Iteration 2-categories and Rational Term Rewriting*

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Rational terms are possibly infinite terms with a finite set of sub-terms. They show up in Theoretical Computer Science whenever some finite cyclic structures (such as data flow diagrams, term graphs, process algebras with recursion) are of concern, and one desires to abstract from the 'degree of folding' of such structures, intuitively identifying those that denote the same infinitary behaviour.

For example, the μ-term $t_1 = \mu_x.ite(B, seq(C_1, x), C_2)$ can be used as a linear representation of a flow-chart modeling the structure of a *while* loop *via* the *if-then-else (ite)* and the *sequentialization (seq)* statements, where the boolean condition B and the statements C_1 and C_2 are left unspecified. The intended meaning of the operator μ_x, when applied to a term $t[x]$ where x occurs free, is of constraining the instantiation of x in t to a copy of $\mu_x.t$, so that μ_x can be considered as a self-instantiation operator. By performing this self-instantiation once in t_1, we get $t_2 = ite(B, seq(C_1, \mu_x.ite(B, seq(C_1, x), C_2)), C_2)$. Now, both t_1 and t_2 are finite representations of the same infinite, rational term, which can be informally written down as $ite(B, seq(C_1, ite(B, seq(C_1, ite(B, seq(C_1, \ldots), C_2)), C_2)), C_2)$. In turn, that term can be regarded as a representative of the equivalence class of μ-terms containing t_1 and t_2.

Infinitary extensions of Term Rewriting have been considered by various authors during the last decade [3–5, 7, 11, 12, 16–18]. Most of those contributions are concerned with the study of the rewriting relation induced by a set of finite term rules on infinite terms, presenting results about the existence of normal forms (possibly reachable after ω steps), confluence and so on. Only a few of them, namely [4, 7, 16, 17], focus on the sub-class of rational terms, regarded essentially as the semantics of some finite but possibly cyclic structures (*term graphs* or *μ-terms*). On the contrary, in our work we consider rational term as the basic structure, and we focus on syntactical and semantical issues concerning the rewriting of their finitary representations.

In [7] we first presented a set-theoretical definition of μ-term rewriting and rational term rewriting (i.e., for the parallel reduction of a possibly infinite, but rational, set of redexes in a rational term) proving that they are sound extensions of the usual definition of rewriting over finite terms. Then, we introduced (one-step) *preiteration* and *rational rewriting logic*, exploiting the formalism proposed

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in [20] for reasoning in logical terms about rewriting. Such logics were presented in the form of sequent calculus, via deduction rules which allow to generate sequents: They are shown to specify sequents which are in one-to-one correspondence with rewrites over μ-term and rational terms, respectively. The added value of this approach is that not only the terms, but also the reductions are now endowed with an algebraic structure, and this allows us to obtain a precise relationship between μ-term and rational term rewriting. In particular, as rational terms are suitable equivalence classes of μ-terms, so rational term rewrites are suitable equivalence classes of μ-term rewrites. Paraphrasing [17], we then infer that 'parallel μ-term rewriting is adequate for rational term rewriting'.

The correspondence between term rewriting systems and 2-categories, where arrows represent terms and cells represent rewriting sequences, was already clarified by the mid-Eighties [21, 22]. Such a correspondence can be extended at the level of the abstract semantics: Given a rewrite system $\mathcal{R}$, the equivalence classes of sequences of rewrites entailed by $\mathcal{R}$, modulo *permutation equivalence* [2], are in a bijective correspondence with cells of a suitable 2-category induced by $\mathcal{R}$. Some results along this line of research can be found in [10, 13, 19, 20, 23]. Recently, it has been shown that the correspondence can be also extended to other data structures, such as *graphs* [14] and *term graphs* [6, 8, 9, 15], simply considering different underlying categories, and lifting their structure at the 2-categorical level. Along this line of research, in this talk we present a *categorical* characterization of μ-term and rational term rewriting. We first extend the correspondence at the logical level, thus considering *sequences* of rewriting steps: As in the case of the 'one-step' variants, we can consider both (full) *preiteration* and *rational* rewriting logic. The resulting logics provide the bridge between the standard presentation of rewriting and the categorical models: Their corresponding categorical presentations are based on *preiteration* and *iteration* 2-categories, respectively [1], and are shown to faithfully implement the logical ones.

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