Computational Logic in Artificial Neural Networks

Ekaterina Komendantskaya

INRIA Sophia Antipolis, France

EPSRC Interview, 6th of March 2008

Outline

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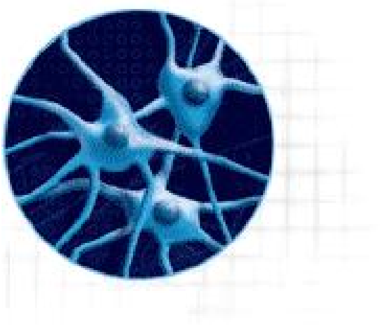
Introduction

Symbolic Logic as Deductive System

- Deduction in logic calculi;
- Logic programming;
- Higher-order proof assistants...

Sound symbolic methods we can trust

Neural Networks



- spontaneous behavior;
- learning and adaptation;
- computational power

Corner-stone Result, [Kalinke, Hölldobler, 94]

Theorem

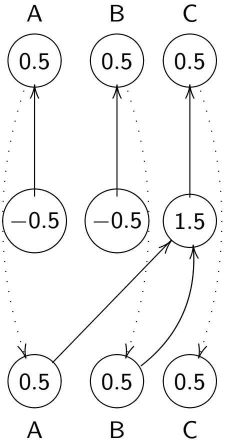
For each propositional program P, there exists a 3-layer feedforward neural network which computes T_P.

We will call such neural networks T_P-neural networks.

A simple T_P-neural network

$B \leftarrow$
$A \leftarrow$
$C \leftarrow A, B$

$T_P \uparrow 0 = \{B, A\}$
$lfp(T_P) = T_P \uparrow 1 = \{B, A, C\}$



Characteristic Properties of T_P-Neural Networks

1. Require infinitely long layers in the first-order case.

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$\Longrightarrow$ Impractical for Computational Logic; not interesting for Neurocomputing audience.

Example 2

$$P(0) \leftarrow$$
$$P(s(x)) \leftarrow P(x)$$
$$T_P \uparrow 0 = \{P(0)\}$$
$$lfp(T_P) = T_P \uparrow \omega =$$
$$\{0, s(0), s(s(0)),$$
$$s(s(s(0))), \ldots\}$$

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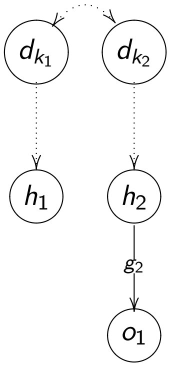
Paradox:
(computability,
complexity,
proof theory)

I propose SLD Neural networks

- They have finite architecture that does not depend on the size of the Herbrand base B_P.
- Their effectiveness is due to several learning functions.
- Allow easy implementation of computational logic.

Example 2 in SLD neural networks



$$P(0) \leftarrow;$$
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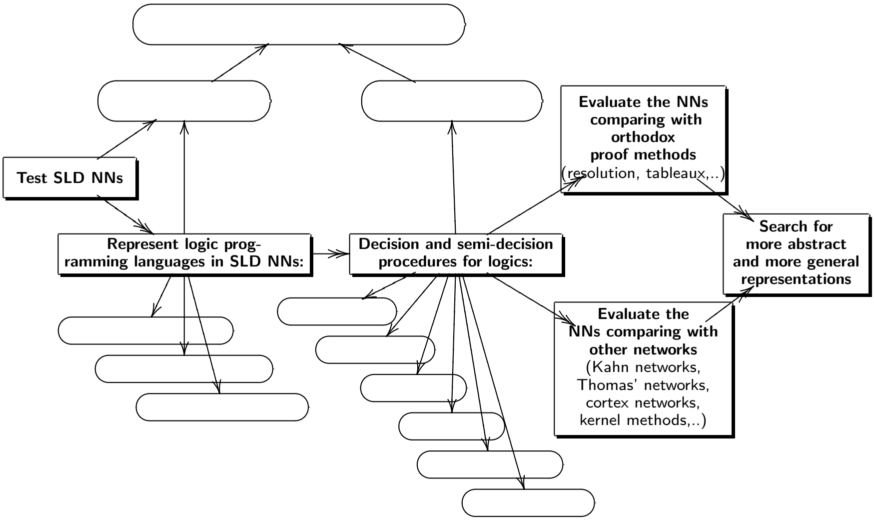
Workplan


Evaluate the NNs comparing with orthodox proof methods (resolution, tableaux,..)
Test SLD NNs
Search for more abstract and more general representations
Represent logic programming languages in SLD NNs:
Decision and semi-decision procedures for logics:
Evaluate the NNs comparing with other networks (Kahn networks, Thomas' networks, cortex networks, kernel methods,..)

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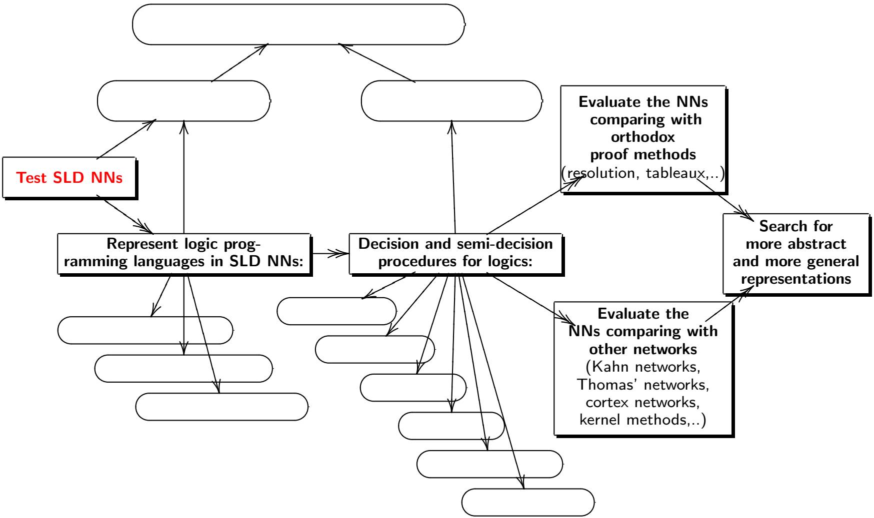
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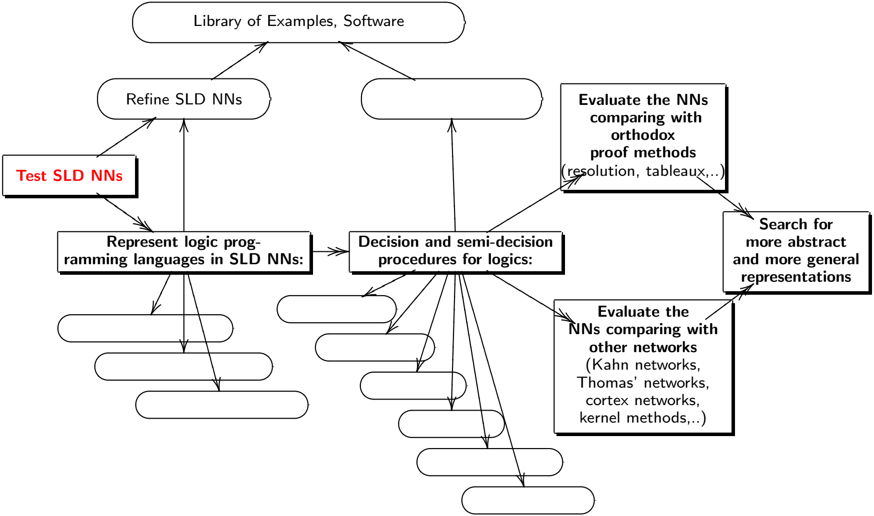
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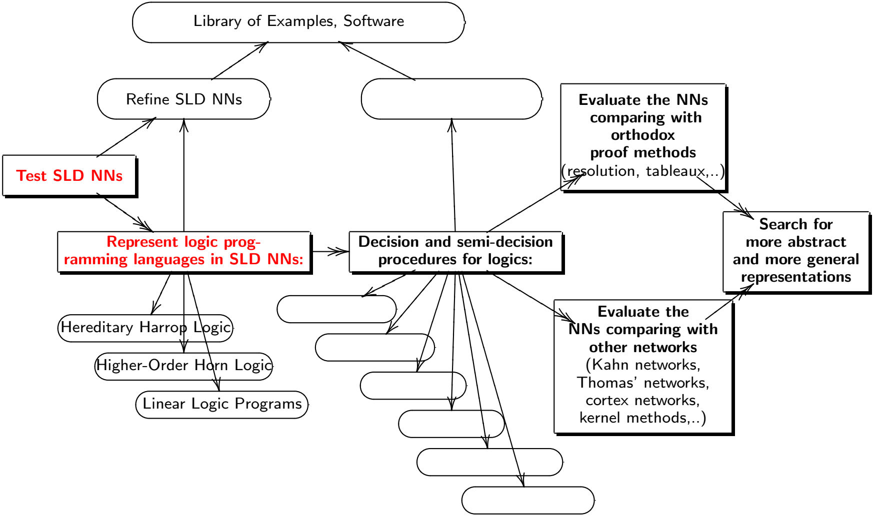
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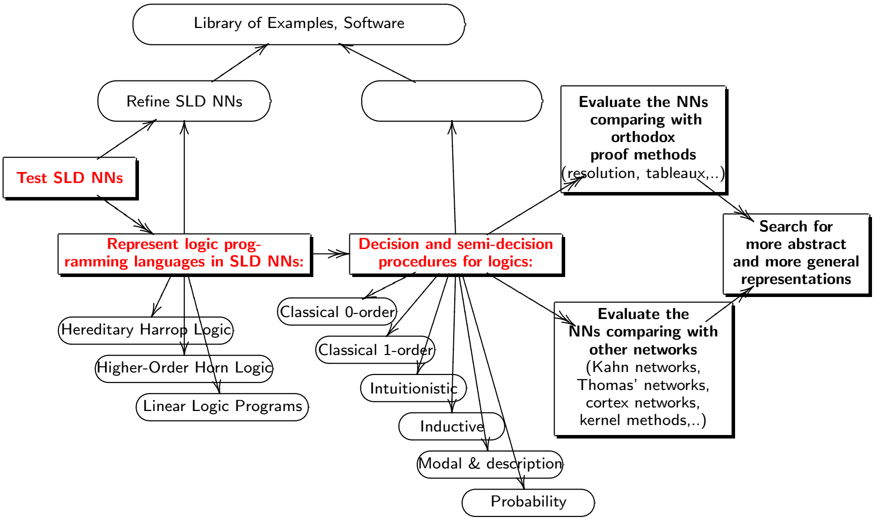

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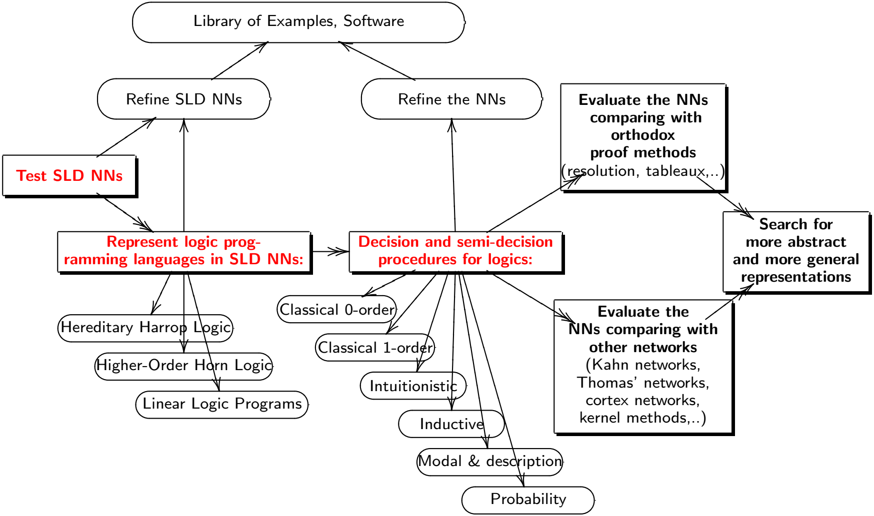

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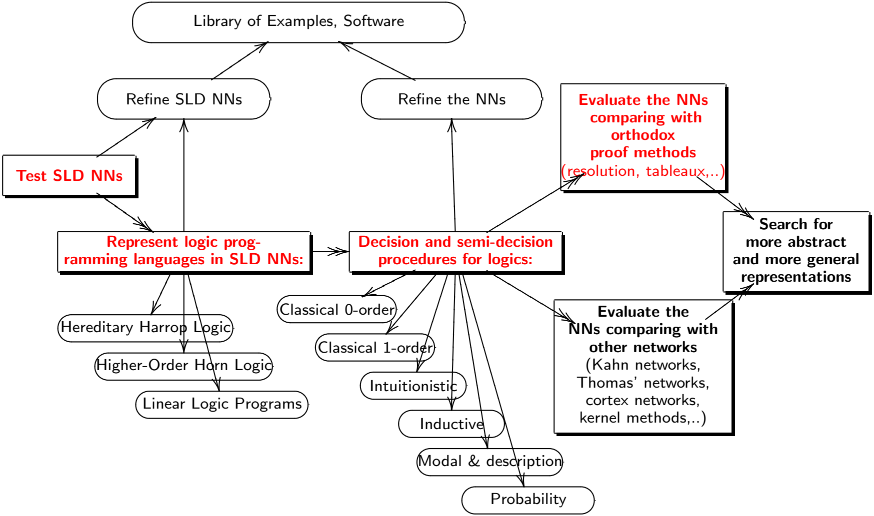
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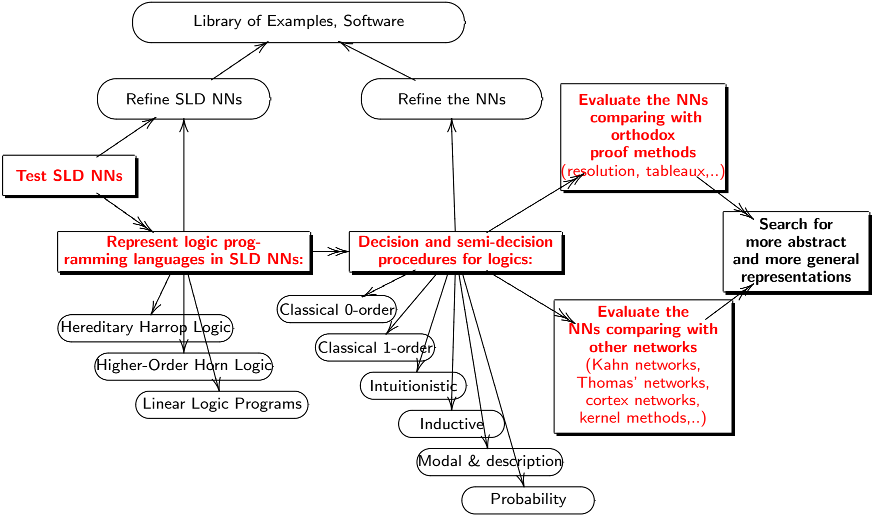
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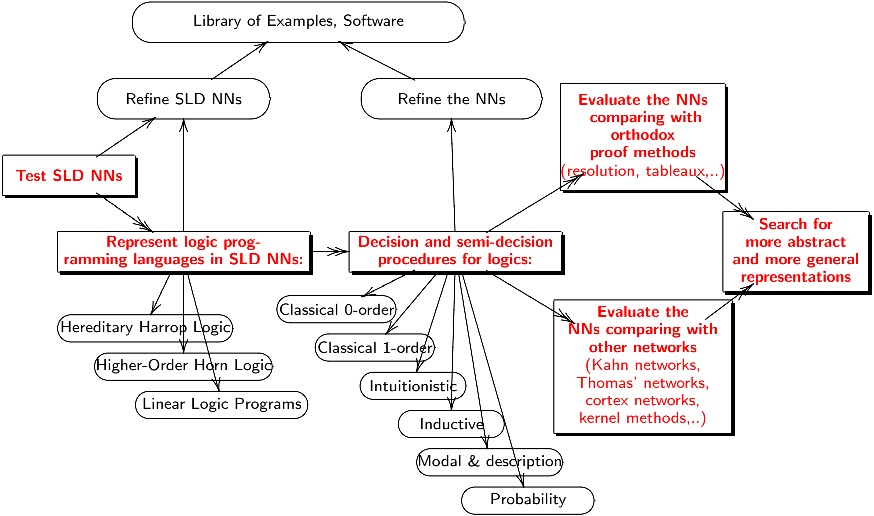
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Timeliness

- Computability Characterisation of Neural Networks;
- A rich body of material accumulated in Connectionism but research vacuum in the field;
- Wide range of NN simulators; numerous research centres working on learning techniques.

Novelty

The project is novel in the aspects of

- **Theory** (Finite representation of Logic Deduction in Neural Networks);
- **Methodology** (SLD resolution rather than semantic operators; finite construction; the use of learning in deduction);
- **Practice** (Evaluation and implementation of neural networks in computational logic).

Potential Beneficiaries

1. Researchers in neuro-symbolic integration, AI;
2. The Computational Logic and Automated Reasoning communities;
3. Individuals and organisations using automated theorem provers;
4. Researchers in different areas of computer science investigating and applying learning techniques.

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Items 2 - 3 $\Longrightarrow$ St Andrews.

St Andrews: Available Expertise

- Computational Logic Group (Roy Dyckhoff): development of proof assistants and theorem provers.
- Search algorithms; experimental methods in CS; constraint satisfaction problems (Ian Gent, Kevin Hammond, Ian Miguel, Tom Kelsey).
- Neural Network Research institutes in Edinburgh (Institute for Adaptive and Neural Computation; the Institute for Perception, Action and Behavior; the Neuroinformatics Doctoral Training Centre).

Thank you!