Formal Specification *(F22HO2)*

Class Test 2 (mock) (Questions)

February 14, 2008

Write so I can easily read it — if I can't read it easily, then I won't read it at all.
Ask about errors or ambiguities in this document. If you don't understand and don't ask, don't complain about the consequences.

1. (10 points) Let $\mathbb{T}$ be a type. Fill in the types:

- $x : \mathbb{T}$ and $x \in S$. (Type S.)
- $\bigcup \mathcal{S}$ and $S \in \mathcal{S}$ and $S : \mathbb{PT}$. (Type $\mathcal{S}$ and $\bigcup \mathcal{S}$.)
- $\varnothing \in S$ and $S : \mathbb{PPPT}$. (Type $\varnothing$.)
- $x \in R$ where $R : \mathbb{Z} \leftrightarrow \mathbb{T}$. (Type x.)
- $S \lhd R$ where $R : \mathbb{T} \leftrightarrow \mathbb{Z}$. (Type S.)
- $x \in l$ where $l : iseq(\mathbb{T})$. (Type x.)
- $x \in f$ where $f : \mathbb{T} \pfun \mathbb{T}$. (Type x.)
- $l \frown l'$ where $l : seq(\mathbb{T})$. (Type l'.)
- $R(\!| S |\!)$ where $R : \mathbb{Z} \leftrightarrow \mathbb{T}$. (Type S; $(\!| - |\!)$ is 'relational image'.)

2. (10 points) Write Z set expressions for (answers must include the symbols {, }, and •, and possibly |; answers that contain { immediately followed by ∀ will score zero marks):

- The empty set of integers.
- The set of even numbers greater than 7 (do *not* mention division; only multiplication).

3. (10 points) Suppose $S, T : \mathbb{PZ}$ and $\mathcal{S} : \mathbb{PPZ}$.

Write down, with full types, Z set expressions for:

- $\bigcup \mathcal{S}$ (the set union of all the elements of S).
- $\bigcap \mathcal{S}$ (the set intersection of all the elements of S).

4. (20 points) The President of France has a string of girlfriends, all gorgeous and of various nationalities. Assuming a pure type $PERSON$, a global variable $italians : \mathbb{P} PERSON$, and a variable $girlfriends?$ (of type $iseq(PERSON)$ or $seq(PERSON)$; your choice), write a predicate to express the English sentence 'the President never has more than two italian girlfriends in a row'.
The type of $girlfriends?$ can be $iseq(PERSON)$ or $seq(PERSON)$. Explain in plain English what implications this choice of Z type has for the President's love life and the nature of Italian women.
Write a schema *ItalianGirlfriends* with state variables exactly $girlfriends?$ (a sequence of the appropriate type), and $nonitalianGirlfriends!$ (of the same type) which outputs the President's non-Italian girlfriends, in order.

5. (10 points) Write down, with full types, a Z set expression for the set of prime numbers. (Do *not* mention division, only multiplication; use the type $\mathbb{N}$ not $\mathbb{Z}$; for this question, 0 and 1 are not prime numbers.)

The whereabouts of Evil Technicians who work in the Hive for the Umbrella Corporation need to be monitored for reasons of world domination.

You may assume a pure type $EVILTECH$ of Evil Technicians.

6. (10 points)

- Using state variables $members : \mathbb{P} EVILTECH$, $inHive : \mathbb{P} EVILTECH$ and $outOfHive : \mathbb{P} EVILTECH$, write a schema for the state of the Hive.
- Write an initial state schema for the state of the Hive. Initially, all members are *outside* of the Hive.

In the question above note that:

- $members$ is just Evil Technicians who are members of the Umbrella Corporation.
- There is no numerical limit on the number of Evil Technicians inside or outside of the Hive (unlike the ClubState example which you have studied).
- No Evil Technician may be both in and out of the Hive.

7. (15 points) Using sets $zombieMembers?$ and $zombieMembers!$ of appropriate type, write a schema *Infects* (with precondition that $zombieMembers? \subseteq members$) such that if there is a zombie Evil Technician outside the hive before *Infects*, then *all* Evil Technicians outside the hive are zombies after; similarly for inside the hive; otherwise there is no change.

8. (15 points) (. . . and a box of pralines.) Suppose the President of France can have more than one girlfriend at a time. Describe, using Z types, how to model his girlfriends in this case, with justification in plain English. Using your types, write a schema which returns his Italian girlfriends as a set (in no particular order).

Apologies to the French Republic, and good luck to the President in the imminent local elections.