Coalgebraic Logic Programming: implicit versus explicit resource handling

Katya Komendantskaya, joint with J. Power and M. Schmidt

School of Computing, University of Dundee, UK

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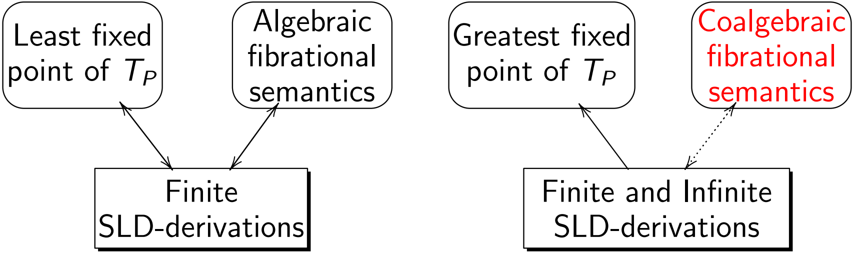
Algebraic and coalgebraic semantics for LP



Example

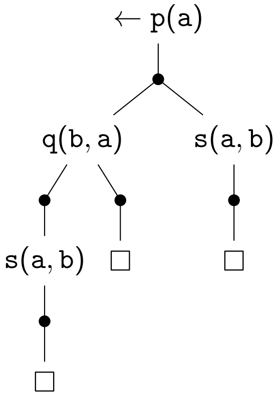
Example

Consider the logic program below .

$$
\begin{aligned}
\mathtt{q(b,a)} &\leftarrow \mathtt{s(a,b)} \\
\mathtt{q(b,a)} &\leftarrow \\
\mathtt{s(a,b)} &\leftarrow \\
\mathtt{p(a)} &\leftarrow \mathtt{q(b,a)}, \mathtt{s(a,b)}
\end{aligned}
$$

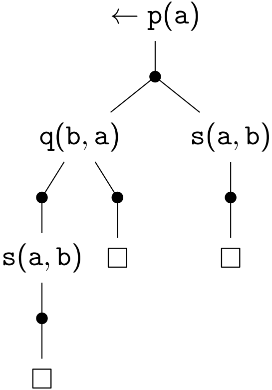
Examples of derivations

The action of
$\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on
`p(a)`



Examples of derivations

The action of $\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on $\mathtt{p(a)}$



Match it? - The SLD derivation

$$
\begin{array}{c}
\leftarrow \mathtt{p(a)} \\
| \\
\leftarrow \mathtt{q(b,a)}, \mathtt{s(a,b)} \\
| \\
\leftarrow \mathtt{s(a,b)}, \mathtt{s(a,b)} \\
| \\
\leftarrow \mathtt{s(a,b)} \\
| \\
\square
\end{array}
$$

Examples of a derivations

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$\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on
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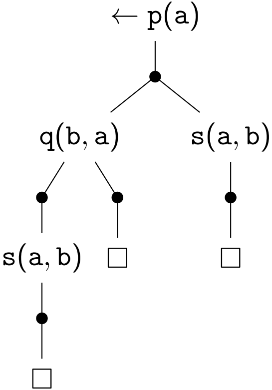
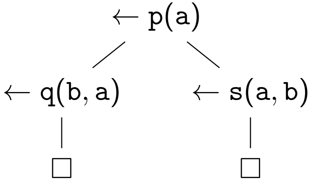


Match it? - The *proof tree*



Examples of a derivations

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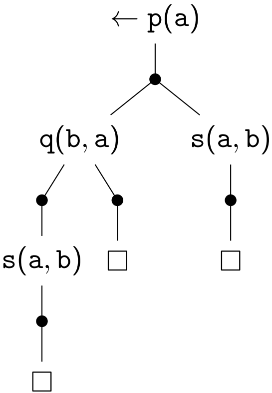
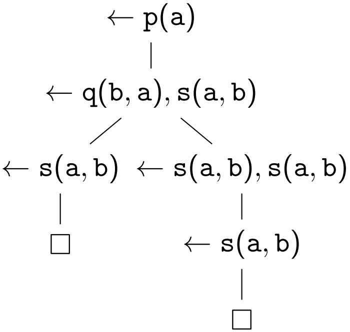


Match it? - The SLD tree



Is there anything at all in practice of Logic Programming that corresponds to the action of $C(P_f P_f)$-comonad?

From the examples above, it's clear that:

Sequential SLD-derivation

is the least suitable...

Proof trees

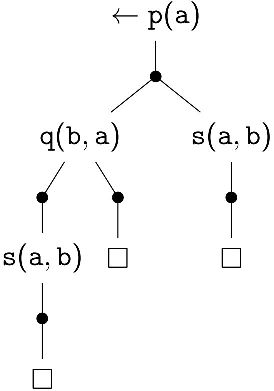
exhibit an and-parallelism in derivations...

SLD-trees

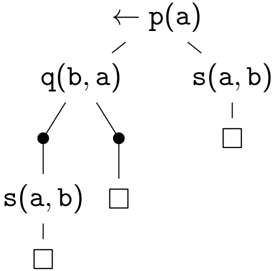
exhibit an or-parallelism in derivations...

It turns out that the answer lies in the combination of the two kinds of parallelism:

$\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on p(a)

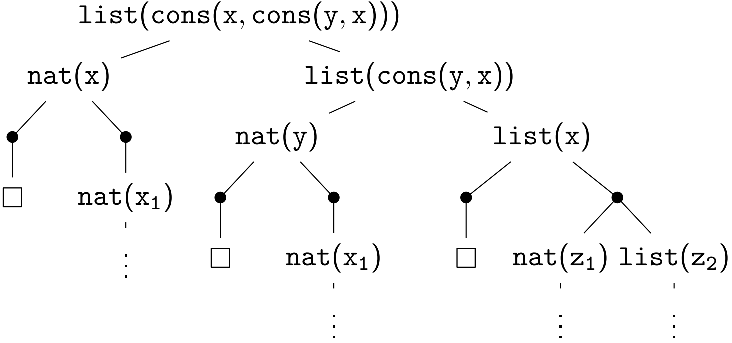


The and-or parallel tree

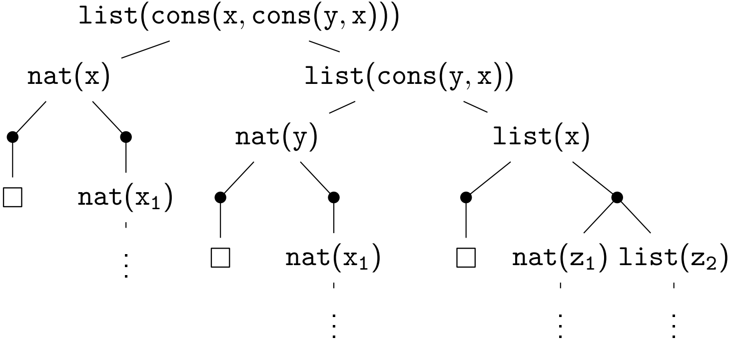


Except for... and-or trees are unsound in the first-order case.

Why unsound?

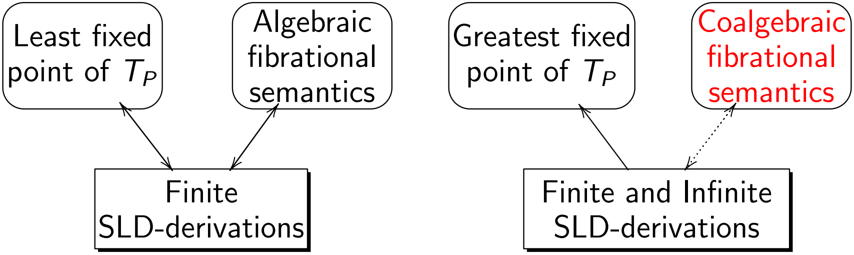

list(cons(x, cons(y, x)))
nat(x)
list(cons(y, x))
nat(y)
list(x)
□
nat(x1)
□
nat(x1)
□
nat(z1)
list(z2)

Why unsound?



This is how we realised we had to come up our own computational model for them.

Algebraic and coalgebraic semantics for LP



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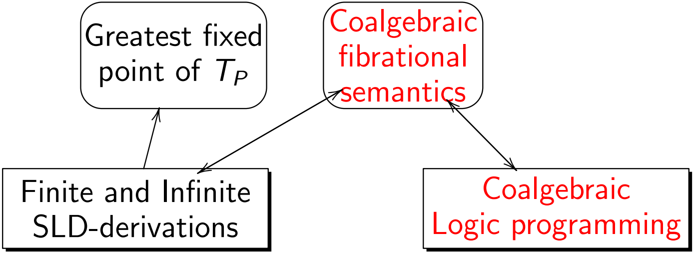

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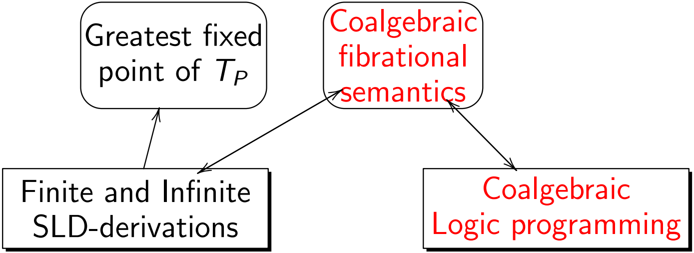

First prototype (by M. Schmidt) is available on the Web.

Recursion and Corecursion in Logic Programming

Example

$$
\begin{aligned}
\texttt{nat(0)} &\leftarrow \\
\texttt{nat(s(x))} &\leftarrow \texttt{nat(x)} \\
\texttt{list(nil)} &\leftarrow \\
\texttt{list(cons x y)} &\leftarrow \texttt{nat(x), list(y)}
\end{aligned}
$$

Example

$$
\begin{aligned}
\texttt{bit}(0) &\leftarrow \\
\texttt{bit}(1) &\leftarrow \\
\texttt{stream(cons (x,y))} &\leftarrow \texttt{bit(x)}, \texttt{stream(y)}
\end{aligned}
$$

SLD-resolution (+ unification and backtracking) behind LP derivations.

Example

$\mathtt{nat(0)} \leftarrow$
$\mathtt{nat(s(x))} \leftarrow \mathtt{nat(x)}$
$\mathtt{list(nil)} \leftarrow$
$\mathtt{list(cons\ x\ y)} \leftarrow \mathtt{nat(x)},$
$\mathtt{list(y)}$

$$\begin{array}{c} \leftarrow \mathtt{list}(\mathtt{cons}(\mathrm{x},\mathrm{y})) \\ | \\ \leftarrow \mathtt{nat}(\mathrm{x}), \mathtt{list}(\mathrm{y}) \end{array}$$

SLD-resolution (+ unification) is behind LP derivations.

Example

```
nat(0) ←
nat(s(x)) ← nat(x)
list(nil) ←
list(cons x y) ← nat(x),
                     list(y)
```

$$\leftarrow \mathtt{list}(\mathtt{cons}(x, y))$$
$$|$$
$$\leftarrow \mathtt{nat}(x), \mathtt{list}(y)$$
$$|$$
$$\leftarrow \mathtt{list}(y)$$

SLD-resolution (+ unification) is behind LP derivations.

Example

```
nat(0) ←
nat(s(x)) ← nat(x)
list(nil) ←
list(cons x y) ← nat(x),
                       list(y)
```

$$\leftarrow \mathtt{list}(\mathtt{cons}(\mathtt{x},\mathtt{y}))$$
$$|$$
$$\leftarrow \mathtt{nat}(\mathtt{x}),\mathtt{list}(\mathtt{y})$$
$$|$$
$$\leftarrow \mathtt{list}(\mathtt{y})$$
$$|$$
$$\leftarrow \Box$$

The answer is x/O, y/nil, but we can get more substitutions by backtracking. We can backtrack infinitely many times, but each time computation will terminate.

Things go wrong

Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
          bit(x), stream(y)
```

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```
bit(0) ←
bit(1) ←
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No answer, as derivation never terminates.

Things go wrong

Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←

          bit(x), stream(y)
```

No answer, as derivation never terminates.

Semantics may go wrong as well.

$$\leftarrow \mathtt{stream(scons(x, y))}$$
$$|$$
$$\leftarrow \mathtt{bit(x), stream(y)}$$
$$|$$
$$\leftarrow \mathtt{stream(y)}$$
$$|$$
$$\leftarrow \mathtt{bit(x_1), stream(y_1)}$$
$$|$$
$$\leftarrow \mathtt{stream(y_1)}$$
$$|$$
$$\leftarrow \mathtt{bit(x_2), stream(y_2)}$$
$$|$$
$$\leftarrow \mathtt{stream(y_2)}$$
$$|$$
$$\vdots$$

Solution - 1 [Gupta, Simon et al., 2007 - 2008]

Use normal SLD-resolution but add a new rule:

If a formula repeatedly appears as a resolvent (modulo α-conversion), then conclude the proof.

Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
            bit(x), stream(y)
```

$$\begin{array}{c} \leftarrow \mathtt{stream}(\mathtt{scons}(x, y)) \\ | \\ \leftarrow \mathtt{bit}(x), \mathtt{stream}(y) \\ | \\ \leftarrow \mathtt{stream}(y) \\ | \\ \leftarrow \mathtt{bit}(x_1), \mathtt{stream}(y_1) \\ | \\ \leftarrow \mathtt{stream}(y_1) \\ | \\ \square \end{array}$$

Solution - 1 [Gupta, Simon et al., 2007 - 2008]

Use normal SLD-resolution but add a new rule:

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Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
            bit(x), stream(y)
```

The answer is: $x/0$, $y/cons(x_1, y_1)$.

$$
\begin{array}{c}
\leftarrow \mathtt{stream}(\mathtt{scons}(\mathtt{x}, \mathtt{y})) \\
| \\
\leftarrow \mathtt{bit}(\mathtt{x}), \mathtt{stream}(\mathtt{y}) \\
| \\
\leftarrow \mathtt{stream}(\mathtt{y}) \\
| \\
\leftarrow \mathtt{bit}(\mathtt{x}_1), \mathtt{stream}(\mathtt{y}_1) \\
| \\
\leftarrow \mathtt{stream}(\mathtt{y}_1) \\
| \\
\square
\end{array}
$$

Explicitly-treated corecursion

To know whether to allow (co-LP) or disallow (standard LP) infinite loops, explicit annotation is needed.

Example

$$
\begin{array}{rcl}
\mathtt{bit}^i(0) & \leftarrow & \\
\mathtt{bit}^i(1) & \leftarrow & \\
\mathtt{stream}^c(\mathtt{scons}(x, y)) & \leftarrow & \mathtt{bit}^i(x), \mathtt{stream}^c(y) \\
\mathtt{list}^i(\mathtt{nil}) & \leftarrow & \\
\mathtt{list}^i(\mathtt{cons}(x, y)) & \leftarrow & \mathtt{bit}^i(x), \mathtt{list}^i(y)
\end{array}
$$

Drawbacks:

- some predicates may behave inductively or coinductively depending on the arguments provided, and such cases need to be resolved dynamically, and not statically; in which case mere predicate annotation fails.
- ... cannot mix induction and coinduction. — All clauses need to be marked as inductive or coinductive in advance.
- Can deal only with restricted sort of structures — the ones having finite regular pattern.

Example

`0:: 1:: 0:: 1:: 0:: ...` may be captured by such programs.
π represented as a stream may not.

- the derivation itself is not really a corecursive process.

Solution - 2. Coinductive LP in [Komendantskaya, Power CSL'11]

- ... arose from considerations valid for coalgebraic semantics of logic programs

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- ... arose from considerations valid for coalgebraic semantics of logic programs
  Technically:
- features parallel derivations;
- it is not a standard SLD-resolution any more, e.g. unification is restricted to term matching;

Coinductive trees

Definition

Let P be a logic program and $G = \leftarrow A$ be an atomic goal. The *coinductive derivation tree* for A is a tree T satisfying the following properties.

- A is the root of T.
- Each node in T is either an and-node or an or-node.
- Each or-node is given by $\bullet$.
- Each and-node is an atom.
- For every and-node A' occurring in T, there exist exactly $m > 0$ distinct clauses $C_1, \ldots, C_m$ in P (a clause C_i has the form $B_i \leftarrow B_1^i, \ldots, B_{n_i}^i$, for some n_i), such that $A' = B_1\theta_1 = ... = B_m\theta_m$, for some substitutions $\theta_1, \ldots, \theta_m$, then A' has exactly m children given by or-nodes, such that, for every $i \in m$, the ith or-node has n children given by and-nodes $B_1^i\theta_i, \ldots, B_{n_i}^i\theta_i$.

An Example

$\texttt{stream(x)} \xrightarrow{\theta_1}$

An Example

$\xrightarrow{\theta_1}$

stream(x)

$\xrightarrow{\theta_2} \ldots \xrightarrow{\theta_3}$

stream(scons(z, y))

bit(z)   stream(y)

An Example

$\mathtt{stream(x)} \xrightarrow{\theta_1}$

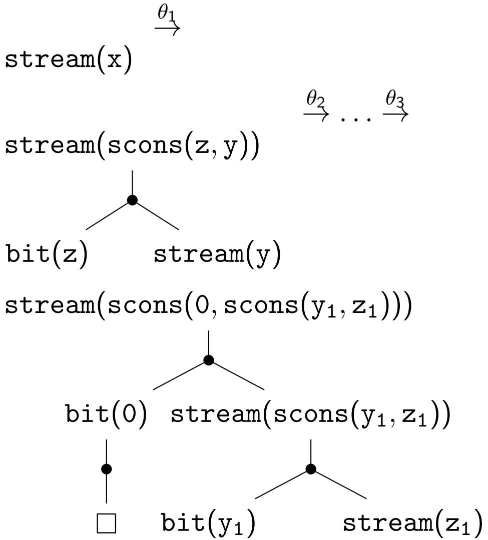
$\mathtt{stream(scons(z, y))} \xrightarrow{\theta_2} \ldots \xrightarrow{\theta_3}$

$\mathtt{bit(z)} \qquad \mathtt{stream(y)}$

Note that transitions θ may be determined in a number of ways:

- using mgus;
- non-deterministically;
- randomly;
- in a distributed/parallel manner.

An Example



Answers for x: *cons*(z, y) and *cons*$(0, cons(y_1, z_1))$. It's a different (corecursive) approach to what a "terminating derivation" is.

Solution - 2. Coinductive LP in [Komendantskaya, Power CSL'11]

Advantages

- Works uniformly for both inductive and coinductive definitions, without having to classify the two into disjoint sets;
- in spirit of corecursion, derivations may feature an infinite number of finite structures.
- there does not have to be regularity or repeating patterns in derivations.

Guarding corecursion

(Co)-Recursion is always dangerous:

... and needs to be guarded against infinite loops. Both in FP and LP, such guards can be given semantically or syntactically ("guardeness-by-construction").

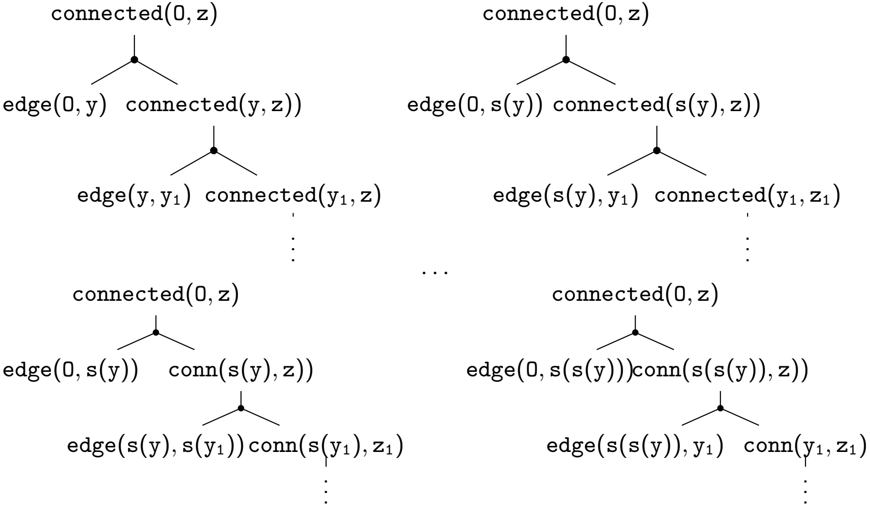
Example

This program is not guarded-by-constructors:

1. $\texttt{connected(x,x)} \leftarrow$
2. $\texttt{connected(x,y)} \leftarrow \texttt{edge(x,z)}, \texttt{connected(z,y)}.$

... and it will produce infinite coinductive trees.

Infinite forests of infinite trees:



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In reality, such programs will be disallowed by the termination checker, and will need to be reformulated.

Guarding corecursion, for example:

Example

$$
\begin{aligned}
\mathtt{connected}(X, cons(Node, Path)) &\leftarrow \mathtt{edge}(X, Node), \mathtt{connected}(Node, Path) \\
\mathtt{connected}(X, nil) &\leftarrow \\
\mathtt{edge}(0, 0) &\leftarrow \\
\mathtt{edge}(X, s(X)) &\leftarrow
\end{aligned}
$$

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\texttt{edge}(X, s(X)) &\leftarrow
\end{aligned}$$

$$\begin{array}{ccccc}
\begin{array}{c} \texttt{conn}(0, \texttt{cons}(\texttt{y}, \texttt{z})) \\ \texttt{edge}(0, \texttt{y}) \quad \texttt{conn}(\texttt{y}, \texttt{z})) \end{array}
& \rightarrow &
\begin{array}{c} \texttt{conn}(0, \texttt{cons}(\texttt{s0}, \texttt{z})) \\ \texttt{edge}(0, \texttt{s0}) \quad \texttt{conn}(\texttt{s0}, \texttt{z})) \\ \Box \end{array}
& \rightarrow &
\begin{array}{c} \texttt{conn}(0, \texttt{cons}(\texttt{s0}, \texttt{nil})) \\ \texttt{edge}(0, \texttt{s0}) \quad \texttt{conn}(\texttt{s0}, \texttt{nil})) \\ \Box \qquad\qquad \Box \end{array}
\end{array}$$

More discipline?

Adapting this sort of programming discipline from lazy functional languages to LP may have its advantages. E.g., it will equally guard against programs that induce infinite SLD-derivations:

Example

1. $\texttt{connected(x,y)} \leftarrow \texttt{connected(z,y),edge(x,z)}$
2. $\texttt{connected(x,x)} \leftarrow$

While currently, it is up to a programmer to manually weed-out such cases.

Corecursion guarding parallelism:

Corecursion FREEING! parallelism:

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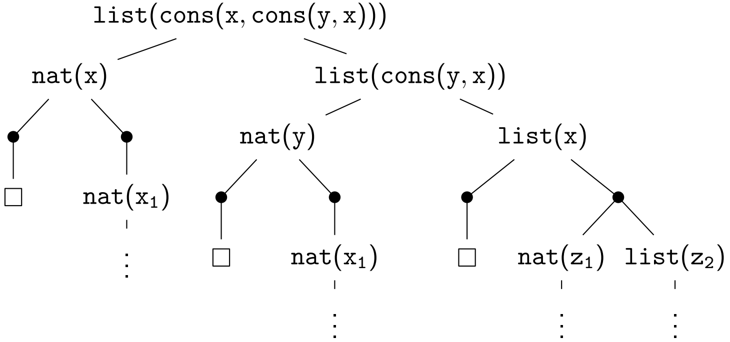
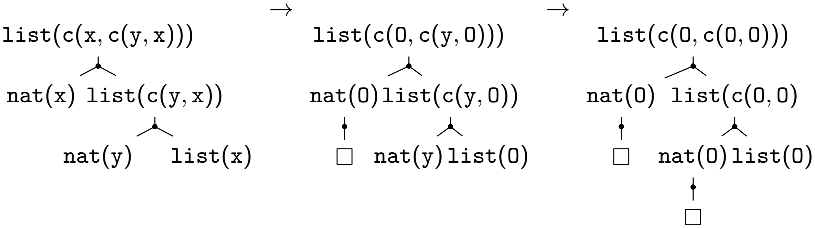


Now, by the same lazy corecursive derivation:



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Both are parallelisable, and LP is free.

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Variable Synchronization?

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Variable Synchronization? ... is no longer in power...

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Variable Synchronization? ... is no longer in ... in use.

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Both are parallelisable, and LP is free.

Variable Synchronization? ... is no longer in ... in use.

Variables can live their own lazy corecursive lives.

[Instead of] Conclusions...

So, what happened to the old Rule?

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Logic Programs = Logic + Control

[Kowalski 1979]

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Logic Programs $=$ *Logic* $+$ *Control*

[Kowalski 1979]

We have new rules:

Corecursive Programs: **LOGIC is Control**

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The End.