Structural Resolution

Katya Komendantskaya (joint work with Patricia Johann)

School of Computing, University of Dundee, UK

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Resolution rule

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- First-order:

$$\frac{C \vee A \quad \neg B \vee D}{\theta(C) \vee \theta(D)},$$

if θ is a unifier of A and B (i.e., $\theta(A) = \theta(B)$.

Restricted Resolution by term-matching

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... incomplete relative to the usual resolution rule...

Structural Resolution

1. (the "term-matching rule"):

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Structural Resolution

1. (the "term-matching rule"):

$$\frac{C \vee A \quad \neg B \vee D}{\theta(C) \vee D},$$

   where θ is a matcher of A and B (i.e., $\theta(A) = B$).

2. plus the rule

$$\frac{C \vee A \quad \neg B \vee D}{C \vee A, \quad \theta(\neg B) \vee \theta(D)},$$

   where θ is a unifier of A and B.

SLD-resolution

... is an instance of standard resolution rule:

$$\frac{C_1 \vee \ldots \vee C_n \vee A \quad \neg B \vee \neg D_1 \vee \ldots \vee \neg D_k}{\theta(C_1 \vee \ldots \vee C_n) \vee \theta(\neg D_1 \vee \ldots \vee \neg D_k)},$$

if θ is a unifier of A and B (i.e., $\theta(A) = \theta(B)$.

Example 1

Example

1.$\texttt{nat(0)} \leftarrow$

2.$\texttt{nat(s(x))} \leftarrow \texttt{nat(x)}$

Computes $x \mapsto 0$, and other natural numbers with the use of backtracking.

$$\leftarrow \texttt{nat(x)}$$

$$|$$

$$\square$$

Example 2

Nat2:

Example

1.$\texttt{nat(s(x))} \leftarrow \texttt{nat(x)}$
2.$\texttt{nat(0)} \leftarrow$

Computes $x \mapsto s(s(s\ldots)))$, the first limit ordinal.

$$\begin{array}{c} \leftarrow \texttt{nat}(\texttt{x}) \\ | \\ \leftarrow \texttt{nat}(\texttt{x}') \\ | \\ \leftarrow \texttt{nat}(\texttt{x}'') \\ | \\ \ldots \end{array}$$

Example 3

Program **Stream**:

Example

```
1.bit(0) ←
2.bit(1) ←
3.stream(scons(x,y)) ←
        bit(x), stream(y)
```

Computes an infinite stream.

$$\leftarrow \mathtt{stream}(\mathtt{scons}(x, y))$$

$$\leftarrow \mathtt{bit}(x), \mathtt{stream}(y)$$

$$\leftarrow \mathtt{stream}(y)$$

$$\leftarrow \mathtt{bit}(x_1), \mathtt{stream}(y_1)$$

$$\leftarrow \mathtt{stream}(y_1)$$

$$\vdots$$

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$$bad(x) \leftarrow bad(x)$$

We are missing a theory to talk about such things...

Nat in S-resolution

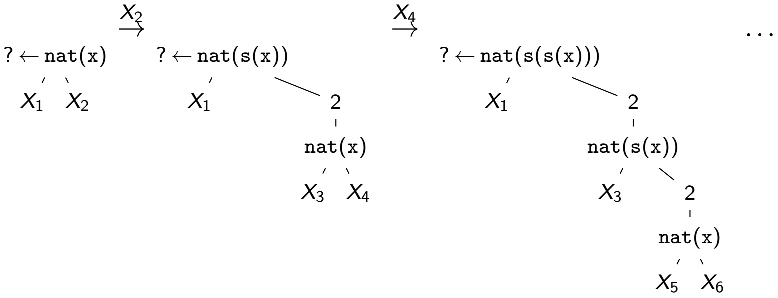
1.$\texttt{nat(0)} \leftarrow$
2.$\texttt{nat(s(x))} \leftarrow \texttt{nat(x)}$

$$\begin{array}{c} ? \leftarrow \texttt{nat(X)} \\ / \quad \backslash \\ X_1 \quad X_2 \end{array} \quad \xrightarrow{X_1} \quad \begin{array}{c} ? \leftarrow \texttt{nat(0)} \\ / \quad \backslash \\ 1 \quad X_2 \end{array}$$

Nat2 in S-resolution

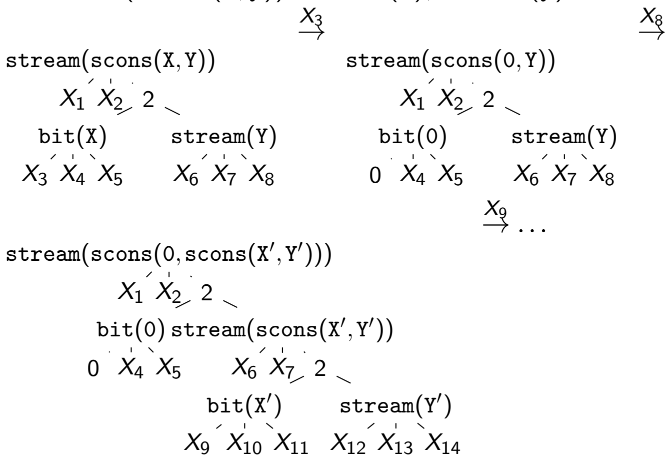
1.$\texttt{nat(0)} \leftarrow$

2.$\texttt{nat(s(x))} \leftarrow \texttt{nat(x)}$



Streams in S-resolution

0. $\texttt{bit}(\texttt{0}) \leftarrow$
1. $\texttt{bit}(\texttt{1}) \leftarrow$
2. $\texttt{stream}(\texttt{scons}(\texttt{x},\texttt{y})) \leftarrow \texttt{bit}(\texttt{x}), \texttt{stream}(\texttt{y})$



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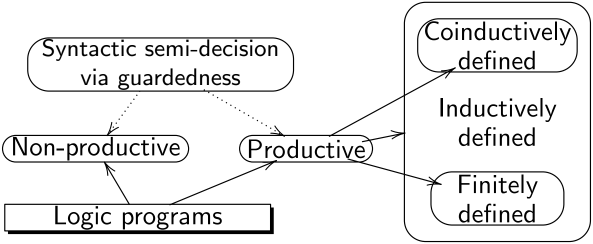
Nat and Nat2	Infinite streams.	Bad recursion.
$\mathtt{nat(s(x))} \leftarrow \mathtt{nat(x)}$ $\mathtt{nat(0)} \leftarrow$ inductive definition	$\mathtt{stream(scons(x,y))} \leftarrow \mathtt{stream(y)}$ coinductive definition	$\mathtt{bad(x)} \leftarrow \mathtt{bad(x)}$ non-well-founded
Productive inductive program	Productive coinductive program	Non-productive program
finite rewriting trees, possibly infinite derivations	finite rewriting trees, necessarily infinite derivations	infinite rewriting trees

Theory of universal Productivity in LP!



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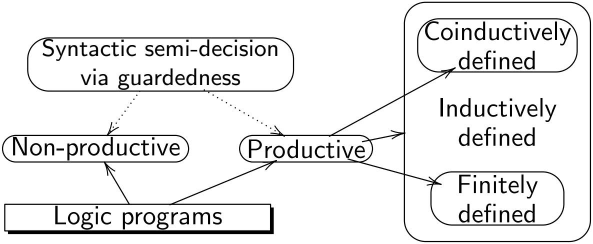

This and more

... in my poster session