Fraser Daly

F.Daly@hw.ac.uk
Department of Actuarial Mathematics and Statistics, Heriot–Watt University

About Me

2001–2008: Nottingham

I graduated from the University of Nottingham in 2005 with an MMath degree in Mathematics.

I stayed in Nottingham and completed my PhD there in 2008, under the supervision of Sergey Utev. My thesis had the title *New bounds on Stein–type operators, with applications*.

2008–2009: Zürich

I spent 16 months as a postdoctoral research fellow at Universität Zürich, Switzerland.

It was here that my son Dominic was born.

2010–2013: Bristol

I moved to Bristol in 2010, where I spent three years working as a Heilbronn Research Fellow.

Robin was born in Bristol in early 2012.

I moved to Edinburgh in August 2013, when I took up my current position as a lecturer in the department of Actuarial Mathematics and Statistics within the School of Mathematical and Computer Sciences at Heriot–Watt University.

When I'm not at work, much of my time is spent being kept on my toes by Dominic (now age 4) and Robin (almost 2).

My Aims for the Crucible

- Realise the interdisciplinary potential of my research by liaising with researchers from other disciplines, discussing potential new applications of my current research and new cross–disciplinary research areas which could benefit from my experience and expertise.
- Find new ideas and perspectives that will help to make my research more innovative.
- Increase my appreciation of the relevance and potential impact of my research and how these can be increased.
- Learn how to effectively communicate my research findings and enthusiasm for my subject area to a wider audience.

Research Interests

My research is in applied probability. I am particularly interested in the following areas.

Limit theorems and probability approximations Many probability distributions that arise in practical problems are too hard to evaluate explicitly, but many can be approximated by simple and common distributions.

The Central Limit Theorem (dating back to de Moivre in the 1730s) tells us that many interesting distributions are approximately Gaussian.

Extensions and applications of the Central Limit Theorem are still an active research area today.

Having bounds on how close a distribution is to Gaussian can help us decide when the approximation is good enough for our purposes.

Explicit error bounds in useful probability approximations can be found in a wide variety of settings.

Stochastic orderings and probability inequalities Aiming to answer questions such as

- How can we compare two probability distributions?
- How can we capture notions of dependence between probability distributions?
- How can assumptions on the structure of a distribution help us find its properties?

Information theory Recent results in both of the above areas have close links with concepts in information theory.

An Example: Pólya's Urn

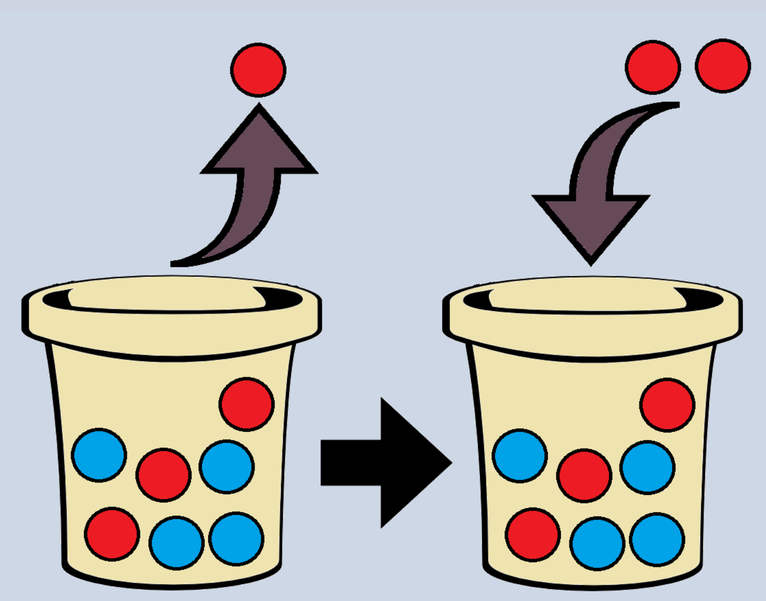
Suppose an urn initially contains N balls, each of which is one of n possible colours.

At each step:

1. Choose a ball at random from the urn and note its colour.
2. Return it to the urn together with another ball of the same colour.

Repeat this for a total of r draws.

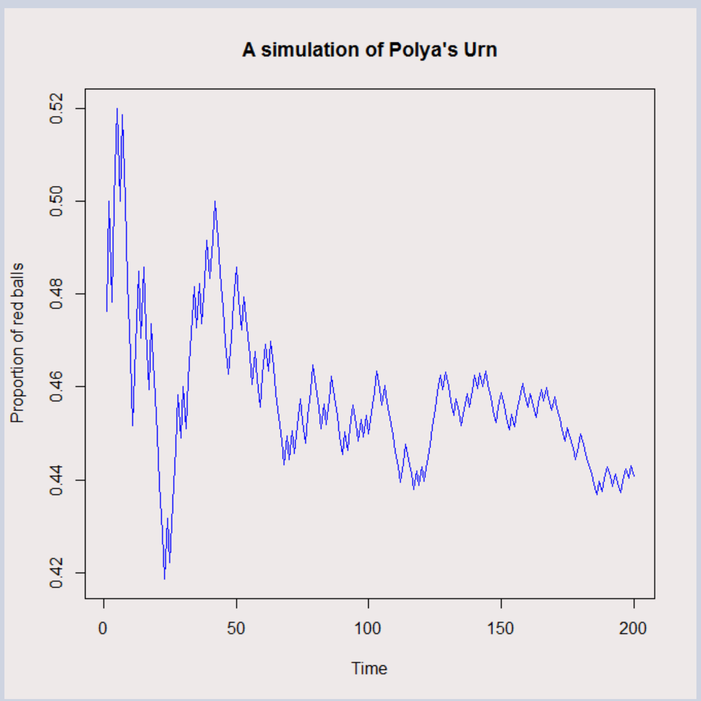
This is Pólya's Urn. This model has applications in the study of epidemics, genetics, computing and communications.



Suppose we are interested in the number of different colours that we haven't seen during these r draws. Let this (random) number be W. We can write

$$W = X_1 + X_2 + \cdots + X_n\,, \qquad \text{where } X_i = \begin{cases} 1 & \text{if colour } i \text{ is never seen,} \\ 0 & \text{otherwise.} \end{cases}$$

An important property here is that $X_1, \ldots, X_n$ are *negatively related*. Roughly speaking, if we know that we haven't seen any red balls in our r draws, that makes it more likely that we did see some blue balls. This negative relation gives us some useful properties of W:



- A bound on the distance between W and a Poisson distribution, another common limiting distribution in probability theory. This error bound depends only on the mean and variance of W, both of which are easy to calculate.
- A useful and simple upper bound for the Shannon entropy of W, an information-theoretic measure of uncertainty.
- A bound for the variance of $g(W)$ for a wide range of functions g.
- Upper bounds for the probabilities $\mathbb{P}(W \geq t)$ and $\mathbb{P}(W \leq t)$ for many t.