Example supplement for “Restoring Natural Language as a Computerised Mathematics Input Method”

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Abstract. Due to a strict page limit for MKM 2007, it was necessary to remove the appendix from the paper [3] and instead provide its contents through an alternative medium. Distributed on the respective web sites of the paper’s authors, the contents are reproduced in this document. The reader will find herein an example in which the method of the above mentioned paper is applied to a typical text. The example is given first in a plain text, then as a version annotated according to the method proposed in this paper.

A Ring theory example

In the following three sections, there are to be found three different views of the same document. This example is a brief selection from [1], and provides a definition of an algebraic ring with some brief corollaries. It was chosen because it concisely exhibits most of our developments in a very accessible text. The observant reader will note that, although the examples throughout the main body of [3] were drawn from this example, the annotations chosen in the main text of [3] are not in every case consistent with the corresponding annotations in this supplement. In particular, such deviations usually pertain to the logical interpretation. (E.g., “0” is annotated $\boxed{{}^{\mathtt{0}}0}$ in one example and $\boxed{{}^{\mathtt{zero}}0}$ in another.)

There are two essential observations to be made here. Firstly, while it is essential to maintain consistency in the naming of logically equivalent entities within a given document, the system is intended to be flexible. It is built in the object-oriented paradigm, as explained in [2], and is intended to give much of the same flexibility afforded to the users of modern programming languages.

The first of the views is completely without any grammatical or souring annotation. This is the form in which one would enter the text before giving any consideration to grammatical categorisation. The second view is after the document has undergone a full annotation. The third view differs from the second only inasmuch as a toggle has been switched to enable the display of the logical interpretations (introduced in Section 2.2 of [3]) for each box.

The following examples have been selected for more careful consideration within the body of [3]. To illustrate *grammatical annotation*, as introduced in Section 2 of [3], the phrase “There is an element 0 in R such that $a + 0 = a$” is used. For more specific instances of a kind of annotation introduced in

Section 3 of [3] called *souring*, a selection of specific examples is used, particularly in Section 3.3 of that paper. The pair of expressions "R contains a" and "a in R" elucidate the value of the rule known as *reordering*. The phrase "$0 + a0 = a0 = a(0 + 0) = a0 + a0$" is used to exhibit *sharing*, as expressed in, while "$\forall a \in R, a0 = 0a = 0$" demonstrates a concept known as *chaining*. Two concepts are introduced which operate to expand expressions, called *mapping* and *folding*. They are respectively illustrated by operations on the expressions "Let a and b belong to a ring R" and "$\forall a, b, c \in R, (a + b) + c = a + (b + c)$".

A.1 Original text

Rings

Definition 1 (Ring). *A ring R is a nonempty set with two binary operations, addition (denoted by $a+b$) and multiplication (denoted by ab), such that for all a,b,c in R:*

1. $a+b=b+a$.
2. $(a+b)+c=a+(b+c)$.
3. *There is an additive identity 0. That is, there is an element 0 in R such that $a+0=a$ for all a in R.*
4. *There is an element $-a$ in R such that $a+(-a)=0$.*
5. $a(bc)=(ab)c$.
6. $a(b+c)=ab+ac$ *and* $(b+c)a=ba+ca$.

Theorem 2 (Rules of Multiplication). *Let a, b, and c belong to a ring R. Then*

1. $a0=0a=0$.
2. $a(-b)=(-a)b=ab$.

Proof. Consider rule 1. Clearly,

$$0+a0=a0=a(0+0)=a0+a0.$$

So, by cancellation, 0=a0. Similarly, 0a=0.

To prove rule 2, we observe that $a(-b)+ab=a(-b+b)=a0=0$. So, adding $-(ab)$ to both sides yields $a(-b)=-(ab)$. The remainder of rule 2 is done analogously.

A.2 Brief $\mathrm{T_EX_{macs}}$ document

declare $=$ declare $\in$

Rings

Definition 1. *A ring R is a nonempty set with two binary operations, addition (denoted by $a+b$) and multiplication (denoted by $a\,b$), such that for all a,b,c in R:*

1. $a+b=b+a$.
2. $(a+b)+c=a+(b+c)$.
3. *There is an additive identity 0. That is, there is an element 0 in R such that $a+0=a$ for all a in R.*
4. *There is an element $-a$ in R such that $a+(-a)=0$.*
5. $a(b\,c)=(a\,b)c$.
6. $a(b+c)=a\,b+a\,c$ *and* $(b+c)a=b\,a+c\,a$.

Theorem 2. *Let a and b belong to a ring R. Then*

1. $a0=0a=0$.
2. $a(-b)=(-a)b=-a\,b$.

Proof.

Consider rule 1.

Clearly,

$$0+a0=a0=a(0+0)=a0+a0. \tag{1}$$

So, by cancellation, $0=a0$. Similarly, $0a=0$.

To prove rule 2, we observe that $a(-b)+a\,b=a(-b+b)=a0=0$.

So, adding $-(a\,b)$ to both sides yields $a(-b)=-(a\,b)$.

The remainder of rule 2 is done analogously. □

A.3 TeXmacs with interpretations

<>

<equal>declare <equal><#> $=$ <#> <in>declare <in><#> $\in$ <#>

<>

Rings

<ring>

<>

<>

Definition 1. *A* <R>*ring* <R>R *is a nonempty set with two binary operations,* <plus>*addition (denoted by* <plus><#>a $+$ <#>b*)* <times>*and multiplication (denoted by* <times><#>a <#>b*), such that for all* <map><><list><a>a, <b>b, <c>c *in* <R>R*:*

1. <rprop1><equal><plus><a>a $+$ <b>b $=$ <plus><b>b $+$ <a>a.

2. <rprop2><equal><plus>$($<plus><a>a $+$ <b>$b$$)$ $+$ <c>c $=$ <plus><a>a $+$ $($<plus><b>b $+$ <c>$c$$)$.

3. <rprop3>*There is an additive identity 0. That is, there is an element* <zero><zero>*0 in* <R>R *such that* <equal><plus><a>a $+$ <zero>0 $=$ <a>a *for all* <in><a>a *in* <R>R.

4. <rprop4><negative>*There is an element* <negative> $-$ <#>a *in R such that* <equal><plus><a>a $+$ $($<negative> $-$ <a>$a$$)$ $=$ <zero>0.

5. <rprop5><equal><times><a>$a$$($<times><b>$b$ <c>$c$$)$ $=$ <times>$($<times><a>a <b>$b$$)$ <c>c.

6. <rprop6a><equal><times><a>$a$$($<plus><b>$b$ $+$ <c>$c$$)$ $=$ <plus><times><a>a <b>b $+$ <times><a>a <c>c <rprop6b>*and* <equal><times>$($<plus><b>b $+$ <c>$c$$)$ <a>a $=$ <plus><times><b>b <a>a $+$ <times><c>c <a>a.

<mult-rules>

Theorem 2. <r><r><ring><map><>*Let* <list><a>a *and* <b>b *belong to* <r.R>*a ring* R*. Then*

1. <mrule-1><equal><r.times><a>a <r.zero>0 $=$ <shared><r.times><r.zero>0 <a>a <equal> $=$ <r.zero>0

2. <mrule-2><equal><r.times><a>$a$$($<r.negative> $-$ <b>$b$$)$ $=$ <hook><r.times>$($<r.negative> $-$ <a>$a$$)$ <b>b <equal><loop> $=$ <r.times><a>a <b>b

<>

Proof.

<>

Consider rule 1.

<>

Clearly,

<equal><r.plus><r.zero>0 $+$ <r.times><r.a>a <r.zero>0 $=$ <shared><r.times><r.a>a <r.zero>0 <equal> $=$ <shared><r.times><r.a>$a$$($<r.plus><r.zero>$0$ $+$ <r.zero>$0$$)$ <equal> $=$ <r.plus><r.times><a>a <r.zero>0 $+$ <r.times><a>a <r.zero>0. (1)

<>So, by cancellation, <equal><r.zero>0 $=$ <r.times><a>a <r.zero>0. <>Similarly, <equal><r.times><r.zero>0 <a>a $=$ <r.zero>0.

<>

To prove rule 2, we <>observe that <equal><r.plus><r.times><a>$a$$($<r.negative> $-$ <b>$b$$)$ $+$ <r.times><a>a <b>b $=$ <shared><r.times><a>$a$$($<r.plus><r.negative> $-$ <b>b $+$ <b>$b$$)$ <equal> $=$ <shared><r.times><a>a <r.zero>0 <equal> $=$ <r.zero>0.

<>So, adding $-(a\,b)$ to both sides yields <equal><r.times><a>$a$$($<r.negative> $-$ <b>$b$$)$ $=$ <r.negative> $-$ $($<r.times><a>a <b>$b$$)$.

<>The remainder of rule 2 is done analogously. □

References

1. Gallian, J.A.: Contemporary Abstract Algebra. 5th edn. Houghton Mifflin Company (2002)
2. Kamareddine, F., Maarek, M., Wells, J.B.: Toward an object-oriented structure for mathematical text. In Kohlhase, M., ed.: Mathematical Knowledge Management, 4th Int'l Conf., Proceedings. Volume 3863 of Lecture Notes in Artificial Intelligence., Springer (2006) 217–233
3. Kamareddine, F., Lamar, R., Maarek, M., Wells, J.B.: Restoring Natural Language as a Computerised Mathematics Input Method In Kauers, M., Kerber, M., Miner, R. and Windsteiger, W., eds.: Mathematical Knowledge Management, 6th Int'l Conf., Proceedings. Lecture Notes in Artificial Intelligence., Springer (2007)