Machine Learning for the Working Logician (Computer Scientist)

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AI meets Formal Software Development

July 2012

Outline

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1. Motivation
2. First experiments

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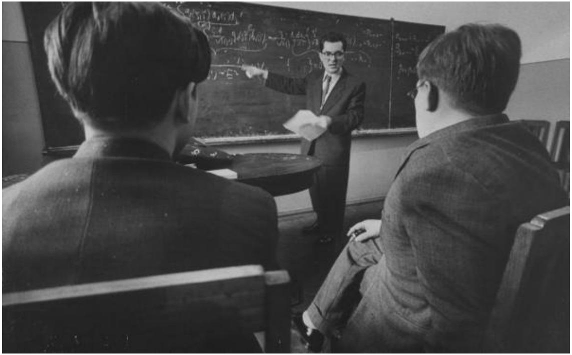

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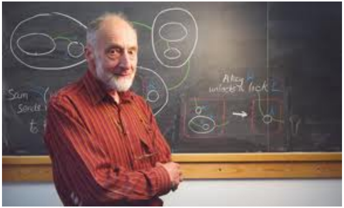

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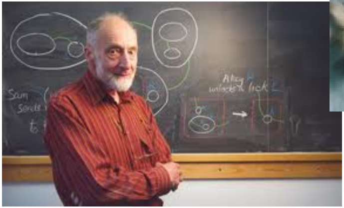

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- ... electronic libraries may be data-mined (Learn2Reason);
- ... proof-search as process, in routine cases, can be statistically analysed (e.g. H.Duncan, Schulz et al.).
- Manual handling of various proofs, strategies, libraries, becomes difficult.

Main applications in Automated Theorem Proving:

Where can we use ML?

ML in other areas of (Computer) Science:

Where data is abundant, and needs quick automated classification:

- robotics (from space rovers to small apps in domestic apploences, cars...);
- image processing;
- natural language processing;
- web search;
- computer network analysis;
- Medical diagnostics;
- etc, etc, ...

In all these areas, ML is a common tool-of-the-trade of the Computer Scientists, additional to their primary research specialisation.

Will this practice come to Automated theorem proving?

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...where AR does not need help

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... where we do not trust them

- new theoretical break-throughs (formulation of new theorems);
- giving semantics to data (cf. Deep learning).

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... will likely to be answered by the community of developers/practitioners, in the long run...

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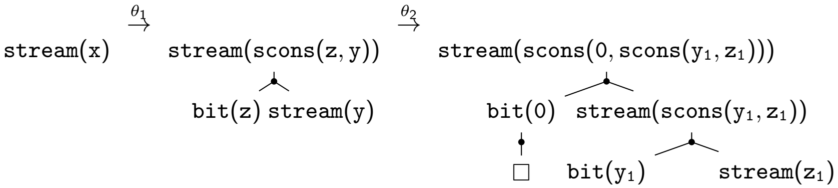
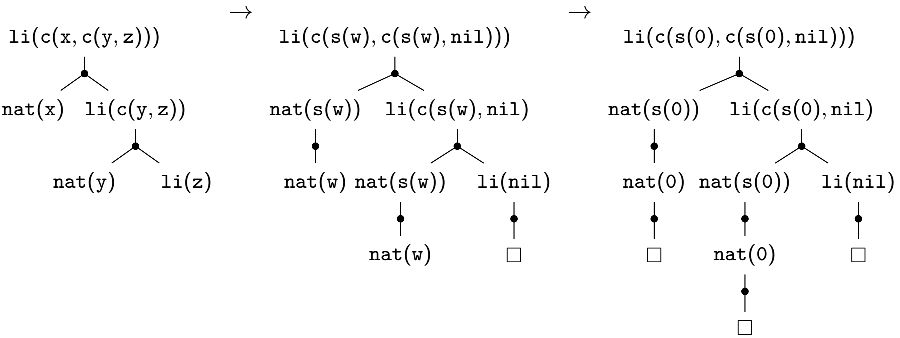
Data: coinductive proof-trees



Coinductive LP:

$$\begin{aligned}
\texttt{bit}(0) &\leftarrow \\
\texttt{bit}(1) &\leftarrow \\
\texttt{stream(scons (x,y))} &\leftarrow \texttt{bit(x)}, \texttt{stream(y)}
\end{aligned}$$



$$
\begin{aligned}
\mathtt{nat(0)} &\leftarrow \\
\mathtt{nat}(s(x)) &\leftarrow \mathtt{nat}(x) \\
\mathtt{list(nil)} &\leftarrow \\
\mathtt{list(cons\ (x,y))} &\leftarrow \mathtt{nat(x)}, \mathtt{list(y)}
\end{aligned}
$$

Experiment setting:

1. Big data sets of the proof trees for various programs;
2. An algorithm is used to convert the Proof-trees into feature vectors;
3. Three-layer back-propagation neural networks used for supervised learning.

All these and more details:

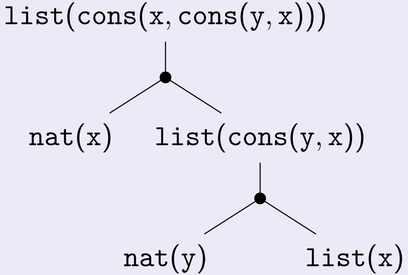
http://www.computing.dundee.ac.uk/staff/katya/MLCAP-man/

Classification task-1: well formed and ill-formed proofs

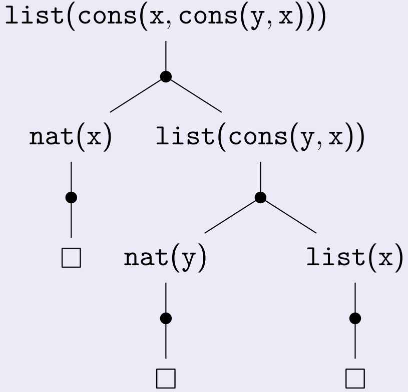
Accuracy of Neural Network recognition:

84.3% for Stream; 76.4% for List.

Well-formed



Ill-formed

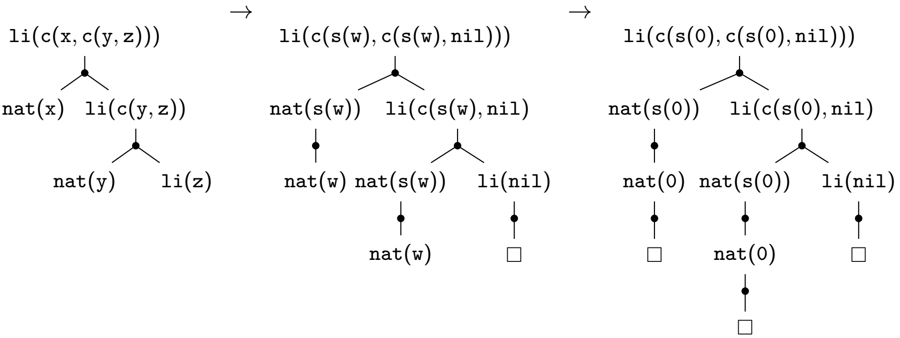


Possible use in Automated Proofs?

Classification task-2: Discovery of Proof-families (among well-formed proofs)

Accuracy of Neural Network recognition:

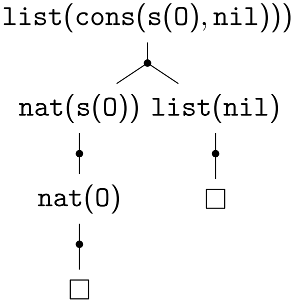
99.1% for Stream; 96.3% for List.

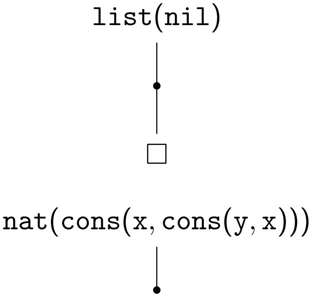


Classification task-2: Discovery of Proof-families

Negative examples

Well-formed trees on the right do not belomg to the proof family generated by the tree on the left.





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- Cutting intermediate and routine proof-steps;
- Proof Speed up;
- Proof un-clattering;
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e.g. `Note: this proof-step belongs to the family of proofs in library N.`

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  - belongs (or not) to a certain desired (previously discovered) proof family;

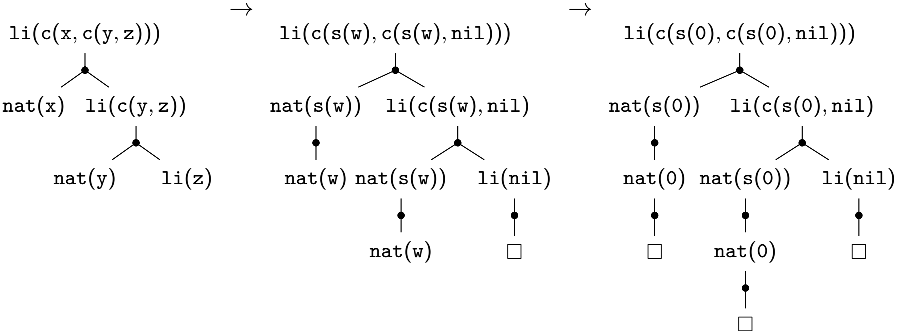
e.g. `Note: this proof-step belongs to the family of proofs in library N.`

  - belongs (or not) to a certain "bad" family discovered before;

Classification task-3: Discovery of Success Proof-families

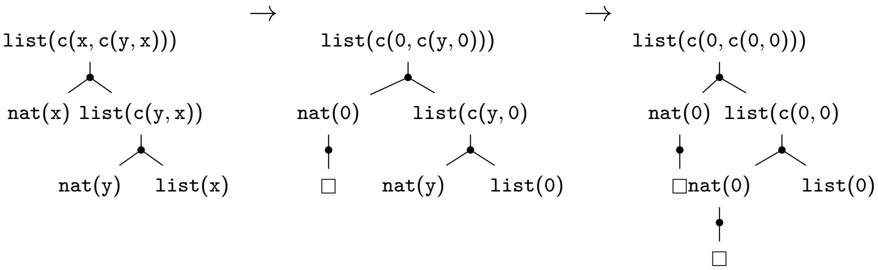
Accuracy of Neural Network recognition:

86% for List.



Classification task-3: Discovery of Success Proof-families

Negative training example:



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This is all about a goal being potentially provable (by the statistical "look" of its proof unfolding). So, could be used for:

- Guiding a proof to success...
- Early warning if the proof step may contain some un-provable content...
- Guarding the proof steps...

Classification task-4: well-typed and ill-typed proofs in a family (among well-formed proofs)

Accuracy of Neural Network recognition:

85.7% for Stream.

$$\mathtt{stream}(\mathtt{x}) \xrightarrow{\theta_1} \mathtt{stream}(\mathtt{scons}(\mathtt{z},\mathtt{y})) \xrightarrow{\theta_2} \mathtt{stream}(\mathtt{scons}(0,\mathtt{scons}(\mathtt{y}_1,\mathtt{z}_1)))$$

$$\mathtt{bit}(\mathtt{z}) \quad \mathtt{stream}(\mathtt{y})$$

$$\mathtt{bit}(0) \quad \mathtt{stream}(\mathtt{scons}(\mathtt{y}_1,\mathtt{z}_1))$$

$$\Box \quad \mathtt{bit}(\mathtt{y}_1) \quad \mathtt{stream}(\mathtt{z}_1)$$

Classification task-4: well-typed and ill-typed proofs in a family

Negative examples:

$$\mathtt{stream}(\mathtt{x}) \xrightarrow{\theta_1} \mathtt{stream}(\mathtt{scons}(\mathtt{y},\mathtt{y})) \xrightarrow{\theta_2} \mathtt{stream}(\mathtt{scons}(\mathtt{scons}(\mathtt{z},\mathtt{z}),\mathtt{scons}(\mathtt{z},\mathtt{z})))$$

$$\mathtt{stream}(\mathtt{scons}(\mathtt{y},\mathtt{y})) \;\to\; \mathtt{bit}(\mathtt{y}) \quad \mathtt{stream}(\mathtt{y})$$

$$\mathtt{stream}(\mathtt{scons}(\mathtt{scons}(\mathtt{z},\mathtt{z}),\mathtt{scons}(\mathtt{z},\mathtt{z}))) \;\to\; \mathtt{bit}(\mathtt{scons}(\mathtt{z},\mathtt{z})) \quad \mathtt{stream}(\mathtt{scons}(\mathtt{z},\mathtt{z}))$$

$$\mathtt{stream}(\mathtt{scons}(\mathtt{z},\mathtt{z})) \;\to\; \mathtt{bit}(\mathtt{z}) \quad \mathtt{stream}(\mathtt{z})$$

$\ldots$

Possible use in Automated Proofs?

... same diagnostic use as with success proof families, but for cases when:

- we do not observe proof leaves: e.g., when ML-tool observes only patches of proofs, not the whole of proof-trees;
- when one works with lazy infinite proofs in coinductive cases;
- when it is on-line step-by-step diagnosis.

Classification task-5: well-typed and ill-typed proofs. (among well-formed proofs)

Accuracy of Neural Network recognition:

82.4% for Stream.

Note: membership in a proof family is not considered.

Possible use in Automated Proofs?

Rating of the classification applications

By accuracy

1. Problem 2.
2. Problem 3.
3. Problem 4.
4. Problem 5.
5. Problem 1.

By usefulness

1. Problem
2. Problem
3. Problem
4. Problem 5?
5. Problem 1.

Summary: applications in Automated Theorem Proving:

Where can we use ML?

1. Early proof dead-end diagnosis;
2. Early success identification; and thus cutting routine cases;
3. Interactive proof-hint generation?
4. Simplification of representation of proofs (by cutting routine/intermediate steps)?
5. Generalisation of proof families to strategies (Gudmund)?

More ideas?

Conclusions

More papers, prototype software, and technical reports on these topics are available on my webpage. Some papers are also uploaded on Dagstuhl meeting Wiki.

You will also find experiments/discussion of various implementation strategies.