

Formal Specification F28FS2, Lecture 12

Implementing schema in ML

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The phone directory spec

Taken from Michael Butler's introductory notes on Z.

Assume types [*Person*, *Phone*].

I don't want to number people, so let's model *Person* by the ML type `string`, and *Phone* by `int`. We could write the following if we want:

```
type Person = string;
type Person = string
type Phone = int;
type Phone = int
```

The directory

The state of the directory:

```
Directory _____  
dir : Person ↔ Phone
```

```
fun Directory (dir:(Person*Phone) list) = true;  
val Directory = fn : (Person * Phone) list -> bool
```

Initialising the directory

Initially the directory is empty:

<i>InitDirectory</i>
<i>dir'</i> : <i>Person</i> ↔ <i>Phone</i>

<i>dir'</i> = {}

```
fun initDirectory () = ([]:(Person*Phone) list);  
val initDirectory = fn : unit -> (Person * Phone)  
list
```

Adding to the directory

We add a name and number pair to the directory:

AddEntry

dir, dir' : Person ↔ Phone

name? : Person

number? : Phone

dir' = dir ∪ {name? ↦ number?}

```
fun AddEntry dir (name:Person) (number:Phone) =  
  (name,number)::dir;  
val AddEntry = fn : (Person * Phone) list -> Person  
-> Phone -> (Person * Phone) list
```

Directory number lookup

An operation to get all the numbers associated with a name:

<i>GetNumbers</i>
\exists <i>Directory</i>
<i>name?</i> : <i>Person</i>
<i>numbers!</i> : \mathbb{P} <i>Phone</i>
$numbers! = \{n : Phone \mid name? \mapsto n \in dir\}$

```
fun GetNumbers ((p,n)::tl) (name:Person) = if
(name=p) then n::(GetNumbers tl name) else
(GetNumbers tl name)
  | GetNumbers [] name = [];
val GetNumbers = fn : (Person * 'a) list -> Person
-> 'a list
```

I did not bother to return `dir`, and I did not bother to restrict the polymorphism on `'a`. Does that make me a bad person?

Directory name lookup

An operation to get all the names associated with a number:

GetNames

\exists *Directory*

number? : *Phone*

names! : \mathbb{P} *Person*

$names! = \{p : Person \mid p \mapsto number? \in dir\}$

```
fun GetNames ((p,n)::tl) (number:Phone) = if
(number=n) then p::(GetNames tl number) else
(GetNames tl number)
  | GetNames [] number = [];
val GetNames = fn : ('a * Phone) list -> Phone -> 'a
list
```

Remove Entry

An operation to remove an entry from the directory:

RemoveEntry

Δ *Directory*

name? : *Person*

number? : *Phone*

$dir' = dir \setminus \{name? \mapsto number?\}$

```
fun RemoveEntry (hd::tl) (name:Person) (number:Phone)
= if (hd=(name,number)) then (RemoveEntry tl name
number) else hd::(RemoveEntry tl name number)
  | RemoveEntry [] name number = [];
val RemoveEntry = fn : (Person * Phone) list ->
Person -> Phone -> (Person * Phone) list
```

Exercises

1. Write a schema to check whether *name?* appears in the phone directory, and return 'true' if it is, and 'false' if it is not (you might like to declare an enumerated type *Bool ::= true | false*).
2. Implement it.
3. Write a schema to return the set of people with more than one number.
4. Implement it.

Exercises

Implement the Birthday Book schema from (see Butler's notes; links on the course webpage, search for 'Butler').