Machine Learning for Proof General: Interfacing Interfaces
(Funded by EPSRC First Grant Scheme)

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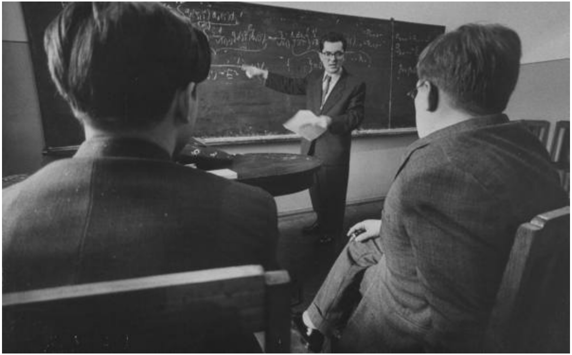

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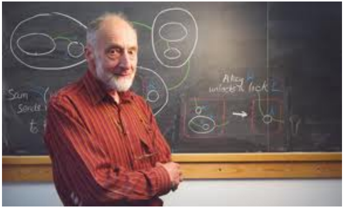

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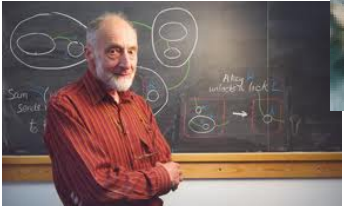

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- ... team-development is hard, especially that ITPs are sensitive to notation;
- ... comparison of proofs and proof similarities across libraries or even within one big library are hard;

Main applications in Automated Theorem Proving:

Where can we use ML?

ML in other areas of (Computer) Science:

Where data is abundant, and needs quick automated classification:

- robotics (from space rovers to small apps in domestic appliences, cars...);
- image processing;
- natural language processing;
- web search;
- computer network analysis;
- Medical diagnostics;
- etc, etc, ...

In all these areas, ML is a common tool-of-the-trade, additional to the primary research specialisation.

Will this practice come to Automated theorem proving?

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...where AR does not need help

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... where we do not trust them

- new theoretical break-throughs (formulation of new theorems);
- giving semantics to data (cf. Deep learning).

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- finding common proof-patterns in proofs across various scripts, libraries, users, notations;
- providing proof-hints, especially in (industrial) cases where routine similar cases are frequent, and proof development is distributed across several programmers.

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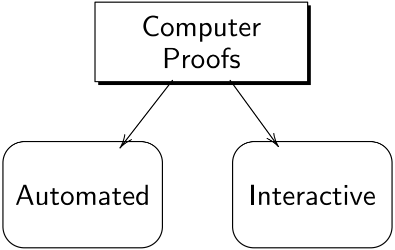

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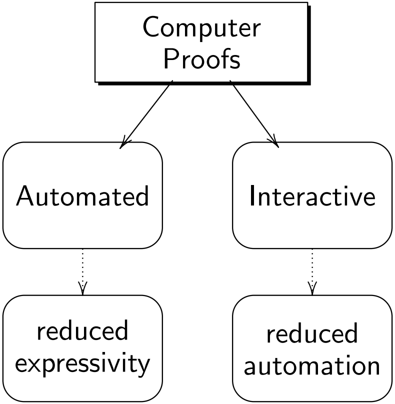

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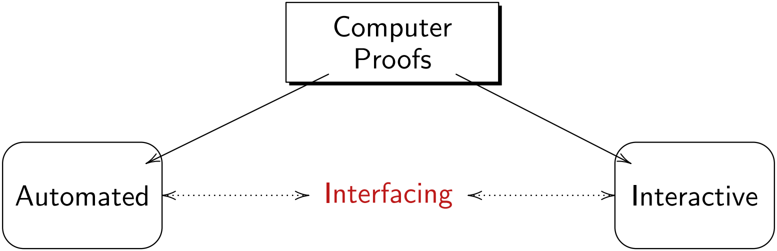

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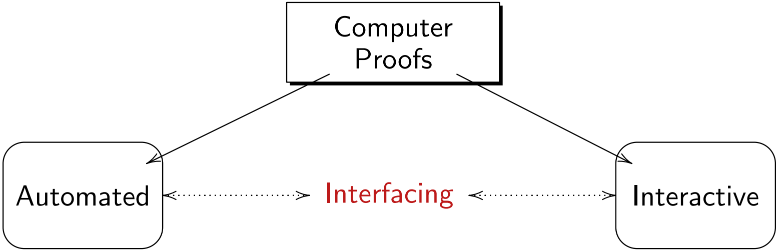
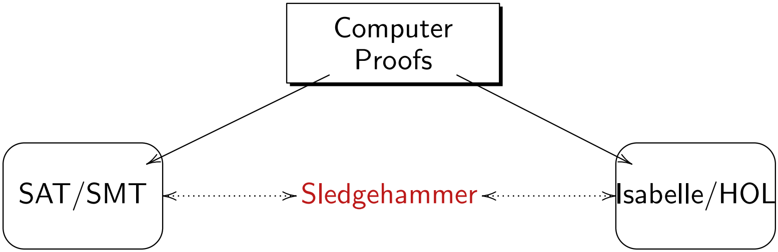


ITP environment allows the user to “call” ATP for generating solutions.

Solution? – Interfacing. Example



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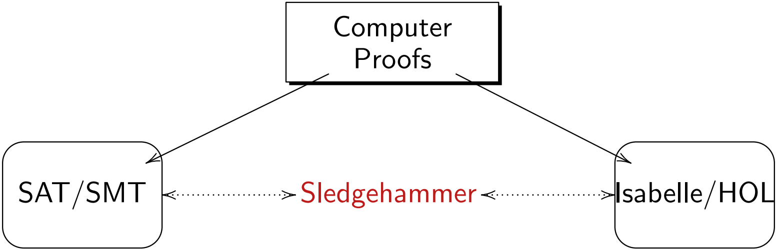
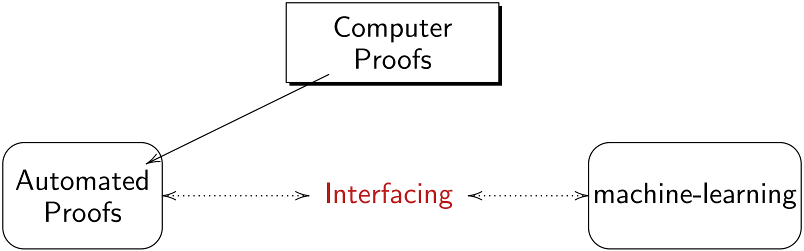


A note: forward interfacing is easier than backwards interfacing.

Less familiar alternative:



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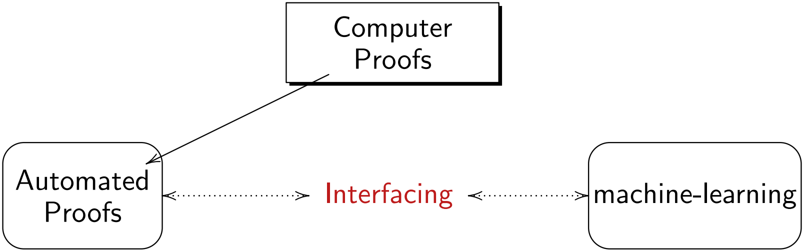

Benefits: learning "proof heuristics", speed up in computations.
Some success: e.g. work by Stephan Shulz, Joseph Urban.

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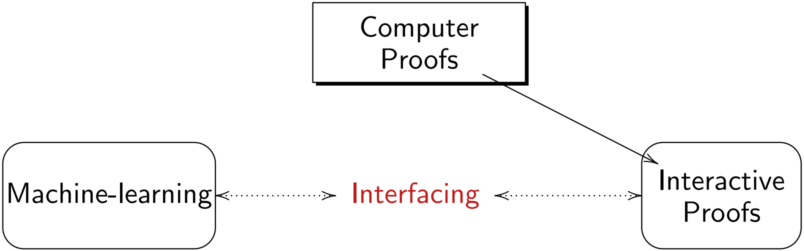

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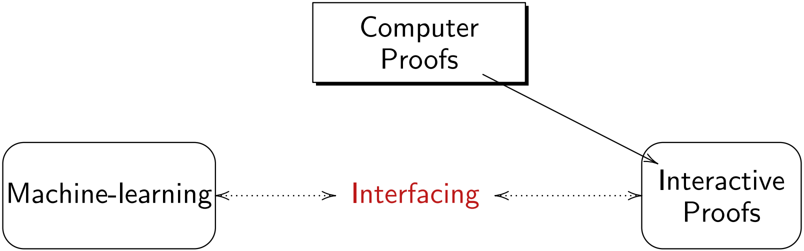

Benefits: helping users to handle big proof developments and libraries. Some attempts: Alan Bundy and Hazel Duncan, current AI4FM project (Edinburgh and Newcastle).

Why machine-learning interactive proofs is harder?

- The richer language reduces the chance of finding regularities and proof patterns by data-mining the syntax alone. Moreover, in ITPs, one and the same goal may have a range of different proofs, whereas different goals can be proven by the same sequence of tactics.

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Note: – similarly –

– huge role user interfaces play in Machine-learning community: MATLAB, WEKA, – are famous interfaces to run a range of statistical algorithms.

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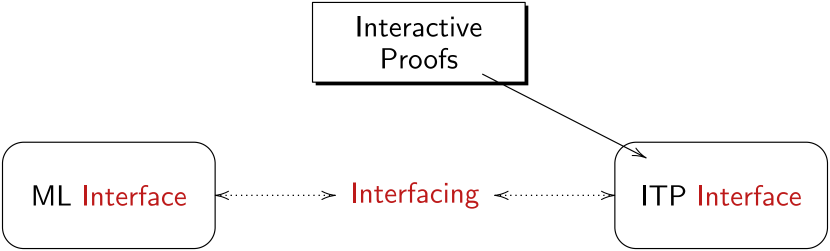

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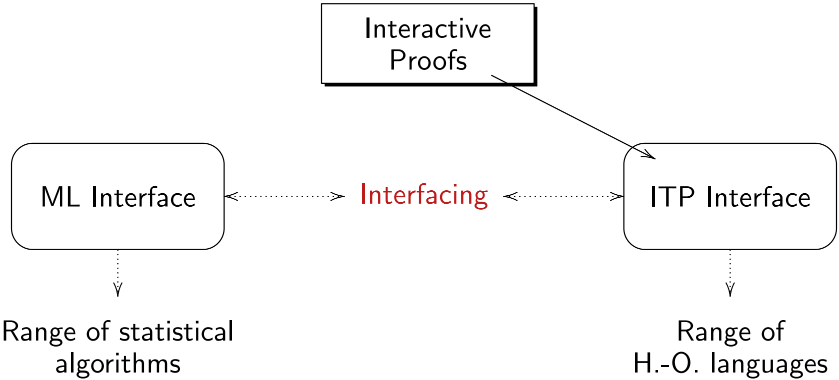

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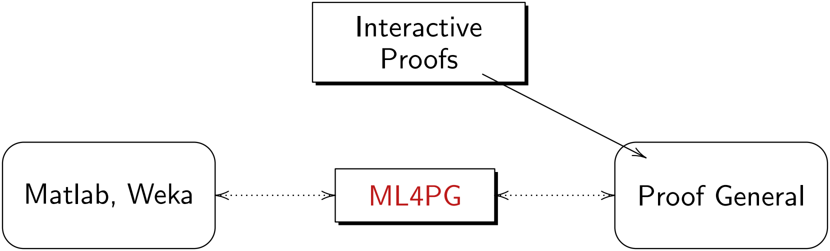

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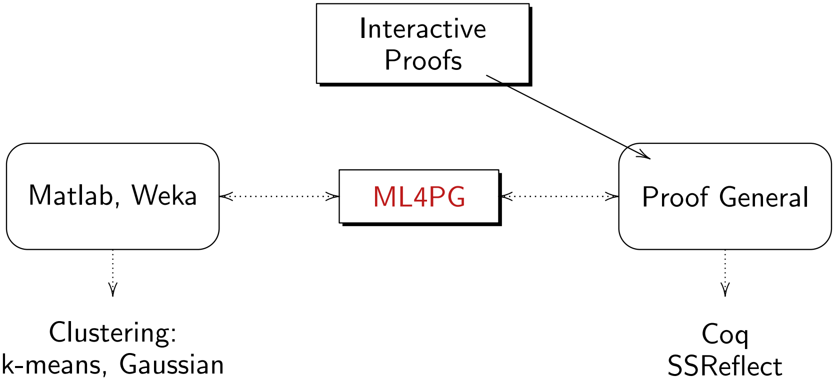

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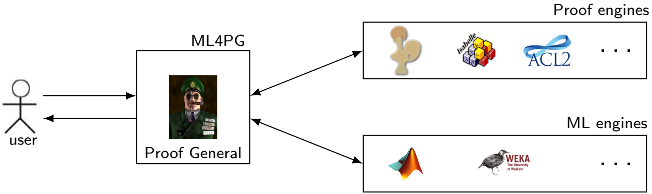
Overall architecture of ML4PG



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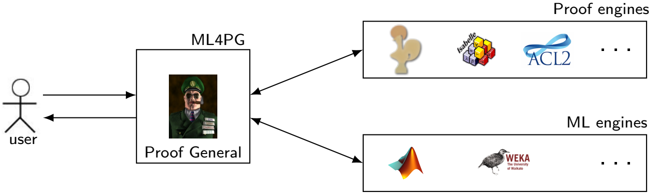

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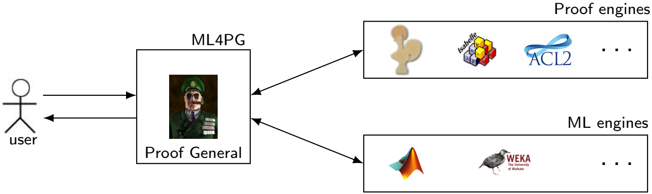

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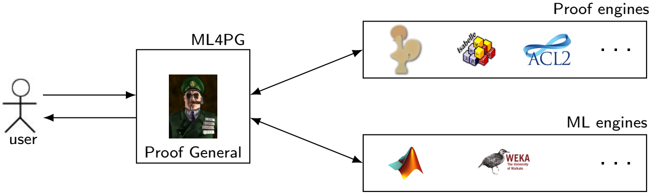

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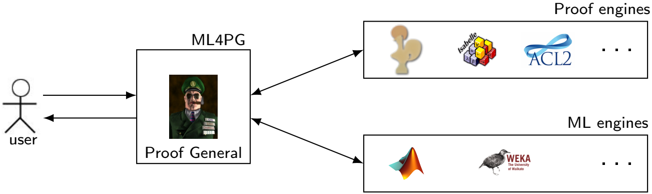

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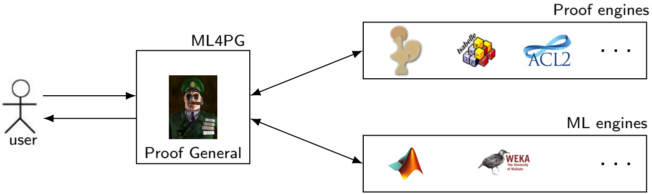

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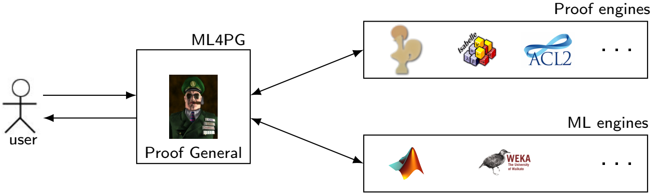

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- ML4PG informs the user of arising proof patterns.

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Problem:

- statistical ML tools expect, as input, a fixed number of features describing all objects to be classified;
- in higher-order proofs, we cannot fix a finite number of goal shapes or proofs configurations to describe all possible proofs;
- we gather statistics based on a fixed number of implicit proof parameters – proof traces.

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- For one proof step, the collection of these parameters is insufficient for meaningful proof-pattern recognition;
- Collection of these features over several proof steps – a proof trace – gives amazing results.

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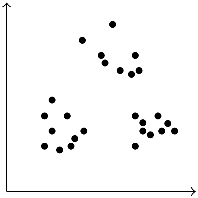

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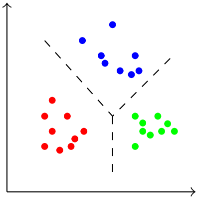

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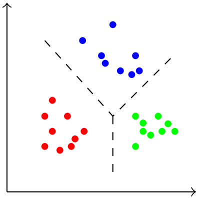

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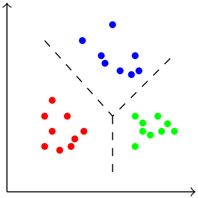

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- Algorithms: K-means, Gaussian Mixture models, simple Expectation Maximisation, ...

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  This means the ML4PG user does not have to analyse the statistics manually!!!

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Demo...

Demo: ML4PG options and various clusters

$$\sum_{1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

$$\sum_{1}^{n} i^3 = \frac{n^4+2n^3+n^2}{4},$$

$$\sum_{1}^{n}(2i-1) = n^2,$$

$$\sum_{1}^{n}(2i-1)^2 = \frac{4n^3-n}{3},$$

$$\sum_{1}^{n}(2i-1)^3 = 2n^4-n^2.$$

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- Applications:
  - Definition of matrix multiplication
  - Binomials
  - Union of sets
  - ...

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Lemma

Let M be a nilpotent matrix, then

$$(1 - M) \times \sum_{0 \leq i < n} M^i = 1$$

where n is such that $M^n = 0$

```
Lemma inverse_I_minus_M_big (M : 'M_m) : (exists n, M^n = 0) ->
  (1 - M) *m (\sum_(0<=i<n) M^i) = 1.
```

Suggestions provided by ML4PG

Theorem (Fundamental Lemma of Persistent Homology)

$\beta_i^{j,k} : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$

$$\beta_n^{k,l} - \beta_n^{k,m} = \sum_{1 \leq i \leq k} \sum_{l < j \leq m} (\beta_n^{j,p-1} - \beta_n^{j,p}) - (\beta_n^{j-1,p-1} - \beta_n^{j-1,p})$$

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Proof

$$\sum_{1 \leq i \leq k} \sum\nolimits_{l < j \leq m} (\beta_n^{j,i-1} - \beta_n^{j,i}) - (\beta_n^{j-1,i-1} - \beta_n^{j-1,i}) \quad =$$

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Proof

$$\begin{aligned}
\sum_{1 \leq i \leq k} \sum_{l < j \leq m} (\beta_n^{j,i-1} - \beta_n^{j,i}) - (\beta_n^{j-1,i-1} - \beta_n^{j-1,i}) \quad &= \\
\sum_{1 \leq i \leq k} ((\beta_n^{l+1,i-1} - \beta_n^{l+1,i}) - (\beta_n^{l,i-1} - \beta_n^{l,i}) + \\
(\beta_n^{l+2,i-1} - \beta_n^{l+2,i}) - (\beta_n^{l+1,i-1} - \beta_n^{l+1,i}) + \\
\dots \\
(\beta_n^{m-1,i-1} - \beta_n^{m-1,i}) - (\beta_n^{m-2,i-1} - \beta_n^{m-2,i}) + \\
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Proof

$$\begin{aligned}
\sum_{1 \leq i \leq k} \sum_{l < j \leq m} (\beta_n^{j,i-1} - \beta_n^{j,i}) - (\beta_n^{j-1,i-1} - \beta_n^{j-1,i}) \qquad &= \\
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(\beta_n^{l+2,i-1} - \beta_n^{l+2,i}) - \cancel{(\beta_n^{l+1,i-1} - \beta_n^{l+1,i})} + &\\
\ldots &\\
(\beta_n^{m-1,i-1} - \beta_n^{m-1,i}) - (\beta_n^{m-2,i-1} - \beta_n^{m-2,i}) + &\\
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Proof

$$\begin{array}{c} \displaystyle\sum_{1\leq i\leq k} \sum_{l<j\leq m} (\beta_n^{j,i-1} - \beta_n^{j,i}) - (\beta_n^{j-1,i-1} - \beta_n^{j-1,i}) \qquad = \\ \displaystyle\sum_{1\leq i\leq k} ((\cancel{\beta_n^{l+1,i-1} - \beta_n^{l+1,i}}) - (\beta_n^{l,i-1} - \beta_n^{l,i}) + \\ (\cancel{\beta_n^{l+2,i-1} - \beta_n^{l+2,i}}) - (\cancel{\beta_n^{l+1,i-1} - \beta_n^{l+1,i}}) + \\ \cdots \\ (\cancel{\beta_n^{m-1,i-1} - \beta_n^{m-1,i}}) - (\cancel{\beta_n^{m-2,i-1} - \beta_n^{m-2,i}}) + \\ (\beta_n^{m,i-1} - \beta_n^{m,i}) - (\cancel{\beta_n^{m-1,i-1} - \beta_n^{m-1,i}})) \end{array}$$

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Proof

$$\sum_{1 \leq i \leq k} \sum_{l < j \leq m} (\beta_n^{j,i-1} - \beta_n^{j,i}) - (\beta_n^{j-1,i-1} - \beta_n^{j-1,i}) = \sum_{1 \leq i \leq k} (\beta_n^{m,i-1} - \beta_n^{m,i}) - (\beta_n^{l,i-1} - \beta_n^{l,i}) = \ldots$$

Suggestions provided by ML4PG

Lemma

If $g : \mathbb{N} \to \mathbb{Z}$, then

$$\sum_{0 \leq i \leq k} (g(i+1) - g(i)) = g(k+1) - g(0)$$

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Lemma

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Suggestions provided by ML4PG

Lemma

Let M be a nilpotent matrix, then

$$(1 - M) \times \sum_{0 \leq i < n} M^i = 1$$

where n is such that $M^n = 0$

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$$(1 - M) \times \sum_{0 \leq i < n} M^i \quad = \quad \sum_{0 \leq i < n} M^i - M^{i+1}$$

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$$\begin{aligned} (1 - M) \times \sum_{0 \leq i < n} M^i &= \\ \sum_{0 \leq i < n} M^i - M^{i+1} &= \\ M^0 - M^1 + M^1 - M^2 + \ldots + M^{n-1} - M^n & \end{aligned}$$

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$$\begin{aligned} (1 - M) \times \sum_{0 \leq i < n} M^i &= \\ \sum_{0 \leq i < n} M^i - M^{i+1} &= \\ M^0 - M^n = M^0 = 1 & \end{aligned}$$

Lemma (Another ML4PG suggestion)

Let M be a nilpotent matrix, then there exists N such that $N \times (1 - M) = 1$

The CoqEAL library

M. Dénès and A. Mörtberg and V. Siles. A refinement-based approach to computational algebra in Coq. In: Proceedings Interactive Theorem Proving 2012 (ITP 2012). Lecture Notes in Computer Science 7406, 83–98. 2012.

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Problem

Decipher the key results which can help us to solve our concrete problems

Fast inverse for triangular matrices

Suppose that we have defined a fast algorithm to compute the inverse of triangular matrices over a field called `fast_invmx`

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Suggestions:

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- Executability of the algorithm

Suggestions:

- Clustering with matrix library of SSReflect and CoqEAL library (˜ 1000)
- 10 suggestions
- Instead of proving:

```
Lemma fast_invmxE : forall m (M : 'M[R]_m), lower1 M ->
      fast_invmx M = invmx M.
```

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Suggestions:

- Clustering with matrix library of SSReflect and CoqEAL library (˜ 1000)
- 10 suggestions
- Prove:

```
Lemma fast_invmxE : forall m (M : 'M[R]_m), lower1 M ->
      M *m fast_invmx M = 1%:M.
```

- Key suggestion:

```
Lemma invmx_is_uniq : forall m (M1 M2 : 'M[R]_m), M1 *m M2 = 1%:M ->
    M2 = invmx M1.
```

Fast inverse for triangular matrices

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Problems:

- Prove the equivalence with the `invmx` algorithm of SSReflect
- Executability of the algorithm

Suggestions:

- CoqEAL suggestion: refine the algorithm to work with sequences instead of matrices
- Clustering with CoqEAL library (˜ 700)
- 7 suggestions all of them related to the refinement from matrices to sequences

Formalisation of the JVM: example suggested by J Moore

Java Virtual Machine (JVM) is a stack-based abstract machine which can execute Java bytecode

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Goal

- Model a subset of the JVM in COQ, defining an interpreter for JVM programs
- Verify the correctness of JVM programs within COQ

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Java Virtual Machine (JVM) is a stack-based abstract machine which can execute Java bytecode

Goal

- Model a subset of the JVM in Coq, defining an interpreter for JVM programs
- Verify the correctness of JVM programs within Coq

This work is inspired by:

H. Liu and J S. Moore. Executable JVM model for analytical reasoning: a study. Journal Science of Computer Programming - Special issue on advances in interpreters, virtual machines and emulators (IVME'03), 57(3):253–274, 2003.

An example: computing 5!

Java code:

```
static int factorial(int n)
{
  int a = 1;
  while (n != 0){
    a = a * n;
    n = n-1;
    }
  return a;
}
```

An example: computing 5!

Bytecode:

0	:	*iconst* 1
1	:	*istore* 1
2	:	*iload* 0
3	:	*ifeq* 13
4	:	*iload* 1
5	:	*iload* 0
6	:	*imul*
7	:	*istore* 1
8	:	*iload* 0
9	:	*iconst* 1
10	:	*isub*
11	:	*istore* 0
12	:	*goto* 2
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JVM model:

counter:

0

stack:

				...

local variables:

5				...

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JVM model:

counter:

1

stack:

1				...

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counter:

2

stack:

				...

local variables:

5	1			...

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JVM model:

counter:

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JVM model:

counter:

4

stack:

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JVM model:

counter:

6

stack:

5	1			...

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JVM model:

counter:

7

stack:

5				...

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counter:

8

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counter:

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counter:

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stack:

1	5			...

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counter:

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stack:

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10	:	*isub*
11	:	*istore* 0
12	:	*goto* 2
13	:	*iload* 1
14	:	*ireturn*

JVM model:

counter:

12

stack:

				...

local variables:

4	5			...

An example: computing 5!

Bytecode:

0	:	*iconst* 1
1	:	*istore* 1
2	:	*iload* 0
3	:	*ifeq* 13
4	:	*iload* 1
5	:	*iload* 0
6	:	*imul*
7	:	*istore* 1
8	:	*iload* 0
9	:	*iconst* 1
10	:	*isub*
11	:	*istore* 0
12	:	*goto* 2
13	:	*iload* 1
14	:	*ireturn*

JVM model:

counter:

2

stack:

				...

local variables:

4	5			...

An example: computing 5!

Bytecode:

. . .

JVM model:

. . .

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12	:	*goto* 2
13	:	*iload* 1
14	:	*ireturn*

JVM model:

counter:

13

stack:

0				...

local variables:

0	120			...

An example: computing 5!

Bytecode:

0	:	*iconst* 1
1	:	*istore* 1
2	:	*iload* 0
3	:	*ifeq* 13
4	:	*iload* 1
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14	:	*ireturn*

JVM model:

counter:

14

stack:

120				...

local variables:

0	120			...

An example: computing 5!

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0	:	*iconst* 1
1	:	*istore* 1
2	:	*iload* 0
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JVM model:

counter:

15

stack:

120				...

local variables:

0	120			...

Formalisation of Java bytecode in Coq

Goal (Factorial case)

$\forall n \in \mathbb{N}$, running the bytecode associated with the factorial program with n as input produces a state which contains $n!$ on top of the stack

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Methodology:

1. Write the specification of the function

```
Definition theta_fact (n : nat) := n`!.
```

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1. Write the specification of the function
2. Write the algorithm (tail recursive function)

```
Fixpoint helper_fact (n a : nat) :=
match n with
| 0 => a
| S p => helper_fact p (n * a)
end.

Definition fn_fact (n : nat) :=
    helper_fact n 1.
```

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1. Write the specification of the function
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3. Prove that the algorithm satisfies the specification

```
Lemma fn_fact_is_theta n : fn_fact n =
    theta_fact n.
```

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1. Write the specification of the function
2. Write the algorithm (tail recursive function)
3. Prove that the algorithm satisfies the specification
4. Write the JVM program

```
Definition pi_fact :=
  [::(ICONST,1%Z);
     (ISTORE,1%Z);
     (ILOAD,0%Z);
     (IFEQ,10%Z);
     (ILOAD,1%Z);
     (ILOAD,0%Z);
     (IMUL, 0%Z);
     (ISTORE, 1%Z);
     (ILOAD, 0%Z);
     (ICONST, 1%Z);
     (ISUB, 0%Z);
     (ISTORE, 0%Z);
     (GOTO, (-10)%Z);
     (ILOAD, 1%Z);
     (HALT, 0%Z)].
```

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Methodology:

1. Write the specification of the function
2. Write the algorithm (tail recursive function)
3. Prove that the algorithm satisfies the specification
4. Write the JVM program
5. Define the function that schedules the program

```
Fixpoint loop_sched_fact (n : nat) :=
match n with
| 0 => nseq 3 0
| S p => nseq 11 0 ++ loop_sched_fact p
end.


Definition sched_fact (n : nat) :=
   nseq 2 0 ++ loop_sched_fact n.
```

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5. Define the function that schedules the program
6. Prove that the code implements the algorithm

```
Lemma program_is_fn_fact n :
  run (sched_fact n) (make_state 0 [::n]
       [::] pi_fact) =
  (make_state 14 [::0;fn_fact n ] (push
      (fn_fact n ) [::]) pi_fact).
```

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7. Prove total correctness

```
Theorem total_correctness_fact n sf :
  sf = run (sched_fact n) (make_state 0
       [::n] [::] pi_fact) ->
  next_inst sf = (HALT,0%Z) /\
  top (stack sf) = (n`!).
```

Where is our tool useful?

Methodology:

1. Write the specification of the function
2. Write the algorithm (tail recursive function)
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Suggestions for `fn_fact_is_theta`:

`fn_expt_is_theta`, `fn_mult_is_theta`, `fn_power_is_theta`

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Suggestions for `program_is_fn_fact`:
`program_is_fn_expt`, `program_is_fn_mult`, `program_is_fn_power`

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Table of Contents

Further work

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- not only trace successful proofs, but also failed and discarded derivation steps;
- increase the number of Interactive Theorem Provers and Machine Learning engines;
- replace local environment with a client-server framework.

"Dundee Fellowship" positions

- University of Dundee is about to announce positions of Dundee Fellows;
- 5-year research fellowship position, becoming a permanent lectureship at the end; starts at 8 point scale;
- Computational was selected as one of a few "named" areas;
- competition will be across several school and departments;
- if you know potential winner – please let me know.

Machine Learning for Proof General: Interfacing Interfaces (Funded by EPSRC First Grant Scheme)

Katya Komendantskaya and Jonathan Heras

University of Edinburgh

4 December 2012