Realizability Models over Linear Combinatory Algebras and the Full Completeness Problem for Typed λ-calculi

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Linear Combinatory Algebras (LCA), [Abr96, Abr97], refine standard combinatory algebras (CA), in the same way as Linear Logic (LL) refines Intuitionistic Logic (IL).

LCAs arise by introducing a unary operation ! on an applicative structure, by asking for the existence of the *linear* combinators B, C, I, and by requiring only restricted forms of the *weakening* combinator K and the *contraction* combinator W. The full power of standard combinatory algebras, i.e. *functional completeness*, is then recovered by introducing further combinators, reflecting the properties of the exponential connective ! in LL. Given an LCA $\mathcal{A}_l$, one can recover a standard CA $\mathcal{A}_s$, by defining $a \bullet_s b = a \bullet_l !b$, in a way that directly reflects Girard's translation of the intuitionistic $\rightarrow$ into LL.

Using the Curry-Howard isomorphism, one can see that LCAs correspond to a "Hilbert style" axiomatization of the $(\multimap, !)$-fragment of LL, in the same way as standard CAs correspond to the implicational fragment of a "Hilbert style" IL.

Many examples of LCAs arise using a *Geometry of Interaction* construction ([Abr96, Abr96a]), or in the setting of *Game Semantics*.

In this talk, we shall first briefly review the theory of LCA's. Then, we will work out simple examples of realizability models on LCA's. In particular, we will see how to construct a *Partial Equivalence Relation* category

$PER_{\mathcal{A}_l}$ on a generic LCA $\mathcal{A}_l$. The category $PER_{\mathcal{A}_l}$ turns out to be a *linear* category, in the sense of [Bie95], i.e. a SMCC with a comonad $(!, der, \delta)$, plus some extra conditions relating it to the monoidal structure. Linear categories are models of LL. Moreover, the co-Kleisli category of $PER_{\mathcal{A}_l}$ is a CCC, and the two categories together form an *adjoint model* ([BW96]), i.e. the adjunction between the two is *monoidal*.

In the second part of the talk, I will address the problem of *Full Completeness* for various typed λ-calculi, including *polymorphic* systems, and possible uses of PER models.

The notion of Full Completeness is the counterpart, for typed λ-calculi and other calculi of proofs for a given logic, of the notion of *Full Abstraction* for programming languages. Roughly, a model of a typed λ-calculus is complete if all semantical elements which inhabit denotations of a type correspond to proofs.

We will discuss examples and axiomatizations of full complete models for various λ-calculi, e.g. simply typed, with a possible non-empty set of basic constants, ML-type polymormism, system F.

References

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