Coalgebraic Logic Programming

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Recursion and Corecursion in Logic Programming

1. In the 70s-80s: Apt, van Emden, Kowalski: study of recursion and least Herbrand model semantics of LP.
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2 in the 80s; Abdallah, van Emden, Lloyd: “perpetual” computations in LP and the greatest fixed point semantics of LP: incomplete, no finite procedure for computations given

3 2000s: Gupta, Simon et al: CoLP: finite derivation procedure for coinductive programs, soundness and completeness for programs describing regular trees.

4 Our work, from 2010, – coalgebraic semantics for LP, and inspired derivation procedures.
Recursion and Corecursion in Logic Programming

Example

\[
\begin{align*}
\text{bit}(0) & \leftarrow \\
\text{bit}(1) & \leftarrow \\
\text{list}(\text{nil}) & \leftarrow \\
\text{list}(\text{cons} (X, Y)) & \leftarrow \text{bit}(X), \text{list}(Y)
\end{align*}
\]

Example

\[
\begin{align*}
\text{stream}(\text{cons} (X,Y)) & \leftarrow \text{bit}(X), \text{stream}(Y)
\end{align*}
\]
SLD-resolution (+ unification and backtracking) behind LP derivations.

Example:

\[
\begin{align*}
nat(0) & \leftarrow \\
nat(s(x)) & \leftarrow nat(x) \\
\text{list(nil)} & \leftarrow \\
\text{list(cons } x \text{ } y) & \leftarrow nat(x), \\
\text{list(y)} & \leftarrow nat(x), \text{list(y)} \\
\end{align*}
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SLD-resolution (+ unification) is behind LP derivations.

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\end{align*}

\begin{align*}
\leftarrow list(cons(x, y)) \\
\leftarrow nat(x), list(y) \\
\leftarrow list(y) \\
\leftarrow \square
\end{align*}

The answer is \(x/O, y/nil\), but we can get more substitutions by backtracking. We can backtrack infinitely many times, but each time computation will terminate.
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The answer is \( x/O, \ y/nil \), but we can get more substitutions by backtracking. We can backtrack infinitely many times, but each time computation will terminate.

Nice, clean semantics: least Herbrand model exists, sound&complete, etc…
Corecursion in LP?

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\begin{align*}
\text{bit}(0) & \leftarrow \\
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\text{stream}(\text{scons}(x, y)) & \leftarrow \\
& \quad \text{bit}(x), \text{stream}(y)
\end{align*}
Corecursion in LP?

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\[
\begin{align*}
\text{bit}(x), \text{stream}(y)
\end{align*}
\]

No answer, as derivation never terminates.
Semantics may go wrong as well: least Herbrand models will contain an infinite term corresponding to stream: so completeness fails.
It can be worse....

Example

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\text{list}(\text{nil}) \leftarrow \\
\]

No answer, as derivation never terminates.
Semantics goes wrong: this time, soundness!
If a formula repeatedly appears as a resolvent (modulo $\alpha$-conversion), then conclude the proof.

Example

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\begin{align*}
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\end{align*}
\]

\[
\begin{align*}
& \quad \text{stream}(X) \\
& \quad \text{bit}(X), \text{stream}(X) \\
& \quad \text{stream}(X) \\
& \quad \square^c
\end{align*}
\]
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Example

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& \quad \text{bit}(X), \text{stream}(Y)
\end{align*}
\]

The answer is: $X / \text{cons}(0, X)$. Requires programs to be regular, in order to be sound and complete.
CoALP: what is it about?

- syntactically – first-order logic programming;
- operationally – lazy (co)recursion;
CoALP: what is it about?

- syntactically – first-order logic programming;
- operationally – lazy (co)recursion;
- inspired by coalgebraic fibrational semantics;
- uses and-or parallel trees, but restricts unification to matching;

**Term-matcher**

A substitution $\theta$ is a term-matcher for $A$ and $B$ is $A\theta = B$. 
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A substitution $\theta$ is a term-matcher for $A$ and $B$ is $A\theta = B$.

- explores the tree-structure of partial proofs – ”coinductive trees”;

**Coinductive tree...**

is an and-or-parallel tree in which unification is restricted to term-matching;
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**Coinductive tree...**

is an and-or-parallel tree in which unification is restricted to term-matching;

- Coinductive trees give a measure for lazy guarded corecursion, (cf. ”clocked corecursion”)

Lazy Corecursion in CoALP: Coinductive trees

\[ \theta_1 \]

\[ \text{stream}(x) \]
Lazy Corecursion in CoALP: Coinductive trees

\[
\begin{array}{cc}
\text{stream}(x) & \text{stream}(\text{scons}(z,y)) \\
\text{bit}(z) & \text{stream}(y)
\end{array}
\]

\[
\theta_1 \Rightarrow \quad \theta_2 \quad \ldots \quad \theta_3
\]
Lazy Corecursion in CoALP: Coinductive trees

\[
\begin{align*}
\theta_1 & \rightarrow & \text{stream(x)} & \rightarrow & \text{stream(scons(z,y))} & \rightarrow & \text{...} & \rightarrow & \theta_3 \\
\text{bit(z)} & & \text{stream(y)} & & \\
\end{align*}
\]

Note that transitions $\theta$ may be determined in a number of ways:

- using mgus;
- non-deterministically;
- in a distributed/parallel manner.
Lazy Corecursion in CoALP

The above would correspond to one-branch of SLD-derivations we have seen! The main driving force: separation of layers of computations into different dimensions.
Computationally essential:

1. for coinductive Stream program, the coinductive-trees are finite!!! – both in depth and in breadth;
2. each tree gives only a partial computation – it is not like eager SLD-trees we have seen earlier;
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1. for coinductive Stream program, the coinductive-trees are finite!!! – both in depth and in breadth;
2. each tree gives only a partial computation – it is not like eager SLD-trees we have seen earlier;

1. ⇒ gives hope for a formalism to describe termination and productivity, as in functional languages
2. ⇒ hints there may be laziness involved...
What do we gain?

1. A coherent theory of termination and productivity of recursion and corecursion in LP
Theory of Productivity in LP

Typeful functional theorem provers:

- Terminating
- Non-terminating
  - Recursion
- Productive
- Non-productive
  - Corecursion
Theory of Productivity in LP

Typeful functional theorem provers:

- Terminating
- Non-terminating
- Productive
- Non-productive

Recursion  \( \uparrow \)

Corecursion  \( \downarrow \)

CoALP

- Coinductive Derivations
- Non-productive
- Productive
- Coinductive
- Inductive
- Finite
What do we gain?

1. A coherent theory of termination and productivity of recursion and corecursion in LP
2. Extension of classes of inductive and coinductive programs we can handle,
Stream of Fibonacci numbers:

Falls into infinite loops in Prolog and CoLP.

1. add(0,Y,Y).
2. add(s(X),Y,s(Z)) :- add(X,Y,Z).
3. fibs(X,Y,cons(X,S)) :- add(X,Y,Z), fibs(Y,Z,S).
4. nth(0,cons(X,S),X).
5. nth(s(N),cons(X,S),Y) :- nth(N,S,Y).
6. fib(N,X) :- fibs(0,s(0),S), nth(N,S,X).
7. fib2(X) :- fib(s(s(0)),X).
Examples of derivations with Fib: lazy step 1

\[
\begin{align*}
\text{fib2}(X) &
\text{fib}(s^2(0), X) \\
\text{fibs}(0, s(0), S) &\quad \text{nth}(s^2(0), S, X) \\
5, S/c(X_1, S_1) &
\end{align*}
\]
Examples of derivations with Fib: lazy step 1

1. \text{add}(0,Y,Y).
2. \text{add}(s(X),Y,s(Z)) :- \text{add}(X,Y,Z).
3. \text{fibs}(X,Y,\text{cons}(X,S)) :- \text{add}(X,Y,Z), \text{fibs}(Y,Z,S).
4. \text{nth}(0,\text{cons}(X,S),X).
5. \text{nth}(s(N),\text{cons}(X,S),Y) :- \text{nth}(N,\text{cons}(X,S),Y).
6. \text{fib}(N,X) :- \text{fibs}(0,s(0),S), \text{nth}(N,S,X).
7. \text{fib2}(X) :- \text{fib}(s(s(0)),X).
Examples of derivations with Fib: lazy step 2

```
add(0,Y,Y).
add(s(X),Y,s(Z)) :- add(X,Y,Z).
fibs(X,Y,cons(X,S)) :- add(X,Y,Z), fibs(Y,Z,S).
nth(0,cons(X,S),X).
nth(s(N),cons(X,S),Y) :- nth(N,S,Y).
fib(N,X) :- fibs(0,s(0),S), nth(N,S,X).
fib2(X) :- fib(s(s(0)),X).
```
Examples of derivations with Fib: lazy step 2

1. add(0,Y,Y).
2. add(s(X),Y,s(Z)) :- add(X,Y,Z).
3. fibs(X,Y,cons(X,S)) :- add(X,Y,Z), fibs(Y,Z,S).
4. nth(0,cons(X,S),X).
5. nth(s(N),cons(X,S),Y) :- nth(N,cons(X,S),Y).
6. fib(N,X) :- fibs(0,s(0),S), nth(N,S,X).
7. fib2(X) :- fib(s(s(0)),X).
Examples of derivations with Fib: lazy step 3

```
fib2(X)  
    |  
    |  
    |  
fib(s^2(0)),X)  
  /  |  
/    |  
fibs(0,s(0),c(X1,c(X2,S2)))    nth(s^2(0),c(X1,c(X2,S2)),X)  
        /  |  
        /    |  
        /      |  
nth(s(0),c(X2,S2),X)  
            /  |  
            /    |  
            /      |  
nth(0,S2,X)  

S2/c(X,S3) →
```
Examples of derivations with Fib: lazy step 4

```
fib2(X)
  ↓
fib(s^2(0)),X)
```

```
fibs(0,s(0),c(X1,c(X2,c(X,S3))))
```

```
nth(s^2(0),c(X1,c(X2,c(X,S3))),X)
```

```
nth(s(0),c(X2,c(X,S3)),X)
```

```
nth(0,c(X,S3),X)
```

```
nth(0,c(X,S3),X)
```

```
x1/0
```

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CoALP
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Examples of derivations with Fib: lazy step 5

\[
\text{fib2}(X) \\
\text{fib}(s^2(0), X) \\
\text{fibs}(0, s(0), c(0, c(X_2, c(X, S_3)))) \\
\text{a}(0, s(0), Z) \\
\text{fibs}(s(0), Z, c(X_2, c(X, S_3))) \\
nth(s^2(0), c(0, c(X_2, c(X, S_3))), X) \\
nth(s(0), c(X_2, c(X, S_3)), X) \\
nth(0, c(X, S_3), X) \\
nth(0, c(X, S_3), X) \\
Z/0 \\
\overset{\rightarrow}{\rightarrow}
\]
Examples of derivations with Fib: lazy step 6

\[
\begin{align*}
\text{fib}_2(X) & \\
\text{fib}(s^2(0),X) & \\
\text{fibs}(0,s(0),c(0,c(X_2,c(X,S_3)))) & \quad \text{nths}(s^2(0),c(0,c(X_2,c(X,S_3))),X)
\end{align*}
\]

\[
\begin{align*}
a(0,s(0),s(0)) & \\
fibs(s(0),s(0),c(X_2,c(X,S_3))) & \\
\text{nths}(s(0),c(X_2,c(X,S_3)),X)
\end{align*}
\]

\[
\begin{align*}
\text{nths}(0,c(X,S_3),X) & \\
\text{nths}(0,c(X,S_3),X) & \quad X_2/s(0)
\end{align*}
\]
Examples of derivations with Fib: lazy step 7

\[
\begin{align*}
\text{fib2}(X) & \\
\text{fib}(s^2(0)), X) & \\
\text{fibs}(0, s(0), c(0, c(s(0), c(X, S3)))) & \text{nth}(s^2(0), c(0, c(s(0), c(X, S3))), X) \\
\text{a}(0, s(0), s(0)) & \text{fibs}(s(0), s(0), c(s(0), c(X, S3))) & \text{nth}(s(0), c(s(0), c(X, S3)), X) \\
\text{a}(s(0), s(0), Z) & \text{fibs}(s(0), Z, c(X, S3)) & \text{nth}(0, c(X, S3), X) \\
\text{nth}(0, c(X, S3), X) & \\
\end{align*}
\]
Examples of derivations with Fib: lazy step 8

```
fib2(X)
```

```
fib(s^2(0)), X)
```

```
fibs(0, s(0), c(0, c(s(0), c(X, S3))))
```

```
nth(s^2(0), c(0, c(s(0), c(X, S3))), X)
```

```
a(0, s(0), s(0))
```

```
fibs(s(0), s(0), c(s(0), c(X, S3)))
```

```
nth(s(0), c(s(0), c(X, S3)), X)
```

```
a(0, s(0), s(0))
```

```
fibs(s(0), s(s(0)), c(X, S3))
```

```
nth(0, c(X, S3), X)
```

```
a(s(0), s(0), s(s(0)))
```

```
X/s(0)
```
Examples of derivations with Fib: lazy step 9

\[ \text{fib2}(s(0)) \]
\[ \rightarrow \]
\[ \text{fib}(s^2(0)), s(0)) \]
\[ \rightarrow \]
\[ \text{fibs}(0, s(0), c(0, c(s(0), c(s(0), S3)))) \]
\[ \rightarrow \]
\[ \text{nth}(s^2(0), c(0, c(s(0), c(s(0), S3))), s(0)) \]
\[ \rightarrow \]
\[ \text{a}(0, s(0), s(0)) \]
\[ \rightarrow \]
\[ \text{fibs}(s(0), s(0), c(s(0), c(s(0), S3))) \]
\[ \rightarrow \]
\[ \text{nth}(s(0), c(s(0), c(s(0), S3)), s(0)) \]
\[ \rightarrow \]
\[ \text{a}(s(0), s(0), s(s(0))) \]
\[ \rightarrow \]
\[ \text{fibs}(s(0), s(0), c(s(0), S3)) \]
\[ \rightarrow \]
\[ \text{nth}(0, c(s(0), S3), s(0)) \]
\[ \rightarrow \]
\[ \text{a}(0, s(0), s(0)) \]
\[ \rightarrow \]
\[ a(s(0), s(0), s(s(0))), Z) \]
\[ \rightarrow \]
\[ \text{fibs}(s(s(0)), Z, S3) \]
\[ \rightarrow \]
\[ \text{nth}(0, c(s(0), S3), s(0)) \]
CoALP Properties:

- Sound and complete with respect to the coalgebraic semantics;
- Finite computations are sound and complete with respect to the least Herbrand model semantics (so, we can do as much as standard Prolog).
- Adequacy result for observational semantics.
## Logic Programming dialects, compared

<table>
<thead>
<tr>
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<th>Parallel Prolog</th>
<th>Co-LP</th>
<th>CoALP</th>
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<tbody>
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<td>No</td>
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<td>coalgebraic</td>
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<tr>
<td><strong>Operational semantics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Logic Programming dialects, compared

<table>
<thead>
<tr>
<th></th>
<th>Prolog</th>
<th>Parallel Prolog</th>
<th>Co-LP</th>
<th>CoALP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fib example</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Execution</strong></td>
<td>Eager</td>
<td>Eager</td>
<td>Eager</td>
<td>Lazy</td>
</tr>
<tr>
<td><strong>Corecursion</strong></td>
<td>No</td>
<td>No</td>
<td>by Regular Loop detection</td>
<td>Guardedness by constructors</td>
</tr>
<tr>
<td><strong>Mode of execution</strong></td>
<td>Sequential</td>
<td>Parallel</td>
<td>Sequential</td>
<td>Parallel</td>
</tr>
<tr>
<td><strong>Declarative semantics</strong></td>
<td>lfwp</td>
<td>lfwp</td>
<td>gfp (restricted)</td>
<td>coalgebraic</td>
</tr>
<tr>
<td><strong>Operational semantics</strong></td>
<td>transitions; states: lists of formulae</td>
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<td>transitions; states: coinductive trees</td>
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Current and future work

1. Using CoALP to formally define a general theory of Termination and Productivity for Recursion and Corecursion in LP
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4. Extension of CoALP with constraints
5. Applications to type inference

... join us!
Thank you!

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