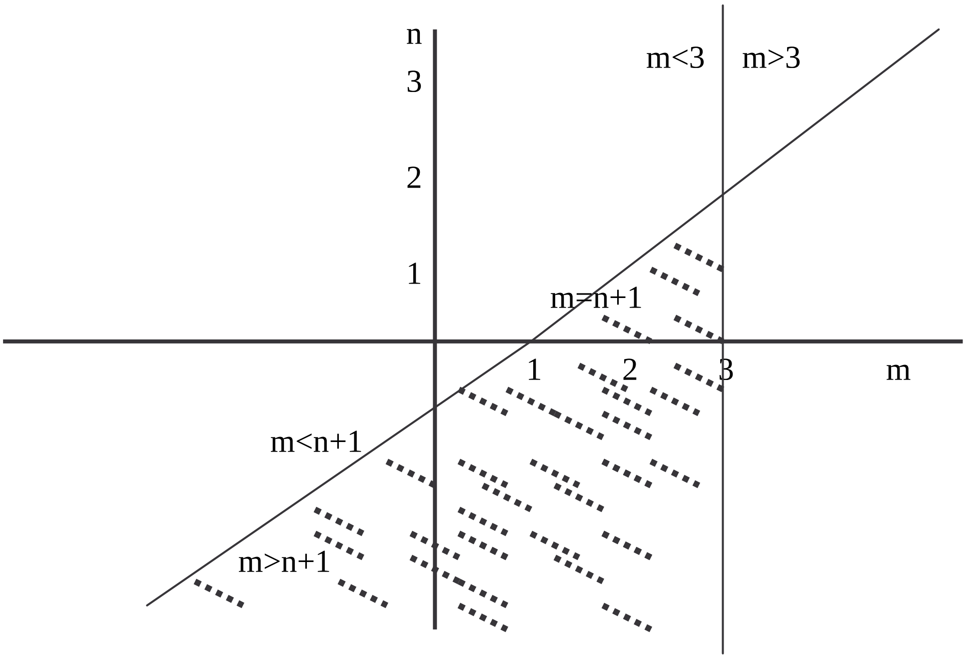
DWC's solutions to exercises in Chapter 8 of Kamareddine & Nederpelt

Section 8.6

8.1

(a) A(2,0) says: 2 < 3 AND 2 > 0+1 -- so it is True;
A(2,1) says: 2 < 3 AND 2 > 1+1 – so it is False, since 2 > 2 is false
A(2,2) says 2 < 3 AND 2 > 2+1 – it is False, since 2 > 3 is false

(b)



(c) A(2,*n*) says 2 < 3 AND 2 > *n* + 1. Since 2 < 3 is always true, this is the same as: True AND 2 > *n* + 1, which is the same as 2 > *n* + 1. We can simplify this by simple algebra to: *n* < 1.

Remainder of this question left as exercises for me to check.

8.2

(a)

$$Woman(Anna) \wedge Man(Bernard) \wedge Younger(Bernard, Anna)$$

(b) $\exists x[Child(x, Bernard)]$ -- this is true and correct – however it is good practice to be clear about the domain of *x*. In this case we know that *x* must be a man or woman, so we can say:

$$\exists x[Man(x) \vee Woman(x) : Child(x, Bernard)]$$

(c) $\forall x[Child(x, Anna) \Rightarrow Woman(x)]$

Again, it would be good practice to include the domain information, just as in (b), however the answer is correct as it stands.

(d) $\forall x[Child(x, Anna) \Rightarrow \neg Child(x, Bernard)]$

An alternative equivalent form (maybe slightly closer to the English version) would be:

$$\neg\exists x[Child(x, Anna) \wedge Child(x, Bernard)]$$

(e)

$$\exists x[Man(x) : Younger(x, Bernard) \wedge \exists y[Child(x, y) \wedge Child(Bernard, y)]]$$

Note this is just the same as:

$$\exists x[\exists y[Man(x) : Younger(x, Bernard) \wedge Child(x, y) \wedge Child(Bernard, y)]]$$

8.3

(a) $\forall x[x \in \mathrm{N} : x > -1]$

(b) $\forall x[x \in \Re : x^2 \geq 0]$

(c) $\forall x[x \in \Re : x > 10 \Rightarrow x > 5]$

(d) $\forall x[x \in \Re : x > 10] \Rightarrow (0 = 1)$

8.4

(a) $\forall x[x \in D : x \neq 0]$

(b) $\forall x[x \in D : x > 10 \wedge x < 25]$

(c) $\forall x[x \in D : x > 10] \vee \forall x[x \in D : x < 10]$

Note that it is NOT right to say this: $\forall x[x \in D : x > 10 \vee x < 10]$ -- that would suggest that D contains every number that is not equal to 10. The wording of 8.4(c) is instead saying that D either contains only the numbers smaller than 10, or it contains only the numbers greater than 10.

(d) $\forall x[x \in D : \forall y[y \in D : x \neq y \Rightarrow |x - y| \geq 1]]$

Note that $|x - y|$ gives the absolute difference – i.e. it is either y-x or x-y, whichever is positive.

Note that Professor Kamareddine’s answer uses a different (better and simpler) notation for universally quantifying two variables that both have the same scope. Either is fine.

8.5

(a) $\exists x[x \in \mathrm{N} : x > 1000]$

(b) $\exists x[x \in \mathrm{N} : x = x^2]$

(c) $\exists x[x \in \Re : x > 0] \wedge \exists x[x \in \Re : x < 0]$

Note that the following would be an incorrect answer to (c) – why?

$\exists x[x \in \Re : x > 0 \wedge x < 0]$

But the following would be fine:

$\exists x[x \in \Re : \exists y[y \in \Re : x > 0 \wedge y < 0]]$

(d) $\neg\exists x[x \in \Re : x > 0 \wedge x < 0]$

8.6

(a) $\forall x[x \in \mathrm{N} : \exists y[y \in N : y - x = 1]]$

(b) $\neg\exists x[x \in \mathrm{N} : \forall y[y \in N : x > y]]$

(c) $\forall x[x \in \mathrm{N} : \forall y[y \in N : x + y > x \wedge x + y > y]]$

(d) $\exists x[x \in \mathrm{N} : \exists y[y \in N : x^2 + y^2 = 58]]$

8.7

(a) $\forall x[x \in D : x \in \Re]$

Note that this is equivalent to: $\forall x[x \in D \Rightarrow x \in \Re]$

(b) $\exists x[x \in D : \forall y[y \in D : x \geq y]]$

(c) $\exists x[x \in D : \forall y[y \in D : x = y]]$ This is quite tricky and not obvious. If we were allowed to use a function like sizeof(D), then we could just say "sizeof(D)==1" – however the question implicitly suggests that we express this without such extra help. The answer says: "There is an *x* in D such that everything in D is equal to *x*" – this is, of course, the same as saying that *D* has precisely one element.

(d) $\exists x[x \in D : \exists y[y \in D : \forall z[z \in D : z = x \vee z = y]]]$ similar reasoning to the above.

Tree structure questions not answered – they are not in the scope of my part of the course.