Designing a small programming language, coalgebraically

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Shonan'13, Coinduction for Computation Structures and Programming Languages
9 October 2013

Outline

1. Motivation: LP key facts

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2. Coalgebraic Semantics and its Implementation
   - Variable-free case
   - General Case

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3. Comparing with the actual state-of-the art...

4. Future: CoALP for Type Inference?

LP Key Facts

- An un-typed declarative language, with eager evaluation.
- ... Based on Predicate logic [*vanEmden and Kowalski, "The Semantics of Predicate Logic as a Programming Language", 1976*]
- ... Set-theoretic semantics given by lfp or gfp of the semantic operator T_P.
- ... Operational semantics given by SLD-resolution. [*Robinson "A Machine-Oriented Logic Based on the Resolution Principle", 1965*]
- Many dialects exist: Prolog, Datalog, etc.
- Applications: Data bases, Proof theory (Automated First Order Theorem Provers), AI, Hindley-Milner style Type inference in Functional languages.

Recursion in Logic Programming

Example

$$
\begin{aligned}
\mathtt{nat(0)} &\leftarrow \\
\mathtt{nat(s(x))} &\leftarrow \mathtt{nat(x)} \\
\mathtt{list(nil)} &\leftarrow \\
\mathtt{list(cons\ x\ y)} &\leftarrow \mathtt{nat(x),\ list(y)}
\end{aligned}
$$

SLD-resolution (+ unification and backtracking) behind LP derivations.

Example

$\mathtt{nat(0)} \leftarrow$

$\mathtt{nat(s(x))} \leftarrow \mathtt{nat(x)}$

$\mathtt{list(nil)} \leftarrow$

$\mathtt{list(cons\ x\ y)} \leftarrow \mathtt{nat(x)},$

$\mathtt{list(y)}$

$$\leftarrow \mathtt{list}(\mathtt{cons}(\mathrm{x}, \mathrm{y}))$$

$$|$$

$$\leftarrow \mathtt{nat}(\mathrm{x}), \mathtt{list}(\mathrm{y})$$

SLD-resolution (+ unification) is behind LP derivations.

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$$\begin{array}{c} \leftarrow \mathtt{list}(\mathtt{cons}(\mathtt{x},\mathtt{y})) \\ | \\ \leftarrow \mathtt{nat}(\mathtt{x}), \mathtt{list}(\mathtt{y}) \\ | \\ \leftarrow \mathtt{list}(\mathtt{y}) \end{array}$$

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Example

```
nat(0) ←
nat(s(x)) ← nat(x)
list(nil) ←
list(cons x y) ← nat(x),
                    list(y)
```

$$\leftarrow \mathtt{list}(\mathtt{cons}(x, y))$$
$$|$$
$$\leftarrow \mathtt{nat}(x), \mathtt{list}(y)$$
$$|$$
$$\leftarrow \mathtt{list}(y)$$
$$|$$
$$\leftarrow \Box$$

The answer is x/O, y/nil, but we can get more substitutions by backtracking. We can backtrack infinitely many times, but each time computation will terminate.

Relation to Coalgebra? – Poor

- Coinduction fails (because of eager evaluation)
- Concurrency fails (because of unification and variable dependencies)

Problems with Corecursion in LP?

Example

$\mathtt{bit(0)} \leftarrow$

$\mathtt{bit(1)} \leftarrow$

$\mathtt{stream(scons\ x\ y)} \leftarrow$

$\qquad\qquad \mathtt{bit(x),\ stream(y)}$

Problems with Corecursion in LP?

Example

```
bit(0) ←
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No answer, as derivation never terminates.

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Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
        bit(x), stream(y)
```

No answer, as derivation never terminates.

Semantics may go wrong as well.

$$\begin{array}{c}
\leftarrow \mathtt{stream}(\mathtt{scons}(x, y)) \\
| \\
\leftarrow \mathtt{bit}(x), \mathtt{stream}(y) \\
| \\
\leftarrow \mathtt{stream}(y) \\
| \\
\leftarrow \mathtt{bit}(x_1), \mathtt{stream}(y_1) \\
| \\
\leftarrow \mathtt{stream}(y_1) \\
| \\
\leftarrow \mathtt{bit}(x_2), \mathtt{stream}(y_2) \\
| \\
\leftarrow \mathtt{stream}(y_2) \\
| \\
\vdots
\end{array}$$

Relation to Coalgebra? – Poor

- Coinduction fails (because of eager evaluation)
- Concurrency fails (because unification and variable dependencies)

Problems with concurrency/corecursion in LP

[A popular trend in the 90s...]

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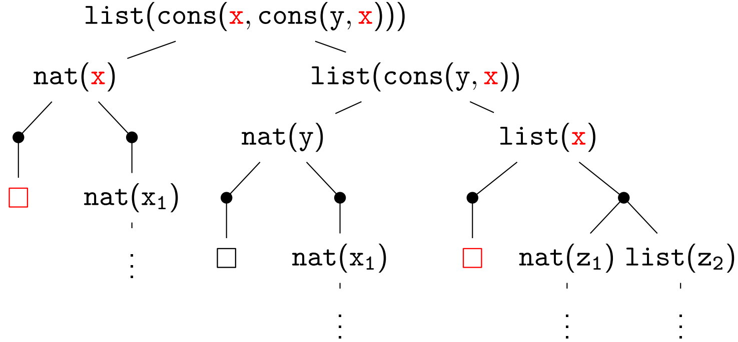
[A popular trend in the 90s...] Unsound and-or parallelism:



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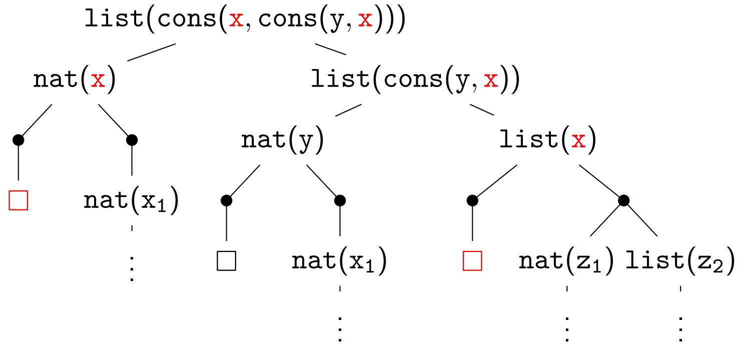

If unsound – lets synchronize variable substitution! – many engineering solutions... Synchronisation breaks parallelisation, in the general case.

Relation to Coalgebra? – Poor

- Coinduction fails (because of eager evaluation)
- Concurrency fails (because unification and variable dependencies)

Lets time-travel to early 70s and fix it...

... create coalgebraically oriented alternative from scratch...

Outline

CoALP semantics bibliography

- E.Komendantskaya and J.Power. Coalgebraic derivations in logic programming. International conference Computer Science Logic, CSL'11.
- E.Komendantskaya and J.Power. Coalgebraic semantics for derivations in logic programming. International conference on Algebra and Coalgebra CALCO'11.
- E. Komendantskaya, G. McCusker and J. Power. Coalgebraic semantics for parallel derivation strategies in logic programming. Proceedings of AMAST'2010.

Current work – implementation and applications...

Coalgebraic Analysis of derivations in Logic Programs

Given a variable-free logic program P, let At be the set of all atoms appearing in P. Then P can be identified with a $P_f P_f$-coalgebra (At, p), where $p : At \longrightarrow P_f(P_f(\mathrm{At}))$ sends an atom A to the set of bodies of those clauses in P with head A, each body being viewed as the set of atoms that appear in it.

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Example

$$
\begin{array}{rcl}
\mathtt{q(b,a)} & \leftarrow & \mathtt{s(a,b)} \\
\mathtt{q(b,a)} & \leftarrow & \\
\mathtt{s(a,b)} & \leftarrow & \\
\mathtt{p(a)} & \leftarrow & \mathtt{q(b,a)}, \mathtt{s(a,b)}
\end{array}
$$

$p(q(b,a)) = \{\{\}, \{s(a,b)\}\}$

Coalgebraic Analysis of ground Logic Programs

Taking $p : \mathrm{At} \longrightarrow P_f P_f(\mathrm{At})$, the corresponding $C(P_f P_f)$-coalgebra where $C(P_f P_f)$ is the cofree comonad on $P_f P_f$ is given as follows: $C(P_f P_f)(\mathrm{At})$ is given by a limit of the form

$$\ldots \longrightarrow \mathrm{At} \times P_f P_f(\mathrm{At} \times P_f P_f(\mathrm{At})) \longrightarrow \mathrm{At} \times P_f P_f(\mathrm{At}) \longrightarrow \mathrm{At}.$$

We inductively define the objects $\mathrm{At}_0 = \mathrm{At}$ and $\mathrm{At}_{n+1} = \mathrm{At} \times P_f P_f \mathrm{At}_n$, and the cone

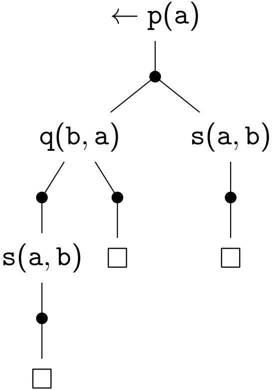
$$\begin{aligned} p_0 &= id : \mathrm{At} \longrightarrow \mathrm{At}(= \mathrm{At}_0) \\ p_{n+1} &= \langle id, P_f P_f(p_n) \circ p \rangle : \mathrm{At} \longrightarrow \mathrm{At} \times P_f P_f \mathrm{At}_n(= \mathrm{At}_{n+1}) \end{aligned}$$

and the limit determines the required coalgebra $\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$.

Semantics, graphically represented

The action of
$\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on
p(a)

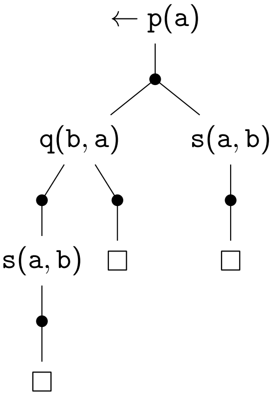
for logic program:




```
q(b,a)  ←  s(a,b)
q(b,a)  ←
s(a,b)  ←
  p(a)  ←  q(b,a),s(a,b)
```

Language design

The action of $\overline{p} : \mathrm{At} \longrightarrow C(P_f P_f)(\mathrm{At})$ on p(a)



We transform this construction verbatim to logic algorithm

- ... corresponds to and-or parallel trees introduced for LP in the 90s
- if we are in the 70s, we “win” 20 years.
- Ready-to-use algorithm for Datalog programs or equivalent finite-model LP fragments.
- Non-determinism – if or-nodes are considered as points of non-determinism

Implementation

[2012, with M.Schmidt] - First prototype in Prolog

[2012-2013, with M.Schmidt, J.Heras] - Parallel implementation in Go (language for concurrency inspired by Hoare logic).
Very nice results in terms of the speed-up.

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Very nice results in terms of the speed-up.

[2013 - , with J.Heras, V.Komendantsky] – Parallel Haskell implementation. We do not have thorough evaluation yet...

Outline

Lawvere theories and the first-order signature Σ

A *signature* Σ consists of a set of *function symbols* $f, g, \ldots$ each equipped with a fixed *arity*. The arity of a function symbol is a natural number indicating the number of its arguments. Nullary (0-ary) function symbols are allowed: these are called *constants*.

Given a signature Σ, construct the Lawvere theory $\mathcal{L}_\Sigma$:

- Define the set $\mathrm{ob}(\mathcal{L}_\Sigma)$ to be the set of natural numbers.
- For each natural number n, let $x_1, \ldots, x_n$ be a specified list of distinct variables.
- Define $\mathrm{ob}(\mathcal{L}_\Sigma)(n, m)$ to be the set of m-tuples $(t_1, \ldots, t_m)$ of terms generated by the function symbols in Σ and variables $x_1, \ldots, x_n$.
- Define composition in $\mathcal{L}_\Sigma$ by substitution.

Example of Lawvere theory generated by a LP

Example

The constants 0 and nil are modelled by maps from 0 to 1 in $\mathcal{L}_\Sigma$, s is modelled by a map from 1 to 1, and cons is modelled by a map from 2 to 1. The term s(0) is therefore modelled by the map from 0 to 1 given by the composite of the maps modelling s and 0; similarly for the term s(nil), although the latter does not make semantic sense.

$$
\begin{array}{rcl}
\texttt{nat(0)} & \leftarrow & \\
\texttt{nat(s(x))} & \leftarrow & \texttt{nat(x)} \\
\texttt{list(nil)} & \leftarrow & \\
\texttt{list(cons x y)} & \leftarrow & \texttt{nat(x), list(y)}
\end{array}
$$

Fibrations modeling the set At

Intuition is to replace At with the functor $At : \mathcal{L}_{\Sigma}^{op} \to Set$ that sends a natural number n to the set of all atomic formulae generated by Σ, set of predicates of the given program and n distinct variables.
Some modifications are needed:

- we need to extend *Set* to *Poset*,
- natural transformations to *lax natural transformations*, and
- replace the outer instance of P_f by P_c - the countable powerset functor (as recursion generates countability).

Fibrations modeling the set *At*

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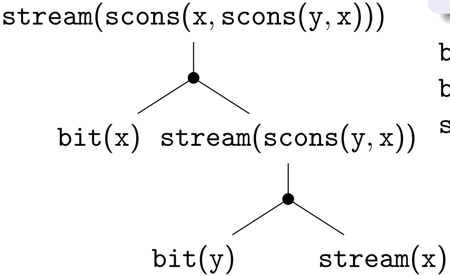
- we need to extend Set to $Poset$,
- natural transformations to *lax natural transformations*, and
- replace the outer instance of P_f by P_c - the countable powerset functor (as recursion generates countability).

Then $p : At \longrightarrow P_c P_f At$ gives a $Lax(\mathcal{L}_{\Sigma}^{op}, P_c P_f)$-coalgebra structure on At.

But cf. Bonchi&Zanasi CALCO'13 paper on getting rid of laxness.

The semantics, graphically:

$A(x, y) \in At(2)$



for a program:

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
          bit(x), stream(y)
```

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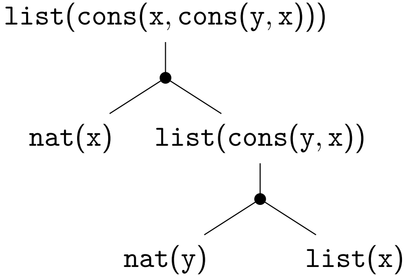

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nat(0) ←
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                 list(y)
```

Language design

- Again, we take the construction of the trees "almost" verbatim in the language design;
- We call the trees arising from the logic algorithm – *coinductive trees*
- The effect of fibrations modelled by term-matching (rather than unification) used in derivations.
- Note the finite tree for `Stream`!!! – looks like lazy evaluation!!!
- As before, we give a parallel implementation for computations of every node.
- Note also because of the "laziness" , a single coinductive tree may not give entire derivation.
- We had a "bad" case of "typing", but the coinductive trees had no unsound substitutions.
- Guardedness ...

Guarding corecursion

(Co)-Recursion

... needs to be guarded against non-termination. Both in FP and LP, such guards can be given semantically or syntactically (later is often known as "guardeness-by-constructors").

`Stream` is guarded by constructors and has finite coinductive trees:

Example

bit(0) $\leftarrow$

bit(1) $\leftarrow$

stream(scons x y) $\leftarrow$, bit(x), stream(y)

Guarding corecursion

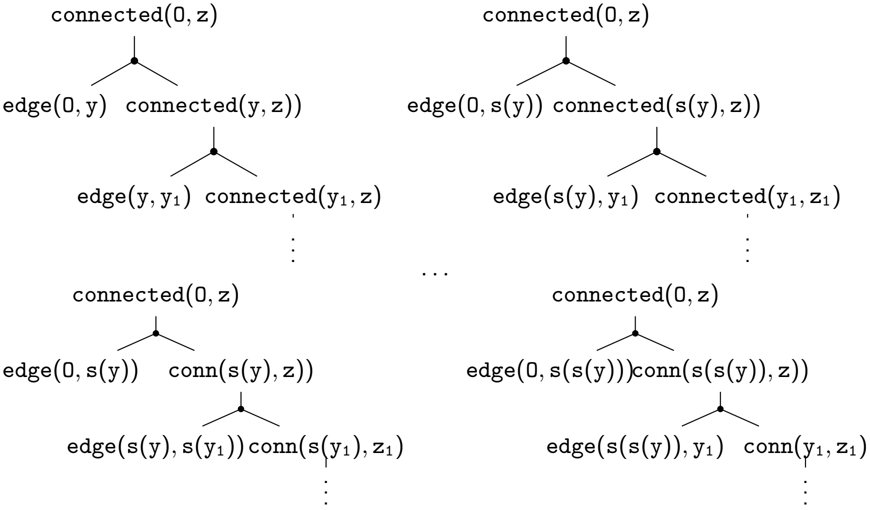
Example

This program (graph connectivity) is not guarded-by-constructors:

1. $\texttt{connected(x,x)} \;\leftarrow$
2. $\texttt{connected(x,y)} \;\leftarrow\; \texttt{edge(x,z)},\; \texttt{connected(z,y)}.$

... and it will produce infinite coinductive trees.

Infinite forests of infinite trees (infinite-breadth and infinite-depth trees):



Guarding corecursion, by constructors:

Example

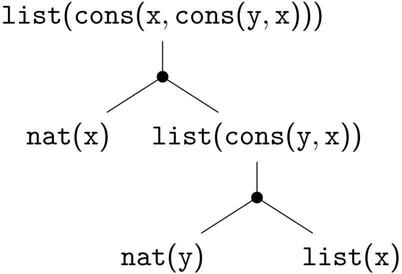
$$\begin{aligned}
\texttt{connected}(X, cons(Node, Path)) &\leftarrow \texttt{edge}(X, Node), \texttt{connected}(Node, Path)\\
\texttt{connected}(X, nil) &\leftarrow\\
\texttt{edge}(0, 0) &\leftarrow\\
\texttt{edge}(X, s(X)) &\leftarrow
\end{aligned}$$

- Cf. Coq/Agda guarding (co)-recursion by constructors;
- There is a bit more to it than that (more conditions needed to keep variable substitution under control in absence of types.)

Maps between fibers, graphically:

$A(x, y) \in At(2)$

Then apply At to the map $(s, s) : 1 \to 2$ in $\mathcal{L}_\Sigma$



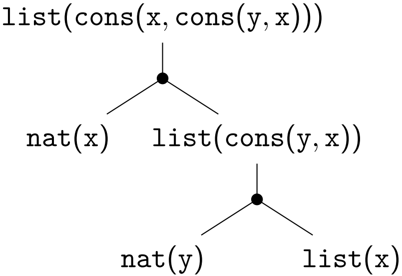
Maps between fibers, graphically:

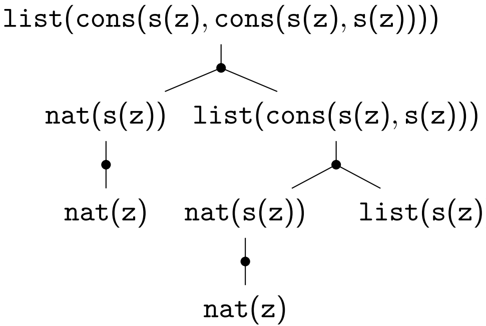
$A(x, y) \in At(2)$

Then apply At to the map $(s, s) : 1 \to 2$ in $\mathcal{L}_\Sigma$

$A(z) \in At(1)$

$At((s, s))(A(x, y))$ is an element of $P_c P_f At(1)$.

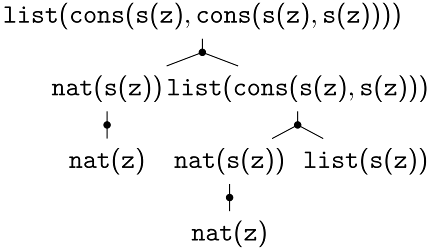




Maps between fibers, graphically:

$A(z) \in At(1)$

Then apply At to the map $O : 0 \to 1$ in $\mathcal{L}_\Sigma$.



Maps between fibers, graphically:

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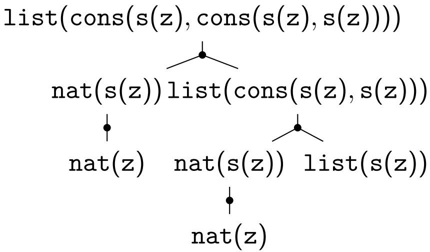
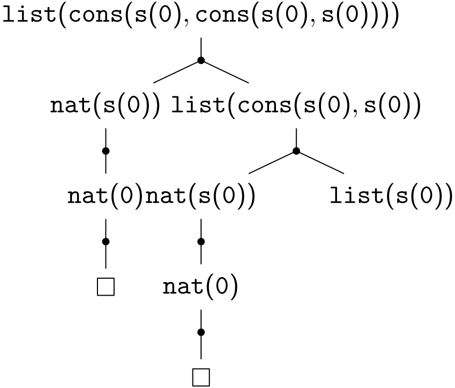
$pAt(0)At((s,s))(A(x,y))$





Language design

- We take the above fiber transitions "almost" verbatim;
- Compute only some maps, not all maps to do derivations;
- Note the gracious way variable dependencies have been handled in "bad typing" case;
- Use the old MGU algorithm (on tree leaves and clauses) to compute substitutions;
- These can be computed in parallel or non-deterministic way...
- Lazy corecursion in full power: potentially infinite transitions between finitely computable coindutive trees;
- Coinductive trees = finite observations;
- Note also the variable length of the size of the finite observations: (i-productivity, but with dynamic i?)

An Example

$$\mathtt{stream(x)} \xrightarrow{\theta_1}$$

An Example

$\texttt{stream(x)} \xrightarrow{\theta_1}$

$\texttt{stream(scons(z, y))} \xrightarrow{\theta_2} \ldots \xrightarrow{\theta_3}$

$\texttt{bit(z)} \qquad \texttt{stream(y)}$

An Example

$\texttt{stream(x)} \xrightarrow{\theta_1}$ $\texttt{stream(scons(z, y))} \xrightarrow{\theta_2} \ldots \xrightarrow{\theta_3}$

$\texttt{bit(z)}$ $\texttt{stream(y)}$

Note that transitions θ may be determined in a number of ways:

- using mgus;
- non-deterministically;
- randomly;
- in a distributed/parallel manner.

An Example

$\mathtt{stream(x)} \xrightarrow{\theta_1}$

$\mathtt{stream(scons(z, y))} \xrightarrow{\theta_2} \ldots \xrightarrow{\theta_3}$

$\mathtt{bit(z)} \quad \mathtt{stream(y)}$

$\mathtt{stream(scons(0, scons(y_1, z_1)))}$

$\mathtt{bit(0)} \quad \mathtt{stream(scons(y_1, z_1))}$

$\square \quad \mathtt{bit(y_1)} \quad \mathtt{stream(z_1)}$

Answers for x: $cons(z, y)$ and $cons(0, cons(y_1, z_1))$.

Guarding corecursive derivations, lazily:

Example

$$\begin{aligned}
\texttt{connected}(X, cons(Node, Path)) &\leftarrow \texttt{edge}(X, Node), \texttt{connected}(Node, Path)\\
\texttt{connected}(X, nil) &\leftarrow\\
\texttt{edge}(0, 0) &\leftarrow\\
\texttt{edge}(X, s(X)) &\leftarrow
\end{aligned}$$

Guarding corecursive derivations, lazily:

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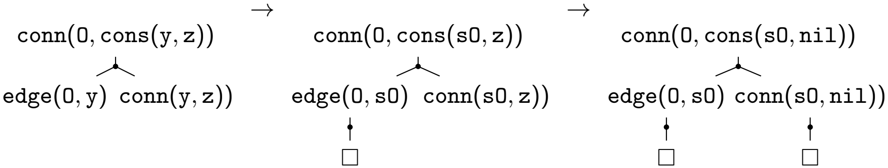

Guardedness: More discipline?

Adapting this sort of programming discipline from lazy functional languages to LP may have its advantages. E.g., it will equally guard against programs that induce infinite SLD-derivations:

Example

1. `connected(x,y)` $\leftarrow$ `connected(z,y), edge(x,z)`
2. `connected(x,x)` $\leftarrow$

While currently, it is up to a programmer to manually weed-out such cases.

Outline

Back to 2013...

... we find out that:

- Parallel LP has been flourishing around 90s, with general-case parallelism still being a problem (took 20 years);
- Coinductive LP was suggested in 2007, with limited data structures (took 30 years)
- LP was adapted in Hindley-Milner Type inference algorithm (70s), but the rest of algorithmic development of type-inference was chaotic, on a "hack-by-need" basis... 2010s showing a need for coinductive and parallel inference algorithms...

Co-LP [Gupta, Simon et al., 2007

Use normal SLD-resolution but add a new rule:

If a formula repeatedly appears as a resolvent (modulo α-conversion), then conclude the proof.

Example

```
bit(0) ←
bit(1) ←
stream(scons x y) ←
            bit(x), stream(y)
```

$$\leftarrow \mathtt{stream(scons(x,y))}$$
$$|$$
$$\leftarrow \mathtt{bit(x), stream(y)}$$
$$|$$
$$\leftarrow \color{red}{\mathtt{stream(y)}}$$
$$|$$
$$\leftarrow \mathtt{bit(x_1), stream(y_1)}$$
$$|$$
$$\leftarrow \color{red}{\mathtt{stream(y_1)}}$$
$$|$$
$$\square$$

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Example

$\texttt{bit(0)} \leftarrow$

$\texttt{bit(1)} \leftarrow$

$\texttt{stream(scons x y)} \leftarrow$

$\qquad\qquad \texttt{bit(x), stream(y)}$

The answer is: $x/0$, $y/cons(x_1, y_1)$.

$$\leftarrow \mathtt{stream(scons(x, y))}$$
$$|$$
$$\leftarrow \mathtt{bit(x), stream(y)}$$
$$|$$
$$\leftarrow \mathtt{stream(y)}$$
$$|$$
$$\leftarrow \mathtt{bit(x_1), stream(y_1)}$$
$$|$$
$$\leftarrow \mathtt{stream(y_1)}$$
$$|$$
$$\square$$

Explicitly-treated corecursion

To know whether to allow (co-LP) or disallow (standard LP) infinite loops, explicit annotation is needed.

Example

$$\begin{array}{rcl}
\mathtt{bit}^i(0) & \leftarrow & \\
\mathtt{bit}^i(1) & \leftarrow & \\
\mathtt{stream}^c(\mathtt{scons}(x,y)) & \leftarrow & \mathtt{bit}^i(x), \mathtt{stream}^c(y) \\
\mathtt{list}^i(\mathtt{nil}) & \leftarrow & \\
\mathtt{list}^i(\mathtt{cons}(x,y)) & \leftarrow & \mathtt{bit}^i(x), \mathtt{list}^i(y)
\end{array}$$

Drawbacks:

- some predicates may behave inductively or coinductively depending on the arguments provided, and such cases need to be resolved dynamically, and not statically; in which case mere predicate annotation fails.
- ... cannot mix induction and coinduction. — All clauses need to be marked as inductive or coinductive in advance.
- Can deal only with restricted sort of structures — the ones having finite regular pattern.

Example

`0::   1::   0::   1::   0::   ...` may be captured by such programs.
π represented as a stream may not.

- the derivation itself is not really a corecursive process.

CoALP vs Co-LP

Advantages

- Works uniformly for both inductive and coinductive definitions, without having to classify the two into disjoint sets;
- in spirit of corecursion, derivations may feature an infinite number of finite structures.
- there does not have to be regularity or repeating patterns in derivations.

Corecursion guarding parallelism:

Corecursion FREEING! parallelism:

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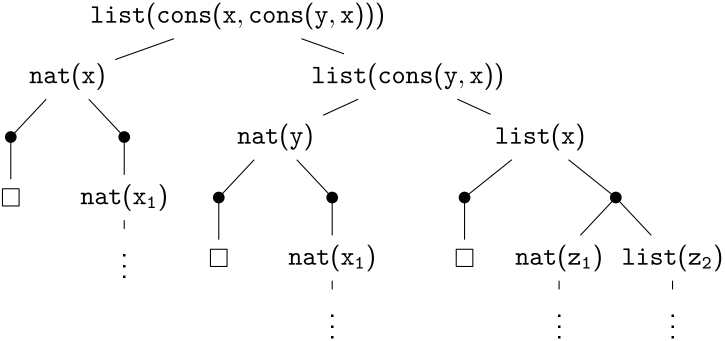
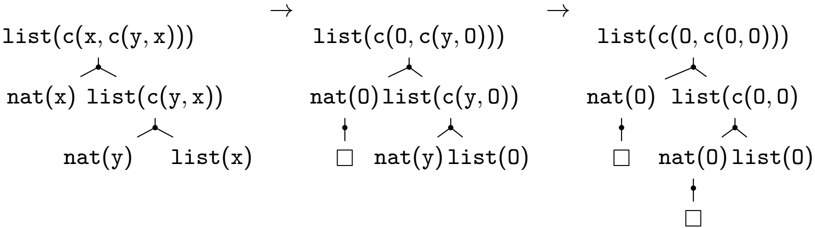
Unification and SLD-resolution are P-complete algorithms. Parallel LP community has to be very inventive in the ways to trick it. In particular, variable synchronization is a huge sequential barrier:

Corecursion FREEING! parallelism:

Unification and SLD-resolution are P-complete algorithms. Parallel LP community has to be very inventive in the ways to trick it. In particular, variable synchronization is a huge sequential barrier:



Now, by the same lazy corecursive derivation:



So, the same guarded corecursive algorithm does the work for free.

Outline

Milner, 1978

"A theory of Type Polymorphism in Programming"

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"A theory of Type Polymorphism in Programming"
An elegant match between polymorphic λ-calculus and type inference by means of Robinson's unification/resolution algorithm.
One made another possible...
Principal type exists, and the Robinson's algorithm is [necessary and] sufficient to compute it.

Constraints and LP in Type inference

- Hindley-Milner Type inference [Milner78, Damas&Milner82] (used in ML, OCAML, Haskel, and some other languages) was based on first-order unification, and simultaneous generation and solving of constraints.

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- In fact, there have been publications on type inference in between, e.g. [Remy & Potier], but not in the direction of LP.

Trend in type inference:

improvement in **expressiveness** of the underlying type system, e.g., in terms of

- *Dependent Types*,
- *Type Classes* [Wadler&Blott89],
- *Generalised Algebraic Types* (GADTs) [Jones&al,06]
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Milner-style decidable type inference does not always suffice (e.g. the *principal type* may no longer exist), and TI requires computation additional to compile-time. Implementations of new type inference algorithms include a variety of first-order decision procedures, notably Unification and Logic Programming (LP) [Jones&al,06], Constraint LP [Odersky,Sulzmann,Schrijvers,Vytiniotis, 1999-2011], LP embedded into interactive tactics (Coq's *eauto*) [Sozeau&Oury,08], and LP supplemented by rewriting [Gonthier&al,11].

What can CoALP do for type inference?

- Practical aspect: hopefully, CoALP's parallelism or corecursion (or some specific combination of the above) will be of some use for new type inference trends;
- Aesthetic: perhaps it is time to bring some harmony into the question of relationship between a type system and the underlying TI algorithm.
  Can we uniformly classify programming languages in terms of extensions of the Hindley-Milner inference algorithm? What impact does it have on operational semantics?

We have minds/hands and EPSRC money to pursue that, so if you see this useful in *your* language, we will be happy to try that.