F71SB2 Statistics 2 Assignment 1

Due: 4 February 2008 at 4.30pm (use AMS course-work box, outside EM 1.25).

INSTRUCTIONS: Attempt ALL FIVE questions. To receive full credit you must **show** your work and **explain your answers**. There are 40 marks available.

1. Consider a random variable X having a negative binomial distribution, and specifically $X \sim \text{Neg.Bin}(2, 0.5)$. Calculate the probability P(X < 4). [4 marks]

2. (a) A pack of 52 cards contains 4 aces. Cards are drawn randomly one at a time, without being placed back in the pack. Find the probability that an ace will be selected for the first time on the fourth draw. [3 marks]

(b) Consider again a pack of 52 cards containing 4 aces. Cards are again drawn randomly one at a time, but this time each card *is replaced in the pack* after it is drawn. Find the probability that an ace will be selected for the first time on the fourth draw in this case.
[3 marks]

3. (a) A class consists of 12 female students and 9 male students. Two students are selected at random as class representatives. Let Y be the random variable showing the number of female students selected as representatives. Determine the probability mass function $f_Y(y)$ of Y. [4 marks]

(b) Determine the cumulative distribution function $F_Y(y)$ of the random variable Y from part (a) above and draw its graph. [6 marks]

4. A quiz contains three questions, each with three possible answers. A group of 40 people take part in this quiz and the frequencies of questions answered correctly by each person are given in the table below.

Number of questions answered correctly (x): 0 1 2 3 Number of people (f_x) : 3 5 17 15

It is assumed that the probability, p, that a person will answer a question correctly is the same for all participants of the quiz.

- (a) Suggest an appropriate binomial distribution for the number of questions answered correctly by a single person. [1 mark]
- (b) Based on the distribution for the number of correctly answered questions that you assumed in part (a) above, use the data in the table to calculate the methodof-moments estimate of the probability, p, that a question is answered correctly. [3 marks]

(c) Calculate the expected frequencies for the number of correctly answered questions per person under the assumed distribution and using your estimate of p from above.

[4 marks]

(d) By comparing the expected numbers in each category with the observed given in the table, comment briefly on the suitability of the assumed distribution and the validity of any other assumptions made. [2 marks]

- 5. The length of time (in days) until people are infected with a specific virus after a disease outbreak in a population is assumed to follow an exponential distribution with parameter $\lambda = 0.35$.
 - (a) Find the probability that a person in the population will not have been infected after a total period of five days since the outbreak of the disease, given that she has avoided infection in the first two days. [4 marks]

(b) Consider now a group of 10 people from the same population that have all avoided infection in the first two days. Find the probability that 8 of them will not have been infected after a total period of five days since the outbreak. [6 marks] BLANK