The Decidability of Context Unification

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Context unification is defined as the problem of finding a second-order unifier where second-order variables are restricted to be instantiated by linear terms. A linear term is a λ-expression $\lambda x_1 \cdots \lambda x_n.t$ where bounded variables x_i occurs exactly once in the body of the expression t. Contrarily to the second-order unification problem, the context unification problem is conjectured to be decidable. This problem has a close relationship with the word unification problem. We prove that, in fact, context unification can be reduce to word unification with regular constraints (Schulz, 1991) if the following fact holds:

> For any context unification problem $t \stackrel{?}{=} u$ and most general unifier, the rank of $\sigma(t)$ is bounded on $f(d)$, where f is a computable function and d is the size of the problem.

Here, the rank of a tree t (similarly for terms) is greater than n if there exist a complete tree t' with depth n and an injective tree embedding from t' to t. Currently, a preliminary proof of this fact still has to be checked.