On the Undecidability of Second-Order Unification

Jordi Levy and Margus Veanes

There is a close relationship between word unification and second-order unification (SOU). This similarity has been exploited, for instance, for proving decidability of monadic SOU, and decidability of linear SOU when no variable occurs more than twice. Trying to prove this second result for (nonlinear) SOU, we found a simple reduction between this problem and the simultaneous rigid E-unification (SREU) problem. This was the starting point for some novel results about the undecidability of SOU. We prove that the number of second-order variables of a problem is not relevant: SOU is undecidable already for one (unary) second-order variable. The number of occurrences per variable can be decisive: SOU is decidable for one occurrence per variable and undecidable for two of them. The total number of variable occurrences is also relevant: SOU is decidable for two occurrences of second-order variables and no first-order variables, and undecidable already for five occurrences. And finally, simplicity, i.e. if no variable occurs below another variable, is not enough for ensuring.

Other questions still remain open. For instance, what is the minimum number of variable occurrences we need to get a class of undecidable problems; or what is the simultaneous upper bound on the number of occurrences and on the arity of variables for ensuring decidability.