Logic and Materialism

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Chapter 1

Logic and materialism

1.1 Introduction

> Clearly then it is the function of the philosopher, ie the student of the whole of reality in its essential nature, to investigate also the principles of syllogistic reasoning.
> Aristotle, The Metaphysics[Aristotle(1933), p161]

Since antiquity, many long lived trans-national state systems have been explicitly founded on some organised religion as their explicit dominant ideology. In the current era, examples include Roman Catholicism, for the Holy Roman, Spanish, Portuguese, and French Empires, Islam, for the Ottoman Empire, and Anglicanism, for the British Empire. With one major exception, there have been no such systems based on secular philosophies; even the United States empire is openly deist[1], if not outright Christian.

That one exception is *dialectical materialism*, the philosophy of the world communist movement since the late 19th century. Just as billions of people were taught the dominant religions in the empires that embraced them, so billions of people have been taught dialectical materialism, in the former USSR and European socialist states until 1989, and in China, and extant socialist states, to the present. The reach of dialectical materialism should not be underestimated. Thus, in 2021, the Anglican church had 85 million communicants[Wikipedia(2021)]: markedly less than the Chinese Communist Party with 95 million members[Statista(2021)].

In its materialist component, dialectical materialism is profoundly progressive, rejecting all forms of idealism, and actively promoting science and rationality as the means for humanity to understand and transform the world. However, as we shall explore, the dialectical component is much more problematic, philosophically and ideologically.

In particular, in the USSR, dialectical materialism was deployed for partisan purposes, both to suppress opposition, and to constrain what were deemed appropriate

[1] *In God We Trust*, as their official motto proclaims.

areas for scientific investigation. The effects on Soviet genetics, with what was effectively Lamarckianism promoted over Mendelism, are well documented[Lecourt(1977)]. Here, we will focus on the less well known impact on the study of formal logic, which, nonetheless, had profound implications for the development of Soviet mathematics, in particular Computer Science and practical computing.

To do so, we will trace:

- the origins of logic alongside dialectics in antiquity, as tools of reasoning and argument;
- the mathematical formalisation of logic from the 19th century;
- the displacement of dialectics by an evolving mathematical logic;
- the conceptualistion of logic and dialectics in Hegelian and then Marxist philosophy;
- and the Marxist critique of formalism.

We will also touch on the Marxist critique of mechanical materialism, as it bears on our argument. In a later chapter, we will look at how computation offers a universalising framework for a resurgent mechanical materialism.

This is a brief account, cherry picking selected aspects of a rich and complex field. Here, the discussion is almost entirely in a European context. While individuals are highlighted, they most certainly were not working in vacuums. Logic, like all science, is very much a community activity. For a comprehensive history of formal logic, including Indian, see Ḃocheński[Ḃocheński(1961)].

1.2 Logic overview

Before we look at the relationships amongst materialism, logic, and dialectics in more depth, we will briefly survey contemporary understanding of logic, to enable us to frame its development. Cockshott et al[Cockshott et al.(2012)Cockshott, Mackenzie, and Michaelson] provides a more detailed account.

A logic is a formalised system for reasoning. It is important to note that logic is about *truth values* (ie true and false) rather than *the truth*. The things we reason about are *assumed* to be true or false independently of logic. They may be factual, or hypothetical, or speculations, or beliefs. Logic cannot in itself establish whether or not something is truthful; rather it is concerned with *correct reasoning*.

When we say logic is *formalised*, we mean that there are rules for:

- constructing statements to reason about, ie grammar or syntax;
- giving meaning to statements, ie semantics;

- manipulating statements to establish their properties, commonly whether they are always, or partially, true or false, ie through proof or evaluation.

We will further explore these below, but we won't exhaustively or formally treat them here.

A logical argument proceeds from *premises*, also known as *assumptions*, to *conclusions*. Reasoning steps are through *rules of inference*, a modern form of the more convoluted *syllogism* of antiquity, discussed in subsequent sections.

A syllogism has the general three term form:

$premise_1$
$premise_2$
$conclusion$

and reads as: given $premise_1$ and $premise_2$, we can conclude $conclusion$.

For example, suppose A and B are statements, and we accept that if A is the case then B is the case. Then, if we take it that A is the case, we will reasonably infer that B is the case as well, that is B logically follows from A. In syllogistic form, this is:

1. A
2. if A then B
3. B

This fundamental rule of contemporary logic is known as *Modus Ponems*, or the method of affirming. For example, consider:

1. Bastet is a cat
2. if Bastet is a cat then Bastet likes fish
3. so Bastet likes fish

This seems entirely reasonable.

In arguments, people may vociferously question whether or not A is the case, or whether or not B actually follows from A. For example, consider:

1. Bastet is a warrior goddess
2. if Bastet is a warrior goddess then Bastet defends the king
3. so Bastet defends the king

We might argue:

- there are no goddesses;

- there are goddesses but no warrior goddesses;
- there are warrior goddesses but Bastet isn't one;
- Bastet is a warrior goddess but there is no king;
- Bastet is a warrior goddess but she doesn't defend the king;
- etc.

But if we accept the truth of 'Bastet is a warrior goddess', and of 'if Bastet is a warrior goddess then she defends the king', then clearly 'Bastet defends the king' is an unimpeachable conclusion.

That is, in the general case, nobody questions the deduction of B, assuming that both A, and A implies B, are the case. Any argument is about the premises, whose truth or falsity is ultimately determined outside of logic[2], not the deduction.

In contemporary logic, we now distinguish *propositional* from *predicate* logic. It is common to refer to both as *calculi*, after George Boole, who called his pioneering system, discussed below, the *Calculus of Logic*[Boole(1854)].

Propositional logic is to do with propositions (ie simple statements) being true or false. Propositions are built from truth values, and variables that abstract over them. Propositions may be *negated* (not), and combined through *disjunction* (or), *conjunction* (and), and *implication* (if...then...).

Predicate logic is then to do with propositions about collections of things, and their members, being true or false. Predicate logic extends propositional logic truth values and variables with *predicates*, which are functions that return truth values. Further, in predicate logic, propositions may be *quantified*, to express *universal* properties, that is all things having some property, and *existential* properties, that is some things (ie at least one) having some property.

We also distinguish *pure* logics, which are not about anything in particular, from *applied* logics, which are about specific domains of things. Just as predicate logic extends propositional logic, an applied logic may be formed by adding additional domain specific rules to a pure logic, typically as domain specific predicates[3]. We will focus on the domains of numbers and of sets, and will talk about number theoretic and set theoretic predicate logic or calculus.

As we shall discuss, from antiquity until the later 19th century, all of these aspects of logic were conflated.

A key philosophical question concerns the status of truth values and rules of inference, as part of a wider question about the status of mathematical entities like numbers and functions. Are they just marks on paper? Do they have some deeper ideal reality? Or are they, as we shall argue, components of materialised mathematical systems, abstracted from, and with strong correspondences, to material reality?

[2]Though perhaps through intermediate logical arguments.
[3]Applied logics may also be constructed *ab initio*.

1.3 Logic and dialectics

Logic and dialectics are core components of the *materialist dialectic*[4]. However, untangling logic and dialectics is a curiously difficult business.

Let's start in the middle. In the medieval European education system, the curriculum was based on the Seven Liberal Arts[Abelson(1906)], codified particularly from Boethius's translations of Aristotle[Marenbon(2009)]. These Arts were divided into the Trivium, consisting of Grammar, Logic and Rhetoric, taught before the Quadrivium, consisting of Arithmetic, Geometry, Astronomy and Music.

The separation of Logic from the mathematical sciences, of Arithmetic, Geometry and Astronomy, is very striking. Note that such distinctions dominated education until the late 18th century. They will have strongly influenced those with the means and opportunities to attend grammar schools[5] and universities, particularly many of the philosophers and scientists discussed above.

Aristotle actually refers to dialectics rather than logic. In *The 'Art' of Rhetoric*[Aristotle(1926)], he distinguishes rhetoric, concerned with informal persuasion, from dialectics, concerned with formal reasoning through the syllogism (p13). This implies that logic is part of dialectics.

Aristotle notes that both rhetoric and dialectics employ syllogistic and inductive reasoning. For reasoning based on the dialectical syllogism, all steps must be made explicit. However, for the rhetorical syllogism, the *enthymeme*, steps may be elided. Further, dialectical induction is based on finding patterns, whereas rhetorical induction is based on concrete examples. Overall:

> The function of Rhetoric, then, is to deal with things about which we deliberate, but for which we have no systematic rules; (p23).

The implication is that dialectics is systematic.

Aristotle also distinguishes sciences, concerned with particular domains, from both rhetoric and dialectic, as universally applicable modes of discourse. This is reflected in the subsequent Trivium/Quadrivium distinction, with the Trivium providing the pure tools for reasoning and arguing about the Quadrivium applied domains.

He further says that, as someone develops richer understandings of a domain:

> ... the more he will unconsciously produce a science quite different from Dialectic and Rhetoric. For if once he hits upon first principles, it will no longer be Dialectic or Rhetoric, but that science whose principles he has arrived at. (p31)

Clearly this applies to the dialectics/logic dynamic itself. As we shall see, as logic became more mathematically grounded, so the space for dialectics shrank, much

[4]We'll later explore how this is related to dialectical materialism, but note that the transposed word use as adjective or noun is significant.

[5]To use the British nomenclature.

as wider scientific advances shrank the space for both religion and philosophy, as discussed in earlier chapters.

In *The Organon (Prior Analytics)* [Aristotle(1962b)], Aristotle further abstracts logic from both dialectics and science, in discussing types of premises for syllogisms (p200 & 202). A demonstrative (ie scientific) premise is true and based on 'fundamental postulates', whereas, for a dialectical premise, a choice may be made between two 'contradictory statements'. Then, for a syllogism, a premise is simply true or false, regardless of origin. That is, for both science and dialectics, once some premise is established, a syllogism may be applied to draw a conclusion.

1.4 Mathematical forms

We saw above that Logic in the Trivium was distinguished from the mathematical sciences of Arithmetic, Geometry and Astronomy in the Quadrivium. Further, in *The Metaphysics* [Aristotle(1926)], Aristotle says that there is hierarchy in mathematics:

> ... mathematics too has divisions, — there is a primary and a secondary science, and others successively, in the realm of mathematics. (p151)

and, in considering how philosophy is layered, distinguishes universal (ie pure) mathematics, from specific (ie applied) mathematical sciences:

> One might indeed raise the question whether the primary philosophy is universal or deals with some one genus or entity; because even the mathematical sciences differ in this respect — geometry and astronomy deal with a particular kind of entity, whereas universal mathematics applies to all kinds alike. (p297)

Aristotle, a Platonist by training, nonetheless appears uncommitted as to the metaphysical status of mathematics, but notes that, for his master, mathematics lies between material reality and pure idea:

> Further, he [Plato] states that besides sensible things and the Forms there exists an intermediate class, *the objects of mathematics* [footnote: ie arithmetical numbers and geometric figures], which differ from sensible things in being eternal and immutable, and from the the Forms in that there are many similar objects of mathematics, whereas each Form is itself unique. (p45)

For Platonists, the *forms* are abstract ideals which nonetheless are real ([Plato(1937), Plato(1962)]). For example, in the *Timaeus*[Plato(1888)], Plato discusses what are known as the Platonic solids: regular polyhedra, like pyramids, cubes and so on. Then, we can observe and make individual Platonic solids of different sizes, which are plainly physically different from each other, and are imperfect. Standing back, we can see that, say, arbitrary physical cubes share the characteristic of six square faces, and we use the same equations to calculate their surface areas or volumes. That is, we use mathematical abstractions to characterise the form of arbitrary physical cubes. Nonetheless, we can consider differences amongst mathematical cubes, for example that one is half or twice the size of another. So, the ideal Platonic cube form is further abstracted from all mathematical cubes.

We will argue that mathematical objects have material being in their physical representations within symbol systems. So, for example, the ideal cube is no more than the mathematical cube, itself a materialised construct that characterises physical cubes.

We will return to this in discussing identity as one of the fundamental *laws of thought*.

1.5 Syllogisms

Aristotle's formulations of syllogisms are key to pre-modern logic. We will now look in slightly more depth at these, but we won't give a formal treatment. For a succinct account, see Smith[Smith(2017)].

Aristotle explores syllogisms in *The Organon*, considering reasoning about properties of individual and collections of things, and about things from particular and general domain. He makes considerable use of concrete examples, which, as we saw above, he called rhetorical induction.

First of all, in *On Interpretation*[Aristotle(1962a)], Aristotle defines the syntax of propositions, but without using any notation. Single sentences are composed of nouns, and verbs that act on them, and may be further conjoined. Propositions are then sentences which are affirmations or denials, and subjects may be universal or singular. *On Interpretation* also introduces the key notion of contradictory propositions.

Thus, Aristotle enunciates four fundamental schemes for propositions, commonly expressed as:

- A: universal affirmative, eg all X are Y;
- I: particular affirmative, eg some X is Y;
- E: universal negative, eg no X is Y;
- O: particular negative, eg some X is not Y;

These schemes all have the structure: *subject* is/are *predicate*. Here, X, Y and Z may be replaced consistently by concrete nouns. The initial letters A, I, E and O are the classical identifiers.

Then the three syllogistic *figures* explored in *Prior Analytics*[Aristotle(1962b)] may be expressed as:

1. P is Q; Q is R; P is R
2. P is Q; P is R; Q is R
3. P is R; Q is R; P is Q

Note that these are second level schema, where P, Q and R may be replaced by one of the four proposition forms A, I, E and O, to give a first level schema.

The validity of these figures then depends on whether the terms we substitute for P, Q and R are universal or particular, and which subjects and predicates are common to which terms.

For example, the first figure where all terms are universal (AAA), is valid:

A: all X is Y
A: all Y is Z
A: all X is Z

Here, the predicate of the first term is the subject of the second term, and the predicate of the second term is the subject of the third term. Note that we accept that this is a valid syllogism, because of how we understand 'all' and 'is'[6].

Note that this validity is independent of what concrete nouns we replace X, Y and Z with. That is, the validity does not depend on the semantics of a particular instance.

However, the first figure with universal first and third terms, but a particular second term (AIA), is invalid:

A: all X is Y
I: some Y is Z
A: all X is Z

The relationships between the subjects and predicates in the terms is the same as in the first example. However, from the second term, some Y are not X, and so, from the first term, there may be an X which is one of those Ys which isn't a Z. Here, our argument depends on the semantics of 'some' implying 'not all'.

Aristotle exhaustively considers all the universal/particular and subject/predicate possibilities for the three figures, dismissing many by deriving contradictions. He also shows how the second and third figure may be reduced to the first figure.

Overall, Aristotle developed a framework both for constructing and analysing arguments, that was in widespread use until the revolutions in logic and mathematics in the mid 19th century.

[6] ie their informal semantics.

1.6 After Aristotle

Aristotle's work was translated to Latin by Boethius [Marenbon(2009)], who lived around 475 to 526 C.E. Subsequently, it was effectively lost to European thought for several hundred years, until it was 'rediscovered' from Arab scholars. Aristotelian philosophy then gained steady traction amongst a significant movement of medieval Catholic scholars, now known as Scholasticism. As we mentioned above, the articulation of the Seven Liberal Arts, and their division into the Trivium and Quadrivium, was core to European education, especially after the establishment of the first Universities from the late 11th century onward under the imprimatur of the Vatican.

France was a centre of syllogistic reasoning, particularly at the University of Paris. In the mid-17th century, the *Port-Royal Logic*[Buroker(2014)] was developed by Arnauld and Nicole, who had studied at the Sorbonne. Though they were clerics, they adhered to Jansenism, a sect in doctrinal conflict with mainstream Catholicism, and were heavily influenced by Descartes.

The *Port-Royal Logic*, published in 1662, retained the A, I, E and O syllogistic forms, but it had a more sophisticated term structure that allowed subordinate propositions. This made both semantics and reasoning more complex. The book became a key text on logic until well into the 19th century. It went into 63 editions, including 10 in English, and was in use at Oxford and Cambridge Universities.

Nonetheless, Aritotelian logic fared less well in England after the Reformation, where the state ideology of Catholicism was displaced by Anglicanism, the English compromise with Protestantism. Francis Bacon (1561-1626), progenitor of what many view as the doleful British tradition of empiricism, rejected formal syllogism entirely. In *Novum Organon*[Bacon(1901)][7], from 1620, he argues that logic is a tool of persuasion, not reason, and promoted induction from observation as the means to understanding:

> XI. As the present sciences are useless for the discovery of effects, so the present system of logic is useless for the discovery of the sciences.
> XII. The present system of logic rather assists in confirming and rendering inveterate the errors founded on vulgar notions than in searching after truth, and is therefore more hurtful than useful.
> XIII. The syllogism is not applied to the principles of the sciences, and is of no avail in intermediate axioms, being very unequal to the subtilty of nature. It forces assent, therefore, and not things.
> XIV. The syllogism consists of propositions; propositions of words; words are the signs of notions. If, therefore, the notions (which form the basis of the whole) be confused and carelessly abstracted from things, there is no solidity in the superstructure. Our only hope, then, is in genuine induction. (p12-4).

[7]Riffing on Aristotle's *Organon*.

Bacon's critique rested on his requirement for a unitary system of reasoning. He argued that Logic, in which he conflates syllogistics and dialectics, cannot furnish this as it is unable to correct for unfounded premises.

Thomas Hobbes (1588–1679), whose philosophy may be characterised as materialist, was more measured than Bacon. In 1655, he gave a succinct account of Aristotle's logic in *De Corpore* [Hobbes(1839)]. While he rejected Aristotelian metaphysics, he viewed syllogistic reasoning as computational, by strong analogy with arithmetic. Thereafter, while logic continued to be taught at Oxford and Cambridge, there was little academic interest in syllogistics until the early 19th century, when its revival was wholly separated from dialectics.

The work of Richard Whately (1787-1863), the Archbishop of Dublin in the Anglican Church of Ireland, was particularly influential. In *Elements of Logic*[Whately(1845)], written in 1826, he robustly defended logic, emphasising its universal applicability, and the need to separate it from its subject matter. In an engaging analogy with arithmetic, he argues:

> All numbers (which are the subject of Arithmetic) must be numbers of *some things*, whether coins, persons, measures, or any thing else; but to introduce into the science any notice of the *things* respecting which calculations are made, would be evidently irrelevant, and would destroy its scientific character: we proceed therefore with arbitrary signs representing numbers in the abstract. So also does Logic pronounce on the validity of a regularly constructed argument, equally well, though arbitrary symbols may have been substituted for the terms; and, consequently, without any regard to the things signified by those terms. And the possibility of doing this (though the employment of such arbitrary/symbols has been absurdly objected to, even by writers who understood not only Arithmetic but Algebra) is a proof of the strictly scientific character of the system. (p34-35)

Thus, Whately asserted that the strength of logic lay precisely in abstraction.

1.7 The Laws of Thought

The same period saw steady progress in the mathematisation of logic, culminating in George Boole's seminal algebraic treatment in *The Laws of Thought* ([Boole(1854)]) in 1854. The book's title derived from the notion that thinking is underpinned by immutable laws. These go to the heart of the fundamental properties of reality, and, hence, why the syllogism can capture unimpeachable reasoning.

Most accounts of the laws of thought describe what Stanley Jevons (1835-1882) termed the *three primary laws* [Jevons(1903)]:

1. The Law of Identity. **Whatever is, is.**

2. The Law of Contradiction. **Nothing can both be and not be.**

3. The Law of Excluded Middle. **Everything must either be or not be.** (p117)

The Scot William Hamilton[8] (1788-1856), a contemporary of Whately and Boole, discussed in detail the laws' origins in Aristotle's writings. In his posthumous *Lectures on Metaphysics and Logic*[Hamilton(1866)], published in 1860, Hamilton reiterated the separation of logic from metaphysics, and logic's universal applicability. For Hamilton, there was something essential about the laws of thought, circumscribing even the deity:

> Whatever violates the laws, whether of Identity, of Contradiction, or of Excluded Middle, we feel to be absolutely impossible, not only in thought but in existence. Thus we cannot attribute even to Omnipotence the power of making a thing different from itself, of making a thing at once to be and not to be, of making a thing neither to be nor not to be. These three laws thus determine to us the sphere of possibility and of impossibility; and this not merely in thought but in reality, not only logically but metaphysically. (p98)

There has been considerable disputation over the status of each law, especially the Law of the Excluded Middle (LEM), and whether or not there are further laws.

1.8 Logical operations and truth tables

Before we discuss the development of modern mathematical logic, we will briefly survey how the semantics of logical constructs may be formalised. Aristotle, and his successors, focused on grammar as central to constructing correct arguments, and depended on meanings and reasoning expressed in everyday language. Thus, much discussion of logic prior to, and indeed after, Boole was about what exactly logical operations meant.

Today, we use *truth tables* to give logical operators precise meanings, as promoted by Ludwig Wittgenstein (1889-1951) [Wittgenstein(1961), pp32-33]. We assume the basic truth values of true and false. Core operations are: negation (not) as reversing true and false premises; conjunction (and) as requiring both premises to be true; and disjunction (or) as requiring either or both premises to be true. See the tables in Figure 1.1:

Note that this form of disjunction is called *inclusive*. We can also define *exclusive* disjunction (xor), which is true if either premise is true, but not both. See Figure 1.2.

In logical reasoning, *material implication*, that is 'implies' or 'if...then...', is central to forming rules of inference. For:

[8] Not to be confused with the Irish mathematician William Rowan Hamilton.

X	not X
true	false
false	true

X	Y	X and Y
false	false	false
false	true	false
true	false	false
true	true	true

X	Y	X or Y
false	false	false
false	true	true
true	false	true
true	true	true

Figure 1.1: Logical operations: negation, conjunction and inclusive disjunction

X	Y	X xor Y
false	false	false
false	true	true
true	false	true
true	true	false

Figure 1.2: Exclusive disjunction

X implies Y

we wouldn't like X to be true at the same time that Y is false. That is, we require the effect of:

not (X and not Y)

Figure 1.3 shows the corresponding truth table. This feels counter intuitive, as 'X implies Y' is true whenever X is false. Here the distinction between logical and real world notions of truth is stark: a logical implication which evaluates to true certainly does not permit us to deduce anything about the real world, unless we know that the first premise is true. Indeed, a true implication resting on a false first premise is termed *vacuously* true.

Overall, there are sixteen operations on two premises. For each of four possible combinations of premise values:

false false
false true
true false
true true

X	Y	not Y	X and not Y	not (X and not Y)
false	false	true	false	true
false	true	false	false	true
true	false	true	true	false
true	true	false	false	true

Figure 1.3: Implication

the value of an operation can be either true or false, so there are 2*2*2*2 combinations. All sixteen can be derived from ‘not and”, which has been exploited in computer hardware design.

1.9 Mathematising logic

While Leibniz (1646-1716) had explored the formalisation of logic well before Boole, his work was lost until the early 20th century[Lenzen(2017)]. Thus, Boole (1815-1864) is now recognised as the progenitor of modern logic, particularly propositional calculus. In *The Laws of Thought*[Boole(1854)], Boole’s intention was:

> ... to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; (p1)

Boole thought that the laws of human thought were quite literally mathematical:

> There is not only a close analogy between the operations of the mind in general reasoning and its operations in the particular science of Algebra, but there is to a considerable extent an exact agreement in the laws by which the two classes of operations are conducted. (p6)

Boole’s key insight was that, if a Universe of discourse is represented as ‘1’, and Nothing as ‘0’, then logical operations on classes of things have arithmetical equivalents. Thus:

- x=x is identity;
- $x \times y$, usually written xy, is things that are both x and y, that is *intersection* of classes;
- x+y is things which are either x or y, that is *union* of classes;
- x-y is those xs which aren’t ys, that is *difference* between classes;
- 1-x is the class of not-x, that is *negation*.

Hence, algebraic techniques may be deployed to manipulate them.

It is straightforward to read off a propositional logic from Boole’s system, where 1 is true and 0 is false. See Figure 1.4, which may be contrasted with Figure 1.1[9].

The information theorist Claude Shannon noted in 1938[Shannon(1938)] that the correspondence between Boole’s calculus and logic could be applied to switching circuits.

[9]Note that Boole did not use truth tables. However, on p76 he effectively presents the truth table for negation.

x	1 - x	x	y	x × y	x	y	x + y
0	1	0	0	0	0	0	0
1	0	0	1	0	0	1	1
		1	0	0	1	0	1
		1	1	1	1	1	1

Figure 1.4: Logical operations as arithmetic: negation, conjunction and inclusive disjunction.

Boole explicitly restricted the values of his calculus to 0 and 1 [Boole(1854), p37][10], Thus the apparently problematic

true or true => 1 + 1 = 2

resolves to 1.

As well as the above notation, based on 1, 0, variables, and arithmetic operators, Boole also introduced 'v' to stand for 'all', so '(1-v)' would mean 'not all', or 'some or none'.

Boole developed his logic with examples drawn from a range of domains, including contemporary economics and theology. He stated premises baldly, without discussion, and focused on what might be logically concluded from them. In so doing, he showed how logical arguments in everyday language might be formalised. He further demonstrated that all the Aristotelian syllogistic figures were subsumed by his approach.

Despite his book's title, Boole barely discussed the three laws of thought considered above. However, he called $x^2 = x$ the 'fundamental law of thought' (p49), and used it to develop the *principle of contradiction* through the following algebraic argument[11]:

1. suppose x is some class;
2. $x = x$, that is, a class is the same as itself;
3. $xx = x$, that is, intersecting a class x with itself doesn't change it;
4. so $0 = x\text{-}xx$, by subtraction from both sides;
5. so $0 = x(1\text{-}x)$, by factorisation.

ie Nothing is x and not x. The final steps depend on classic algebraic techniques.

Overall, while Boole was critical of scholastic logic, he acknowledged its importance:

[10] This is reminiscent of programming languages like C, where false is 0 and true is any positive integer.

[11] reconstructed from pp 49-51

> ... [scholastic logic] is not a science, but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest. It does not, however, follow, that because the logic of the schools has been invested with attributes to which it has no just claim, it is therefore undeserving of regard. A system which has been associated with the very growth of language, which has left its stamp upon the greatest questions and the most famous demonstrations of philosophy, cannot be altogether unworthy of attention. (pp241-2)

Boole's conceptualisation still conflated reasoning about things and about collections of things, and his notation, though cunning, is clumsy. Still, he enabled arithmetic certainty in chains of reasoning: that is, with Boole, logic truly became a matter of computation, as Hobbes had sought.

Boole's novel approach met with opposition. Thus, Jevons[Jevons(1903)] wrote:

> ... Dr Boole regarded Logic as a branch of Mathematics, and believed that he could arrive at every possible inference by the principles of algebra. The process as actually employed by him is very obscure and difficult; and hardly any attempt to introduce it into elementary text-books of Logic has yet been made. (p191)

The Aristotelian separation of logic based on syllogistic figures from mathematics persisted well into the 20th century. For example, Williams'[Williams(1913)] popular book on logic from 1913 was little changed from Jevons. It still foregrounded the laws of thought, and didn't mention Boole. Nonetheless, despite infelicities in Boole's system, his work has proved foundational, and marked a fundamental break with the longstanding Aristotelian tradition.

1.10 Frege and the foundations of mathematics

The German matematician Gottlob Frege (1848-1925) turned Boole's work on its head, and sought to found arithmetic, and then mathematics, on logic. The mathematical logic that underpins Computing ultimately flows from Frege's.

Frege's *Begriffsschrift*[12] [Frege(1967a)], published in 1879, was subtitled:

> a formula language, modeled upon that of arithmetic, for pure thought

The emphasis on 'pure thought' is very much in the Aristotelian tradition of separating reasoning from that which is reasoned about.

Here, Frege introduced a number of fundamental innovations. While, like Boole, he used variables and arithmetic operators, he replaced the Aristotelian notions of subject and predicate with those of *argument* and *function*, so 'X has the property

[12] Literally 'conceptual writing'.

Y' would be written '$Y(X)$', and 'X and Y are related by R' as '$R(X, Y)$'. Further, he based all rules on material implication and negation, showing how to derive the other logical operators, and all syllogistic forms, from them. Finally, he introduced an operator for universal quantification, that is for talking about all members of a class having some property.

Frege deployed a novel syntax-graph notation[13] to elucidate chains of reasoning, augmented with abbreviations which behave rather like macros in programming, that is, rules for textual substitution. This notation was subsequently replaced by the more usual equational form.

In *The Foundations of Arithmetic*[Frege(1953)], from 1884, Frege elaborated his *logicist* philosophy of mathematics. He explicitly sought to disassociate logic from subjective philosophy, which he called psychology, and asserted the strong connection with mathematics. However, he also criticised mathematics for accepting incomplete proofs, requiring that every step should be made explicit. Today this is a characteristic of computer based proofs.

In *Basic Laws of Arithmetic*[Frege(1964)], published in 1893, Frege was frank that logic could not be justified by external appeal:

> The question why and with what right we acknowledge a logical law to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer. (p15)

In the *Basic Laws of Arithmetic*, Frege mounted a sustained critique of psychological logic. His ostensible target was Benno Erdmann (1851-1921), whose book of elementary logic was published in 1892 [Erdmann(1907)]. However, he used his critique to expand his arguments for the separation of logic from idealist philosophy, and for its conceptual unity with mathematics:

> And that is how our thick logic books come into being; they are bloated with unhealthy psychological fat that conceals all more delicate forms. Thus a fruitful cooperation between mathematicians and logicians is made impossible. While the mathematician defines objects, concepts and relations, the psychological logician is spying upon the origin and evolution of ideas, and to him at bottom mathematician's defining can only appear foolish, because it does not reproduce the essence of ideation. He looks into his psychological peep-show and tells the mathematician: 'I see nothing at all of what you are defining'. And the mathematician can only reply: 'No wonder, for it is not where you are looking for it'. (p24-5)

In *Begriffsschrift*, Frege explored how to base the notion of number on relative positions of symbols within abstract sequences. He further developed this approach in *The Foundations of Arithmetic*, which was fully formalised in *Basic Laws of Arithmetic*. We will later discuss the crisis in mathematical logic at the end of the 19th

[13]The introduction of novel notations has bedeviled the development of programming languages.

N	sum 1 to N	=	N*(N+1)/2	=
1		1	1*2/2	1
2	1+2	3	2*3/2	3
3	1+2+3	6	3*4/2	6
4	1+2+3+4	10	4*5/2	10
...				

Figure 1.5: Summing

century, when Russell showed that Frege's system gave rise to paradoxes of self-reference.

1.11 Numbers and induction

Aristotle was concerned with systematising reasoning about properties of collections of things. We have explored the syllogistic, or *deductive* approach, based on syllogistic rules of inference from assumed properties. Aristotle distinguished this from *inductive* reasoning, that is identifying patterns in collections of things.

Inductive reasoning was placed on a formal footing by Giuseppe Peano (1858-1932), who unified notions of number, set and induction. As we shall see, Russell and Whitehead pursued Frege's project of founding all mathematics on logic by re-formulating Frege's system in Peano's terms.

Peano's approach[Peano(1967)], from 1889, was based on a very simple conception of *successive* numbers, starting with one[14] and repeatedly adding one. Thus:

2 = 1+1
3 = 2+1 = 1+1+1
4 = 3+1 = 2+1+1 = 1+1+1+1
...

Peano next presented having a property as akin to being in a collection with well defined characteristics, much like Boole and Frege. He then defined induction over collections of numbers by considering common properties of their members regarded as a sequence.

In general, for an inductive proof, we assume that some property holds for 1. Then, suppose N is an arbitrary number. If assuming that the property holds for N, we can prove that it holds for N+1, then the property must hold for all numbers, as we can work our way forward from 1 to any number. That is, the property is an inductive pattern for the number sequence.

As an example, consider summing numbers, as shown in the first two columns of Figure 1.5. Suppose we hypothesise that the sum to N is N times N+1 divided by 2, as shown in the third column. We can see this holds for 1:

[14] It is now usual to start at zero.

$1*(1+1)/2 = 1*2/2 = 2/2 = 1$

And, with a little manipulation, the sum to N+1 will be:

$$(N+1)*((N+1)+1)/2 =$$
$$(N+1)*(N+2)/2 =$$
$$(N*(N+2)+1*(N+2))/2 =$$
$$(N*N+N*2+N+2)/2 =$$
$$(N*N+N+N*2+2)/2 =$$
$$(N*(N+1)+2*(N+1))/2 =$$
$$\mathbf{N} * (\mathbf{N} + \mathbf{1})/\mathbf{2} + (N+1)$$

That is, the sum to N+1 is the sum to N, ie $N*(N+1)/2$, plus N+1.

This style of inductive proof is central to many areas of mathematics, complementing the traditional proof by contradiction. It is also the basis of the fundamental programming technique of *recursion*, where computations over collections are defined in terms of computations over sub-collections, down to an empty collection.

Now, in our example, we have *verified* the hypothesis that the sum from one to N is half the product of N and N+1. But we didn't explain where that hypothesis came from. As with deduction, induction is a process of formalising a property once we have one to reason about. Coming up with an inductive property in the first place, that is identifying hypothetical patterns, is to do with the practice of mathematics.

Induction appears to justify counting without limit. How then might we characterise such seeming infinity?

1.12 Infinity and infinitesimals

In the *Metaphysics*[Aristotle(1933)], Aristotle distinguishes between *potential* and *actualised* infinities:

> Infinity and void and other concepts of this kind are said to 'be' potentially or actually in a different sense from the majority of existing things. e g that which sees, or walks, or is seen. For in these latter cases the predication may sometimes be truly made have without qualification, since "' that which is seen is so called sometimes because it is seen and sometimes because it is capable of being seen: but the Infinite does not exist potentially in the sense that it will ever exist separately in actuality it is separable only in knowledge. (Book IX, Part 6, p447).

Zeno of Elea (495-430 BC) had constructed paradoxes that revolve around being able to divide things indefinitely, and hence into an infinite number of components [Dowden()]. Aristotle dealt with such paradoxes at length in *Physics*. In *Metaphysics*, he deployed the distinction between potential and actual infinities, to contest indefinite division:

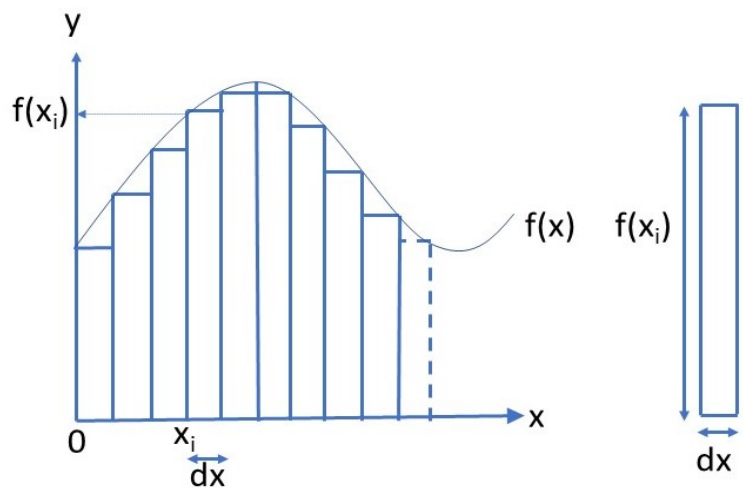

Figure 1.6: Integration by summing areas

> For the fact that the process of division never ceases makes this actuality exist potentially, but not separately. (Book IX, Part 6, p447)

The calculus of Newton and Leibniz raised acute problems of divisabilty [Tiles(1989)]. In particular, integration involves summing the values of a function for successive values of vanishingly small difference. Consider Figure 1.6, which shows the curve for some function `y = f(x)`.

We can approximate the area under the curve by dividing it into rectangles of width `dx` and height $\mathtt{f(x}_i\mathtt{)}$, and summing the areas:

$$\mathtt{f(0)*dx+f(x_1)*dx+f(x_2)*dx+...+f(x_i)*dx...}$$

Of course, for each rectangle, there remains a vaguely triangular shape, between it and the curve, which is not accounted for. These accumulate as an error value. If we can make `dx` smaller then the error becomes smaller. How small can `dx` become?

This boils down to how many values there are in between the start and end values of the integration. If there are an infinite number, then the difference between them is zero. But then we have a paradox of summing an infinity of zeros giving a non-zero result: equivalent to the paradoxes of repeatedly dividing time and space that Zeno explored.

The practical solution is to sum the function for smaller and smaller values of `dx`, until the difference between successive sums is small enough not to be concerned about. That is, the integration converges towards some acceptable value. There are precise analytic solutions for some classes of function, and the resulting values are explained as ‘at the limit’ of the equivalent sums of differences. However, the limit is often treated as if it is at infinity, that is the as if the range of integration is an actualised infinity.

1.13 To infinity, and beyond

The characterisation of infinitesimals and infinity was central to the work of Georg Cantor (1845-1918), which underpinned the crisis in mathematical logic in the late 19th and early 20th centuries. Cantor's work was refined over several decades, so here we will present a summary rather than recapitulating his arguments.Tiles provides a thorough account[Tiles(1989)]. We will go into rather more detail than earlier, as we will need this for subsequent discussion.

First of all, we will distinguish the integers, which have finite representations as sequences of digits, from the *rational* numbers, which have finite representations as the ratios of two integers, but may have infinite expansions if an attempt is made to divide the numerator by the denominator. For example:

```
10/3 ==> 3.3333333333...
```

Those numbers which cannot be represented as the ratio of two integers are called the *real* numbers. Examples are π and e. Such numbers also have infinite expansions, but are expressed as formula for calculating them, for example by summing series of rational numbers.

Now, if unbounded division is acceptable, then there appears to be an infinity of real numbers between two rational numbers. Cantor sought to characterise this *continuum* using set theory.

We will write sets as between the braces { and }, with elements separated by commas, for example the set of even numbers:

{2,4,6,8,10}

The empty set is {}. Note that we consider sets with the same elements as identical, and only include unique members, so {1,2} is regarded as the same as {2,1}, and {1,1} is strictly just {1}.

We distinguish *ordinal* and *cardinal* numbers. Ordinal numbers may be used as indices into ordered sets. So, for the set of even numbers, the 1st element is 2, the 2nd element is 4, the 3rd element is 6, and so on. Here we use the ordinals 1, 2, 3...to select elements of the set, in order. Cardinal numbers are used for sizes of set. So the cardinality of the set of even numbers above is 5.

We further distinguish *finite* and *infinite* sets. For finite sets, like that of even numbers above, the cardinality is also the ordinal number for the last element. But, for an infinite set, the cardinality can't be an ordinal number, as there is no last element by definition.

Cantor was particularly concerned with properties of infinite sets. Now, while an infinite set clearly cannot be constructed in finite time, some infinite sets can be given finite characterisations. In particular, we can specify *algorithms* to *enumerate* some infinite sets, that is to generate successive members. And an algorithm is a finite materialised description.

For example, the infinite set of numbers can be enumerated by starting at 1 and repeatedly adding 1. Note that the previous sentences is a finite description. However, it specifies a potentially infinite computation, which cannot be completed by a materialised system, which must necessarily be finite.

A set is said to be *countable*, or *denumerable*, if its members can be put into one to one correspondence with some set of integers. For example, for our set of five even numbers:

$1 \leftrightarrow 2$
$2 \leftrightarrow 4$
$3 \leftrightarrow 6$
$4 \leftrightarrow 8$
$5 \leftrightarrow 10$

Thus, an enumerated set is in this sense countable, even though it may be infinite, as its members can be put into one to one correspondence with the infinite set of integers. For example, to generate the set of even numbers, successively double the integers. So the set of all even numbers can be put into one to one correspondence with the set of all integers. This may seem counter intuitive as only half of a finite set of integers are even.

Many other infinite sets derived from the integers can be put into one to one correspondence with them. This includes the rational numbers, which we can systematically enumerate as follows:

$1 \leftrightarrow 1/1$
$2 \leftrightarrow 1/2$
$3 \leftrightarrow 2/1$
$4 \leftrightarrow 2/2$
$5 \leftrightarrow 1/3$
$6 \leftrightarrow 3/1$
$7 \leftrightarrow 2/3$
$8 \leftrightarrow 3/2$
$9 \leftrightarrow 3/3$
...

In order to compare properties of infinite sets, Cantor used the Hebrew symbol $\aleph$ (*aleph*) with successive subscripts to denote their cardinalities as *transfinite* numbers. Thus, the set of integers has cardinality $\aleph_0$ (*aleph nought*), as do all countable sets.

However, using a *diagonal* argument, Cantor sought to demonstrate that the real numbers are not enumerable, and hence not countable. First of all, a real number can't be expressed as a ratio of two integers, so must have an infinity of decimal digits. Each of of those decimal digits can be put into one-to-one correspondence with the integers, so we can index them with ordinal numbers. Suppose we could enumerate the real numbers, so the digits of the ith real number d_i were d_i^1, d_i^2, d_i^3 and so on,

$$
\begin{array}{llllll l}
1. & 0. & \mathbf{d_1^1} & d_1^2 & d_1^3 & d_1^4 & ... \\
2. & 0. & d_2^1 & \mathbf{d_2^2} & d_2^3 & d_2^4 & ... \\
3. & 0. & d_3^1 & d_3^2 & \mathbf{d_3^4} & d_3^4 & ... \\
4. & 0. & d_4^1 & d_4^2 & d_4^3 & \mathbf{d_4^4} & ... \\
... & & & & & &
\end{array}
$$

Figure 1.7: Diagonalistion

set	power set	cardinality
{1}	{{},{1}}	2
{1,2}	{{},{1},{2},{1,2}}	4
{1,2,3}	{{},{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}	8
...		

Figure 1.8: Power sets

as in Figure 1.7. We can make a new real number $0.n^1n^2n^3n^4...$ where the first digit n^1 is different to the first digit of the first real d_1^1, the second digit n^2 is different to the second digit of the second real d_2^2, the third digit n^3 is different to the third digit of the third real d_3^3, and so on. This new real can't be in the enumerated sequence, so there must be more real numbers than $\aleph_0$, so the reals are not enumerable. We'll come back to the cardinality of the reals shortly.

Now, as well as sets of integers, it seems legitimate to make sets of sets. For example, we could make the countability of our set of five even numbers explicit as an ordered set of sets of integers and their doubles:

{{1,2},{2,4},{3,6},{4,8},{5,10}}

In particular, given a set, we can construct a set of all of its subsets, known as the *power set*. We start with the empty set {}, and then add all the single elements, pairs of elements, triples of elements and so on, ending with the whole set. See Figure 1.8.

A power set always has greater cardinality, that is more members, than the original set: for a set of N elements, the power set has 2^N elements: see the third column of Figure 1.8. In particular, if the set of integers has cardinality $\aleph_0$, then the power set of integers has cardinality $2^{\aleph_0}$, denoted $\aleph^1$ (*aleph one*). Thus, the powerset of integers is uncountable, as $\aleph_1$ is necessarily bigger than $\aleph_0$. Cantor then demonstrated that the set of real numbers also has cardinality $\aleph_1$.

From an Aristotelian perspective, Cantor's diagonalisation argument is illegitimate, because it presupposes an actualised infinity of real numbers, to arbitrary precision. For a finite set of 'real' numbers, represented to fixed precision, diagonalisation produces a number which is already present.

1.14 Summary

We motivated this chapter with the observation that logic had proved problematic for dialectical materialism, the world view of the communist movement. Dialectical materialism is derived from the materialism of Marx and Engels, so we have briefly followed the development of logic from Aristotle to the period when Marx and Engels were active in the middle and late 19th century. In particular, we have seen how logic, as reasoning, became increasingly separated from dialectics, as premise formulation, as it was placed on a mathematical basis.

We have said little about the world views of the logicians we have considered. These will become far more pertinent in Chapter 2, when we consider Engels' reactions to post-Aristotelian developments in logic, and how this influenced dialectical materialism. In Chapter 3, we will explore the crisis in mathematical logic brought about by the representation of logic in itself, and the responses of logicians.

Chapter 2

Logic and dialectical materialism

2.1 Introduction

> ...to regard the syllogism as merely consisting *of three judgments* is a formalistic view that ignores the relation of the determinations which alone is at issue in the inference. It is altogether a merely subjective reflection that splits the connection of the terms into isolated premises and a conclusion distinct from them:
>
> > All humans are mortal,
> > Gaius is a human
> > Therefore Gaius is mortal.
>
> One is immediately seized by boredom the moment one hears this inference being trotted out, a boredom brought on by the futility of a form that by means of separate propositions gives the illusion of a diversity which is immediately dissolved in the fact itself.
> G. Hegel, *The Science of Logic*[Hegel(2010), p592-3]

We will next turn to the treatment of formal logic in the Marxist tradition. We will say less here about logic itself, and focus on interpretations, and their practical consequences. The whole discussion is framed by dialectics, discussed in earlier chapters, and by wider ideological struggles over its status in the Soviet Union. The latter is lucidly explored in Helena Sheehan's excellent *Marxism and the Philosophy of Science*[Sheehan(1985)].

2.2 Hegel and logic

The idealism of Georg Hegel (1770-1831) is central to European philosophy. Unlike the British tradition, which broke with Aristotle before reviving logic, dialectics is at the heart of Hegel's epistemology. Here we will explore Hegel's approach to logic.

The title of Hegel's book *The Science of Logic* [Hegel(2010)] seems strange from an Aristotelian perspective. As we saw, Aristotle counterpoises dialectics to science, as a form of persuasion rather than an autonomous way of understanding how the world works, and to syllogistic logic, as a source of premises rather than a method for reasoning about them. Still, it is salutary that Hegel does view logic as being amenable to scientific exegesis.

As the quote at the start of this chapter suggests, Hegel was highly sceptical about the value of separating the form of a syllogism from its context of application. That is, the syllogism itself is of no interest. Rather, its importance lies in how it connects particular premises (determinations) to particular conclusions, which in turn depends on prior reasoning to establish relationships amongst premises. As we shall see, this attitude is a recurrent feature of dialectical materialism.

Nonetheless, Hegel does systematically explore the syllogistic forms[1], and considers how they relate to each other, using dialectical transformations. But he deploys no formal notation other than denoting the terms in syllogisms as universal (U), singular (S), or particular (P), a simplification of the earlier practice. Like his predecessors, Hegel uses terse argument and examples.

Hegel's attitude to syllogistic logic is further illuminated in the *Prologue* to *The Science of Logic*. First of all, he noted that science and commonsense had displaced metaphysics, as well as the notion that logic taught one how to think. Nonetheless, logic was retained amongst the sciences 'probably for the sake of a certain formal utility', though 'its shape and content have remained the same throughout a long inherited tradition' (p8).

For Hegel, thought is held in language and that the form of language determines thought, for which German is clearly superior(!) (*Preface to Second Edition*, p12). And logic is central to the human condition:

> So much is logic natural to the human being, is indeed his very *nature*. If we however contrast nature as such, as the realm of the physical, with the realm of the spiritual, then we must say that logic is the supernatural element that permeates all his natural behavior, his ways of sensing, intuiting, desiring, his needs and impulses; and it thereby makes them into something truly human, even though only formally human – makes them into representations and purposes. (p12)

Here, he explicitly disagrees with Aristotle's position:

> 'In so many respects', says Aristotle in the same context, 'is human nature in bondage; but this science, which is not pursued for any utility, is alone free in and for itself, and for this reason it appears not to be a human possession'. (p14, citing Aristotle, *Metaphysics*, 982b)

Hegel argues that, on the contrary, logic is about abstract thought which is why it is taught to young people, with concrete matters coming later. Here, quite apart from

[1]For over 30 pages in the cited translation.

Hegel's unawareness of how privileged such education was, he reflects the distinction between the Trivium and Quadrivium, which must have still been current.

For Hegel, a major benefit of logic lies in its utility as an abbreviation because of its universality. However, because logic shorn of content cannot attain truth, it is hardly surprising that it has been rejected by common sense as barren:

> Regarding the formulas that define the rules of inference which in fact is a principal function of the understanding, however mistaken healthy common sense might be in ignoring that they have their place in cognition where they must be obeyed, and also that they are essential material for rational thought, it has nonetheless come to the equally correct realisation that such formulas are indifferently at the service just as much of error as of sophistry, and that, however truth may be defined, so far as higher truth is concerned, for instance religious truth, they are useless – that in general they have to do only with the correctness of knowledge, not its truth. (p18)

Here, Hegel's position is reminiscent of Hobbes'.

Hegel argues that the separation of form and content is illusory:

> It is soon evident that what in ordinary reflection is, as content, at first separated from the form cannot in fact be in itself formless, devoid of determination (in that case it would be a vacuity, the abstraction of the thing-in-itself); that it rather possesses form in it; indeed that it receives soul and substance from the form alone and that it is this form itself which is transformed into only the semblance of a content, hence also into the semblance of something external to this semblance.(p18-9)

Hegel goes on to considerably expand on this position in the *Introduction*. Herein lie the roots of the rejection of formalism, that is the study of logic independently of content, in dialectical materialism.

2.3 Engels and logic

The revolutionary world view of Karl Marx (1818-1883) and Frederic Engels (1820-1895) is premised on scientific materialism. Their inversion of Hegel's idealism, transforming Hegelian dialectics into a methodology for changing as well as interpreting the world, is discussed above. Here, we will explore how they retained Hegel's rejection of formalism, with profound implications for the practice of logic in actually existing socialism.

Marx wrote practically nothing about dialectics or syllogistic logic. In the *Introduction* to *Grundrisse* [Marx(1973)], he makes a passing analogy between Hegel's Universal-Particular-Individual syllogistic form and production-distribution/exchange-consumption. Stuart Hall views this as satirical[Hall(1973)].

Engels, in contrast, made a number of references to logic in his later work. In *Anti-Dühring*[Engels(1954b)], from 1877, he strongly asserted the subordination of logic in its union with dialectics. First of all, he reasserts Aristotle's position that, once a science is sufficiently specialised, it becomes independent of any wider epistemology. Syllogistic logic, now termed formal, and dialectics, as the laws of thought, are independent of philosophy:

> That which still survives, independently, of all earlier philosophy is the science of thought and its laws — formal logic and dialectics. Everything else is subsumed in the positive science of nature and history. (p40)

This formulation was repeated in the 1880 *Socialism: Utopian and Scientific*[Engels(1951a)], which was extracted from *Anti-Dühring*. And Engels reiterated this in 1886, in *Ludwig Feuerbach and the End of Classical German Philosophy* [Engels(1951b)]:

> For philosophy, which has been expelled from nature and history, there remains only the realm of pure thought, so far as it is left: the theory of the laws of the thought process itself, logic and dialectics. (p363)

Returning to *Anti-Dühring*, Engels argues that mathematics has to borrow axioms from logic, which are 'expressions of the scantiest thought-content' (p60). Following Hegel, he says that that logical axioms alone 'do not cut much ice', and to make progress it is necessary to draw on geometry (p61). His main plaint is that Dühring emphasises the independence of pure mathematics from experience, rather than acknowledging its abstraction from reality (p61).

Engels says that dialectics is more powerful than formal logic:

> Even formal logic is primarily a method of arriving at new results, of advancing from the known to the unknown — and dialectics is the same, only much more eminently so; moreover, since it forces its way beyond the narrow horizon of formal logic, it contains the germ of a more comprehensive view of the world. (p186).

Further, formal logic is only significant for elementary mathematics. In higher mathematics, indeed in all new science, dialectics is needed to advance:

> Elementary mathematics, the mathematics of constant quantities, moves within the confines of formal logic, at any rate on the whole; the mathematics of variables, whose most important part is the infinitesimal calculus, is in essence nothing other than the application of dialectics to mathematical relations. In it, the simple question of proof is definitely pushed into the background, as compared with the manifold application of the method to new spheres of research. But almost all the proofs of higher mathematics, from the first proofs of the differential calculus on, are from the standpoint of elementary mathematics strictly speaking, wrong. And this is necessarily so, when, as happens in this case, an attempt is made to prove by formal logic results obtained in the field of dialectics. (p186-7)

Nonetheless, his characterisation of formal logic involving constants rather than variables suggests a fundamental misunderstanding. This was also Marx's view of mathematics, as shall see below. And Engels betrayed a curious contempt for mathematics:

> The abstract requirement of a mathematician is, however, far from being a compulsory law for the world of reality. (p75)

Finally, in the posthumously published *Dialectics of Nature*[Engels(1954a)], Engels revisited these themes. Once again, dialectical logic is superior to formal logic, because it integrates both analysis and reasoning:

> Dialectical logic, in contrast to the old, merely formal logic, is not, like the latter, content with enumerating the forms of motion of thought, ie the various forms of judgment and conclusion, and placing them side by side without any connection. On the contrary, it derives these forms out of one another, it makes one subordinate to another instead of putting them on an equal level, it develops the higher forms out of the lower. (p296)

Engels acknowledges that, despite developments in natural and historical sciences, formal logic has an essential and knowable quality:

> The number and succession of hypotheses supplanting one another – given the lack of logical and dialectical education among natural scientists – easily gives rise to the idea that we cannot know the essence of things (Haller and Goethe).[189][2] This is not peculiar to natural science since all human knowledge develops in a much twisted curve; and in the historical sciences also, including philosophy, theories displace one another, from which, however, nobody concludes that formal logic, for instance, is nonsense. (p319)

Nonetheless, once again, Engels firmly rejects mathematical abstraction:

> What they [mathematicians and natural scientists] charge Hegel with doing, viz., pushing abstractions to the extreme limit, they do themselves on a far greater scale. They forget that the whole of so-called pure mathematics is concerned with abstractions, that all its magnitudes, strictly speaking, are imaginary, and that all abstractions when pushed to extremes are transformed into nonsense or into their opposite. (p359)

Finally, in his notes for *Dialectics of Nature*, Engels distinguishes between mathematical operations, which could be proved by 'material contemplation', and logical deductions, which could only be proved by deduction:

---

[2]This note refers to poems by Haller and Goethe.

> Calculative reason—calculating machine!—Curious confusion of mathematical operations, which are capable of material demonstration, of proof because they are based on direct, even if abstract, material contemplation, with purely logical ones, which are capable only of proof by deduction, hence are incapable of the positive certainty possessed by mathematical operations—and how many of them wrong! Machine for integration; cf. Andrews' speech, Nature, Sept. 7, 76.317[3] [Engels(1987)]

The nature of infinity had also concerned both Engels and Dühring. Dühring's writings about infinity, were rather tetchily dismissed by Cantor:

> It is peculiar that Dühring himself admits on page 126 of his paper that for the possibility of 'unlimited synthesis' there must be a reason, which he designates as 'for understandable reasons, utterly unknown'. This seems to me a contradiction[Cantor(1996), p917]

Engels, in turn, criticised at length Dühring's notion of infinity. However, Engels confuses countabilty with counting:

> But what of the contradiction of "the counted infinite numerical series"? We shall be in a position to examine this more closely as soon as Herr Dühring has performed for us the clever trick of counting it. [Engels(1954b), p74]

We will return to infinity in Chapter 4.

2.4 Dietzgen, Dialectical Materialism and logic

Joseph Dietzgen (1828-1888) met Marx during the 1848 German revolution, and subsequently became his friend. Self-educated, Engels credited him, in *Ludwig Fuerbach and the end of Classical German Philosophy*, with independently discovering the materialist dialectic:

> And this materialist dialectic, which for years has been our best working tool and our sharpest weapon, was, remarkably enough, discovered not only by us but also, independently of us and even of Hegel, by a German worker, Joseph Dietzgen. (2)[4][Engels(1951b)] (p350-1)

Note the formulation 'materialist dialectic'. This is sounds like a form of dialectic grounded in materialism. But this does not exclude materialism grounding other modes of analysis. So we may admit a non-dialectical materialist science.

[3]Thanks to Slava Gerovitch for this reference, indirectly from [Gerovitch(2002)] (p48) after Yanovskaya.

[4]Note 2 refers to Dietzgen's *The Nature of Human Brainwork, Described by a Manual Worker*[Dietzgen(1902)] from 1869

Burns[Burns(2002)] argues that Dietzgen coined the expression ‘dialectical materialism’, having read Engels on Feurbach, in his *Excursions of a Socialist into the Domain of Epistemology*[Dietzgen(1887)] from 1887 [5]. This apparent point of trivia highlights how slippery these notions are.

‘Dialectical materialism’ sounds like a variety of materialism that is grounded in dialectics, that is one which admits no other forms of analysis. As we shall see, dialectics and science are increasingly conflated in subsequent Marxist philosophy, and it was disputed that one could be separated from the other. Scientific materialism that was not avowedly dialectical was termed *mechanical*.

To return to logic, in his most mature work, *The Positive Outcome of Philosophy*[Dietzgen(1906b)], Dietzgen conflated the premises of formal logic with the deductions made from them. He credits Aristotle with developing syllogistic logic, using Modus Ponems as an example:

> He showed clearly and definitely, excellently and substantially, how logical deductions should be made in order to arrive at positive understanding. All dogs are watchful. My pug-dog is a dog, therefore it is watchful. What can be more evident? (XIII)

But Dietzgen confuses critiquing premises with critiquing the deduction:

> The premise from which he deducted the watchfulness of dogs in general, was handed down by tradition and had been accepted on faith. But was it founded on fact? Could there not be some dogs who lacked the quality of watchfulness, and might not our pug-dog be very unreliable, in spite of all exact deductions? (XIII)

Dietzgen goes on to cite Bacon’s (and Descartes’) rejection of syllogistic logic:

> Both of them were disgusted with Aristotle and with his formal logic, particularly with the subtleties of scholasticism. It did not satisfy this new epoch to found positive understanding on traditional contentions and exact deductions therefrom. (XIII)

As with Hegel, what is needed is a new synthesis of logic and dialectics. Dietzgen’s great strength lies in making explicit the class perspective of dialectics, in particular in an earlier collection of letters to his son *Letters on Logic Especially Democratic-Proletarian Logic*[Dietzgen(1906a)]. Here, he criticises class (ie bourgeoisie) logic for rationalising exploitation as natural, by emphasising difference:

> The enemies of democratic development, in attacking the idea of freedom and equality, point to the manifoldness of nature, the individual differences of men, the distinctions between weak and strong, wise and fools, men and women, and consider it tyranny to attempt to equalise

[5] Thanks to Ken Macleod for referring me to Burns.

> that which nature has made different. They cannot understand that like things may be different and different things alike. They are blinded by their class logic which sees only the differences, but not the unity, not the transfusion of all classes. (Eleventh Letter)

This leads Dietzgen to associate formal logic with class logic, because it is based on reasoning from premises as separated aspects of things. However, he rejects the second law of thought, that contradictions demonstrate invalid premises, in an argument that is explicitly transcendental:

> Class logic teaches that contradictory things cannot exist. According to it, a thing cannot be genuine and false at the same time. This class logic has a narrow conception of existence. It has only observed that there are many different things in nature, but has overlooked the fact that all these things have also a general nature. We, on the other hand, recognise that every thing, every person, is a part of the infinite world and partakes of its general nature, is eternal and perishable, true and untrue, great and small, one sided and many fold, in short contradictory. (Eleventh Letter)

However, Dietzgen has no time for logicians, because, echoing Hegel, they separate logic from its content:

> The formal logicians are as ignorant as they are roguish, when they persist in discussing the intellect or thought in the traditional manner as if they were isolated things, while ignoring the necessary connection of the object of the logical study with the world of experiences. This interconnection leads to an explanation of truth and error, of sense and nonsense, of god and idols, and this is very inopportune for the professors. For this reason this unwelcome problem is handed over to the mystical departments, to metaphysics and religion, so that these venerable pillars of official wisdom may continue their services to the ruling classes. (Fourth Letter)

Early on in *Letters*, Dietzgen deploys an analogy of a potter to reject formal logic, saying that thoughts cannot be separated from actions, nor form from content:

> These adherents of formal logic may be compared to a maker of porcelain dishes who would contend that he was simply paying attention to the form of his dishes, pots, and vases, but that he did not have anything to do with the raw material, while it is evident that he is compelled to form the body in trying to embody forms. These things can be separated by words only, but not by actions. In the same way as body and form, the finite and infinite or so-called celestial spheres, the physical and the metaphysical, are inseparable. (Third Letter)

Here, Dietzgen seems to forget Marx's famous observation in *Capital* Volume 1[Marx(1970)]

> But what distinguishes the worst architect from the best of bees is this, that the architect raises his structure in imagination before he erects it in reality. (p174)

Familiar with Aristotelian syllogistic logic, Dietzgen makes the reasonable criticism that it could not adequately capture multifaceted properties of things:

> Gold and sheet iron are unlike metals, but they have the same metallic likeness. That like things are different and different things alike, that it is everywhere only a question of the degree of difference, of formal differences, this is overlooked by 'formal' logic and by all who seek truth in any logical diagram or fetich, instead of in the eternal, omnipresent existence of the inseparable universe. (Eighth Letter)

Invoking his revolutionary principles, he says in the later *The Positive Outcome of Philosophy*

> The philosophers should abandon their old hobby of trying to prove anything by syllogisms. Nowadays, a case is not substantiated by words, but by facts, by deeds. The sciences are sufficiently equipped, and thus the 'possibility of understanding' is demonstrated beyond a doubt. (XIII)

While Dietzgen is no longer part of the mainstream Marxist canon, he was highly influential before the Bolshevik revolution. In the Introduction to the 1902 combined edition of *The Nature of Brain Work*, *Letters on Logic* and *Positive Outcome of Philosophy*, Anton Pannekoek wrote:

> ... a thorough study of Dietzgen's philosophical writings is an important and indispensable auxiliary for the understanding of the fundamental works of Marx and Engels.[Dietzgen(1906b), p7ff]

We will explore Marxist responses to logic after the Bolshevik Revolution. First, we shall backtrack to key developments in formal logic at the start of the 20^{th} century, the formulation of a common research programme for logic, and the emergence of different logical schools.

2.5 Russell's paradox and Principia Mathematica

At the end of the 19^{th} century, Frege's system was seen as the pinnacle of formal logic, on the high road to formalising mathematics. Alas, this was short lived. In 1902, Bertrand Russell (1872-1970) wrote to Frege[Russell(1967)] identifying a fundamental contradiction at the heart of his system.

In Frege's system, predicates are characterised by sets of items that satisfy some property. Russell's paradox involves observing that predicates may or may not apply to themselves. Equivalently, sets may or may not have themselves as members.

Suppose we write the set:

{peach, pear, plum}

we are drawn to interpret these symbols as fruit names. But they have no necessary interpretations. Maybe they're the names of guinea-pigs. We might equally notice that they all start with the letter 'p' and are in alphabetic order. We could equally well have written:

{parsnip, pea, potato}

Suppose we do interpret these sets as of names of fruit and vegetables, with the common property of being edible. Then, we could make a new set of sets of names of edible things:

{{peach, pear, plum}, {parsnip, pea, potato}}

If we name the sets:

fruit = {peach, pear, plum}
vegetables = {peach, pear, plum}

we can make sets of set names, for example:

edible = {fruit, vegetables}

We may interpret a name as an invitation to replace it with what it stands for, but we are not obliged to do so.

Now, a predicate applying to itself, represented as a set containing itself, sounds like it should lead to an infinite expansion. However, this may be avoided through the use of its name instead of its contents. That is, there are finite representations of apparently infinite constructs.

Clearly, none of these sets contain themselves. Thus, we can make a set of sets that do not include themselves:

not-self = {fruit, vegetables, edible}

In turn, this set does not include itself, so maybe it should:

not-self = {fruit, vegetables, edible, not-self}

But if it is included in itself, then it does include itself and so it shouldn't.

This is not merely an academic exercise: we can demonstrate this on any computer with a folder system. It is commonplace to make links from folders to other folders. Then, one way to make it easier to get from the bottom of a large folder back to the top is to make the last entry a link to the folder itself. Thus, we can make a new folder with links to folders that don't have links to themselves. Should that folder link to itself or not?

Russell's paradox drew into question the whole prospect of formalising mathematics in logic. As Frege responded to Russell [Frege(1967b)]

> The second volume of my *Grundgesetze* is to appear shortly. I shall no doubt have to add an appendix in which your discovery is taken into account. If only I already had the right point of view for that!

Nonetheless, Russell, and his former teacher Alfred North Whitehead (1861-1947), embarked on trying to systematically ground mathematics in a formal logic that was broadly equivalent to Frege's or Cantor's. The three volumes of *Principia Mathematica (PM)* [Whitehead and Russell(1910, 1912, 1913)] appeared between 1910 and 1913, and reconstructed a significant portion of mathematics. However, their 'ramified theory of types', an attempt to formalise restrictions on how sets might be nested, to avoid paradoxes of self reference, was quickly deemed unsuccessful. While *PM* is now little read, not least because of its non-standard notation, it has long been heralded as foundational.

2.6 Hilbert's Programme

Hilbert's Programme, formulated by David Hilbert (1862-1943), framed the conduct of formal logic in the 20^{th} century, and to this day. The 'programme' was not a settled statement of purpose like a manifesto: rather, it was codified from Hilbert's evolving conceptions.

We can see the roots of Hilbert's programme in his 1904 response to Russell's paradox, *On the foundations of logic and arithmetic*[Hilbert(1967)]. He began by characterising leading mathematicians' views on the foundations of mathematics. Leopold Kronecker (1823-1891) was called a *dogmatist* for accepting integers as implicitly existent, without recourse to foundations. Herman van Helmholtz (1821-1894) was termed an *empiricist* for only accepting existence derived from experience, thus ruling out thought experiments as the basis for theories. Elwin Christoffel (1829-1900) was termed an *opportunist*. While he had opposed Kronecker's rejection of irrational numbers, he sought positive reasons for accepting them. Frege's work was acknowledged as foundational, but his system was criticised for a lack of rigour, giving rise to paradoxes of self reference. Here, Hilbert writes:

> *Rather, from the very beginning a major goal of the investigations into the notion of number should be to avoid such contradictions and to clarify these paradoxes.*[6] (p130)

Hilbert went on to characterise Richard Dedekind (1831-1916) as following a *transcendental* method, as he assumed actualised infinities of objects. We will return to this position, which Aristotle had criticised. Finally, Hilbert said that Cantor, while distinguishing consistent and inconsistent sets, gave no criteria for distinguishing them, necessitating *subjectivist* assumptions[7].

[6]Italics in original.

[7]Arguably, Hilbert's potted summaries gave fuel to the Marxist opponents of formal logic, for

Hilbert concludes these remarks by saying that all these difficulties could be overcome by what he called the *axiomatic method*. Further, the paradoxes might be avoided by acknowledging the co-dependence of logic and arithmetic, and concurrently developing their laws.

Formal systems are based on *axioms*, elementary formula which are true for all arguments, and *rules of inference*, for constructing or proving *theorems*, additional true formula, from axioms and other theorems. The Laws of Thought, derived from Aristotle, might be seen as progenitors of axioms. Then, the axiomatic method involves finding sets of independent axioms that, together with appropriate rules, are adequate for elaborating all of some domain. For example, Hilbert had already thoroughly applied this approach to formalising geometry in 1899[Hilbert(1950)].

Hilbert placed great stress on the *consistency* of axiomatic systems, that is that it should not possible to use them to derive contradictions. The impossibility of establishing consistency was to prove key to the later crisis in mathematical logic.

In a subsequent paper, *Axiomatic Thought*[Hilbert(1996a)], Hilbert first referred to a 'programme'[8], developing his objectives in greater detail. As well as consistency, he wished to determine the solvability in principle of arbitrary mathematical questions, to be able to check the results of mathematical activity, and to determine whether or not there might be simpler proofs. Here, Hilbert also returned to the old Aristotelian problem that so exercised Marxists, that of 'the relationship between *content* and *formalism* in mathematics' (p1113).

Notably, the ability to decide whether a not a mathematical question was solvable in a finite number of steps:

> ... goes to the essence of mathematical thought. (p1113)

This is still commonly referred to as the *Entscheidungsproblem* - the decision problem.

There are lots of other decision problems, for example, whether or not two formulae are equivalent, but proof of properties in a finite number of steps was seen as key. Indeed, in demonstrating the impossibility of so doing, Alan Turing laid the basis for digital computing. And, by not taking in the practical significance of Turing's theoretical work, quite openly promoted by John von Neumann, the Soviet Union was late to develop computers.

2.7 Meta-theory and logical schools

Hilbert thought that mathematical abstractions should, and could, be explored independently of any content of application. But Hilbert's programme required that mathematics itself should be subject to mathematical reasoning. That is, there should

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whom terms like *dogmatist*, *empiricist* and *subjectivist* had strong philosophical and political resonances.

[8] *[55] To be sure, the execution of this programme is at present still an unsolved task.* (p1115)

be a *metamathematics*, with mathematics as its contents, but only to establish the consistency of axioms[Hilbert(1996b)].

Now, mathematics is expressed in a language[9], with symbols, syntax and semantics. Thus, mathematics was to become its own *metalanguage*, a language for talking about language.

In the decades following Russell's paradox, three schools of formal logic emerged, reflecting different responses. We may characterise them according to what they accepted as admissible mathematical entities, and what forms of mathematical reasoning about them was admissible, that is, how meta-mathematics might be conducted [Kleene(1952)]. This boiled down to their attitude to infinity.

The *logicists*, exemplified by Frege and Russell, accepted both infinite mathematical entities and infinitory reasoning, like Cantor's. They were Platonists, in that they accorded existence to ideal mathematical entities. Hence, they were idealists.

Next, the *formalists*, like Hilbert, accepted infinite mathematical entities as objects of study, but sought to only use finite reasoning. The existential status of mathematical entities was not of concern.

Finally, the *intuitionists* took the most radical stance. Building on Kronicker, they would only accept finite constructions in both mathematics and meta-mathematics, appealing to mathematical intuition. They also rejected the Law of the Excluded Middle (LEM), that is that it is not possible for something and its negation to be simultaneously true.

There were profound, and sometimes vituperative, disagreements between proponents of these different schools. Nonetheless, their formal systems were actually very closely related. In particular, Ewald[Ewald(1996)] argues that 'formalist' is a misleading term for Hilbert (p1106), as well as one he latterly rejected (p1107). Given Hilbert's insistence on finitary reasoning, his disputes with the intutionists was effectively 'an internal feud among constructivists' (p1116).

2.8 Intuitionism

One response to Cantor was to entirely reject non-finitary methods, and, in effect, real numbers. Thus, in 1886, Kronecker argued that particular results found by manipulating infinite sequences were only admissible if they could be reconstructed without going 'beyond the concept of a *finite* series', using arithmetic over integers [Kronecker(1996), p947].

Subsequently, Brouwer's intuitionism was concerned with the reconstruction of formal logic from a small number of 'intuitive' concepts, using constructive, finite techniques. In 1908, Brouwer published a critique of classical logic, whose themes recur in all his later writing [van Atten and Sundholm(2017)]. First of all, Brouwer thought that logic separate from mathematics led to unfounded conclusions:

[9] Or, perhaps, many slightly different, but heavily connected, languages.

> ... logical argumentations, which, after all, consist in mathematical transformations in the mathematical system that makes [the observations] intelligible, may derive unlikely conclusions from scientifically accepted premises, when carried out independently of observation. (p38)

Against the Formalists, Brouwer thought that logic should be grounded in mathametics, itself grounded in both observation and primordial intuitions of basic truth:

> ... all paradoxes disappear, when one restricts oneself to speaking only of systems that explicitly can be built out of the Ur-intuition, in other words, when instead of letting mathematics presuppose logic, one lets logic presuppose mathematics (p40)

Nonetheless, he suggests that, independently of mathematics, argument by syllogism and contradiction are both acceptable (p27,29). However, he rejects the LEM[10], that it is not possible for both something and its negation to be true (p29ff). This implies that one or other must be true, not allowing for the status of either to be uncertain. Similarly, Brouwer rejects double negation as cancelling, because something not being not true may still leave its status indeterminate.

Brouwer further argues that LEM:

> ... demands that every supposition is either correct or incorrect, mathematically: that of every supposed fitting in a certain way of systems in one another, either the termination or the blockage by impossibility, can be constructed. The question of the validity of the principium tertii exclusi [ie LEM] is thus equivalent to the question concerning the *possibility of unsolvable mathematical problems*. (p42)

Brouwer was happy with LEM in finite cases, as it may be checked exhaustively. He was also happy with its application to infinite cases, so long as they may be constructed by induction. However, Brouwer objected to arguments from entities which are assumed to exist, if their existence cannot be demonstrated. In particular, he rejected assumptions of completed infinities.

In 1921, in an overview of intuitionist set theory[Brouwer(1998a)], Brouwer further objected that the Axiom of Comprehension:

> ... on the basis of which all things with a certain property are joined together into a set ... (p23)

is unreliable because it is not constructive. That is, enunciating a predicate does not guarantee that anything exists that satisfies it. This ability to argue about sets with assumed properties was central to formal logic, but also led to paradoxes like Russell's. It is certainly not possible to construct a set of all sets, which requires a

[10] *tertii exclusi*

completed infinity. So is it legitimate to base an argument on the set of all sets which are not members of themselves?

Bouwer's notion of 'ur-intuition' is plainly idealist. His subjectivism is made explicit in his later work. For example, in *Mathematics, Science and Language* [Brouwer(1998b)] from 1929, he writes:

> Obviously, a causal sequence has no existence other than that of a correlate of a tendency of the human will towards mathematical acting; there is no question of an existence of a causal coherence in the world independent of man. On the contrary, the so-called causal coherence of the world is the outward-acting force of human thought, serving a dark function of the will, making the world more or less defenseless, like the snake that renders its prey powerless through its hypnotic stare or the inkfish through its darkening spray. (p46)

2.9 From the Bolshevik revolution to Menshevising Idealism

It is not possible to do justice to the catastrophic events of the first quarter of the 20^{th} century, killing millions of people and devastating the lives of millions more, without appearing to trivialise them. Nonetheless, from our current perspective, the key outcome of the 1914 to 1919 world wide war was the establishment of the Soviet Union[Carr(1950-1978)], the first state governed by a mass party explicitly committed to materialism.

Following the Civil War (1917-1923), the Soviet focus was on reconstruction, and immediately bettering people's lives. Once it was clear that wider international revolution had stalled, the overwhelming priority was to stabilise and strengthen the Union. For this, and for building towards a Communist future, science was deemed central.

With a shortage of experts, lost through war or emigration, the state could not initially afford to place too much premium on the ideological rectitude of the remaining non-Communist intelligentsia, provided their loyalty was assured. At the same time, the expanding education system, under Communist direction, steadily produced a new generation of 'Red expert' scientists and engineers, who were explicitly committed to Soviet objectives, but who necessarily worked alongside the pre-Revolutionary cohorts.

Mathematics had a central role, and this period saw the growth of two world class mathematical centres, in Moscow and Leningrad. While pure mathematics research continued, the emphasis was on applied mathematics. Overall, formal logic was not prominent.

Nonetheless, the status of formal logic was still contested, even amongst polarisations within the Bolsheviks over the direction that the Soviet Union should take. For example, in 1921, Lenin[Lenin(1965)], in discussing Trotsky's Trade Unions proposal, attacked Bukharin for his neglect of dialectical logic[11]:

> When Comrade Bukharin speaks of "logical" grounds, his whole reasoning shows that he takes—unconsciously, perhaps—the standpoint of formal or scholastic logic, and not of dialectical or Marxist logic.

Still respecting divisions deriving from Aristotle, Lenin saw formal logic as part of a lower level education, and criticised it for not going beyond form:

> Formal logic, which is as far as schools go (and should go, with suitable abridgements for the lower forms), deals with formal definitions, draws on what is most common, or glaring, and stops there.
> ...
> Dialectical logic demands that we should go further.

As with Engels and Dietzgen, Lenin sought dialectical unity of logic with its contents:

> ...dialectical logic holds that "truth is always concrete, never abstract", as the late Plekhanov liked to say after Hegel.
> ...
> His [Bukharin's] approach is one of pure abstraction: he makes no attempt at concrete study,...

After Lenin's death in 1924, the struggles between different Bolshevik factions became acute. These took place against the background of the New Economic Policy (NEP), which, contrary to socialist aspirations, had introduced a substantial market component, to try to accelerate recovery following the exigences of the pragmatic command economy of War Communism. To over simplify, the 'left' faction promoted a speedy transition from the NEP to a planned economy, whereas the 'right' faction sought a slower change. And these struggles were deeply entangled with jockeying for position, and a settling of scores.

As Sheehan[Sheehan(1985)] systematically explores, the relationship of science to philosophy became an important component of these disputes, at both ideological and practical levels. Two positions developed. On the one side, the relevance of dialectics to science was questioned. This was in keeping with the Aristotelian tradition that mature sciences developed autonomously of their dialectical roots. This position was characterised by opponents as mechanist, descended from the mechanical materialism that Marx and Engels had opposed. On the other side, was a renewed emphasis on dialectics. This appeared to be in keeping with the mainstream Marxist tradition, drawing on Hegel.

Initially, ideological struggles within scientific discourse were against the 'right' tendency, characterised rhetorically as Menshevik, Here, the dialecticians under Deborin gained the upper hand against the mechanists. Within mathematics, this led

[11]It is hard to imagine a contemporary head of state arguing about the status of logic in a debate about Trade Unions, let alone during a Civil War.

to an increased repudiation of formal logic. The dialecticians' ascendancy was short lived. During subsequent struggles against the 'left' tendency, an over dependence on dialectics was characterised as idealist. As Friedmann subsequently wrote, in the telling titled *Revolt Against Formalism in the Soviet Union* [Friedmann(1938)]:

> But the critics of mechanism, carried away by their zeal, fell into the opposite extreme of idealism. This required a 'struggle on two fronts' as the theoreticians of the Party call it. Apparently it was faults in practice which here too called attention to the theoretical problems. (p307)

However, the defeat of this tendency did not result in the rehabilitation of logic. Rather, both positions were conflated as *Menshevising idealism*, and formal logic in the Soviet Union increasingly stalled until the early 1950s.

2.10 Menshevising Idealism and logic

The outstanding mathematician Andrei Kolmogorov (1903-1987) exemplified the new generation of Red experts. His 1925 paper *On the principle of the excluded middle*[Kolmogorov(1967)] offered a critique of formalism and intuitionism, but also showed how classical and intuitionist logic might be reconciled, within an intuitionist framework.

Brouwer had argued that it was illegitimate to use both the LEM and transfinite premises to establish finitary results. Kolmogorov demonstrated that such finitary results still stood without recourse to either.

With the intuitionists, Kolmogorov accepted that, in the absence of other evidence, contradictory terms should be regarded as indeterminate. Further, with the intuitionists, he questioned whether transfinite premises had any meaning, even if they might be used to reach finitary results.

Kolmogorov's characterisation of formalism, as uncommitted to choice of premises, sounds akin to Aristotelian dialectics. Noting that a contradiction may be resolved by adding one of the opposed terms as an axiom, he observed that, from a formalist perspective:

> The selection of the formula taken as the new axiom, from each pair of contradictory formulas, is thus subject only to considerations of convenience. (p417)

Further, while formalist logic attributed no meaning to axioms, intutionism was based on axioms that 'express facts given to us' (p417). This is the Hegelian, and also dialectical materialist, position of an integrated, content-full logic. However, Kolmogorov still identified 'mathematical logic' as a distinct component of mathematics:

> ... we do not isolate a special 'mathematical logic' from general logic, but we admit only that the originality of mathematics as a science creates for

> logic special problems, that are investigated by a specialised 'mathematical logic'. (p418)

Here, Kolmogorov did not take an explicitly partisan philosophical, as opposed to mathematical, stance.

However, as Vucinich[Vucinich(1999)] explores, in that same year of 1925, Soviet ideologues explicitly attacked the idealism underpinning Cantor's results, in the philosophical journal *Under the Banner of Marxism*. Vucinich notes that no attempt was made to find a materialist alternative to Cantor (p117-8), and that, while less attention was paid to Hilbert, dialectical logic was counterpoised to formal logic (p118-9). Nonetheless, the attacks were against Cantor's set theory, rather than Soviet mathematicians (p122). The former Leningrad School mathematician G. G. Lorentz[Lorentz(2002)] notes that, in the same period, an algebraic school in Kiev was closed, under the direction of the Ukrainian Communist party, and its scholars dispersed to other centres.

The Red experts Ernst Kolman (1892-1979) and Sofia Yanovskaya (1896-1966) provided a clear statement of the dialectical materialist position in 1931, in *Hegel and Mathematics*, also published in *Under the Banner of Marxism*[Kolman and Yanovskaya(1931)]. Repeatedly citing *Anti During*, and the recently available *Dialectics of Nature*, they emphasised the roots of dialectical materialism in Hegel, while criticising his dialectics.

Kolman and Yanovskaya summarised and rejected the schools of logic we identified above: intuitionism for depending on *a priori* assumptions; logicism for unifying logic and mathematics, and identifying the laws of reason with axioms and theorems; and formalism for treating logic independetly of content. They further reject 'mechanistic empiricists' who see mathematics as part of physics, and Mach for psychologising. Overall:

> Thus none of these philosophical schools, which all grasp one and only one side of reality, is in a position to understand the link between mathematics and practice and its laws of development. Hegel alone gave mathematics a definition such as grasped the essence of the matter, a definition which, quite independently of Hegel's views, is actually profoundly materialist.

Nonetheless, taking a class standpoint on mathematics does not involve rejecting it:

> ... on the contrary it [bourgeois mathematics] must be subjected to a reconstruction, since it represents the material world, albeit one-sidedly and distortedly, nevertheless objectively.

Kolman and Yanovskaya identify attempts to reduce analysis to arithmetic as ultimately leading to:

> ... the well-known paradoxes of set theory which destroyed the whole structure, not only of mathematical but also logical (sic), which had been specially erected for that purpose.

Thus, though for very different reasons, they shared the intuitionist suspicion of Cantor's infinities.

At the time, Yanovskaya was translating and editing Marx's recently rediscovered manuscripts on mathematics, which appeared in 1933[Marx(1983)]. In discussing Taylor's theorem, Marx wrote:

> This leap from *ordinary algebra*, and besides *by means of ordinary algebra*, into the *algebra of variables* is assumed as *un fait accompli*, it is not proved and is prima facie *in contradiction to all the laws* of conventional algebra ... In other words, the starting equation ... is not only *not proved* but indeed knowingly or unknowingly assumes a substitution of *variables* for *constants*, which flies in the face of all the laws of algebra - for algebra, and thus the algebraic binomial, only admits of constants, indeed only two sorts of constants, *known* and *unknown*. The derivation of this equation from algebra therefore appears to rest on a deception. (p117)

It is as if Marx can only accept variables as place holders for actually existing values, rather than for values in general. This muddleheadedness gets to the heart of the Hegelian critique of formalism in logic: that it is illegitimate to remove content from logic, here constants, by abstraction to variables, that may in turn be replaced by arbitrary values. Engels' assertions about variables and abstraction, noted above, may well derive from discussions with Marx.

Of course it is perfectly legitimate to abstract over any constants, or indeed formulae, in an equation, provided such abstraction is made explicit[12]. However, Kolman and Yanovskaya quote Marx without further comment.

In 1932, Kolmogorov explored an approach to intuitionism that was acceptable to dialectical materialism. In *On the Interpretation of Intuitionist Logic*[Kolmogorv(1998)], he reformulated intuitionistic logic as a *calculus of problems*, and showed that it is formally equivalent to Heyting's formalisation of Brouwer's logic. Cunningly, Kolmogorov avoided the critique of form without content, and that of variables generalising constants, by focusing on problems, which are grounded in reality:

> ...intuitionistic logic should be replaced by the calculus of problems, for its objects are in reality not theoretical propositions but rather problems. (p328)

That is:

> ... the concepts 'problem' and 'solution of a problem' can be employed without misunderstanding in all cases that occur in the concrete areas of mathematics. (p329)

[12] This proved a key feature of subsequent mathematical logic, and is now a widely used technique in programming.

This slight of hand substitutes abstraction over concrete problems for abstraction over premises.

Interestingly, Kolmogorov refers to the rules of his logic as 'computational' (p334). The idea of mathematics as computation is central to our conception of materialism.

Thus, intuitionism was the foundation of the Soviet school of constructive logic. Ironically, constructivism is now a core approach for the theory and practice of programming languages.

2.11 Menshevising Idealism and British Marxism

The Communist Party of Great Britain (CPGB) was formed in 1920. A Marxist-Leninist party, the CPGM was explicitly aligned with the Communist Party of the Soviet Union, and promoted the dialectical materialist world view. While it never enjoyed the mass membership, or electoral success, of other Western European communist parties, it had considerable influence in the trades unions.

The CPGB is often seen as a relatively philistine organisation, more focused on day to day struggles than theory. Nonetheless, a significant number of what are now known as public intellectuals were members. These regularly appeared on the radio, and in popular print media, and their books were produced by mainstream, non-aligned publishers, as well as the Party press. Werskey's *The Visible College*[Werskey(1978)] provides a through account of this milieu in the 1930s. Of particular interest to us are the mathematicians Hyman Levy (1889-1975) and David Guest (1911-1938).

Guest, who died fighting with the International Brigades in Spain in 1938, seems to have been the British Marxist who most closely studied the debates around foundations of mathematics. In 1929, in *Mathematical Formalism*[Guest(1939a)], he observed that Polish logicians had found contradictions in Russell and Whiteheads's *Principia Mathenmatica*. He also suggested that Hilbert's desire to formally demonstrate the consistency of mathematics was floored because the only way to do so was to 'produce a set of mental objects satisfying them' (p210). Ultimately, this depended on being able to minimally characterise the finite integers, a deep problem for Hilbert and the intuitionists.

In *The 'Understanding' of the Propositions of Mathematics*, from 1930[Guest(1939b)], Guest critiqued the LEM on the familiar grounds that it is meaningless to simultaneously consider a proposition and its negation. Further, he said that mathematical propositions are like empirical propositions, in that they may be overturned by new evidence. In contrast, properties of concrete instances may be established by carrying out processes, giving as an example trying to establish whether or not a specific number is prime. This seems similar to Kolmogorov's view of logical rules as computational.

Shortly before his death, Guest appeared sympathetic to intuitionism. In a review of E. T. Bell's *Men of Mathematics* [Guest(1939c)], he contrasted the reformism of

Principia Mathenmatica with the ‘revolutionary challenge of the ‘Intuitionists”[13]:

> But what is this but the spirit of dialectics breaking through the hard shell of formal logic, within which so much scientific thought has been imprisoned in the past! (p256).

At the International Congress of the History of Science and Technology, held in London in 1931, the dialectical materialist position on logic was presented by Kolman. In his paper *The Present Crisis in the Mathematical Sciences and General Outlines for Their Reconstruction*[Kolman(1931)], Kolman surveyed what he saw as the current contradictions of mathematics:

> All the profound contradictions of mathematics–the contradiction between the singular and the manifold, between the finite and the infinite, the discrete and the continuous, the accidental and the necessary, the abstract and the concrete, the historical and the logical, the contradiction between theory and practice, between mathematics itself and its logical foundation–all are in reality dialectical contradictions.

While acknowledging some value in Hilbert’s approach, Kolman criticised formal logic for ignoring the historical necessity of the concept of number:

> As for Hilbert’s axiomatics, it is true that it is of use in explaining the logical connections between individual mathematical concepts, but, since it represents a construction post factum it, too, is unable to give a correct picture of development.

And, as before, Kolman dismisses both logical atomism and intuitionism as idealist:

> It is a matter of indifference whether the world of mathematical concepts is regarded as a world of rigid immovable universals, as it is by the logists, or whether it is looked upon as the sphere of action of the free becoming as it is by the intuitionists. ... The most refined finesses of finitism, of metalogic, of mathematical atomistics, merely express the anxiety of bourgeois mathematicians to separate themselves from matter and dialectics by the veil of formal logic, guiding them directly into the desert of scholasticism.

Levy and Guest were both present at the Soviet sessions. The dialectical materialist position on analysing science more widely was presented by Boris Hessen (1893-1936) in a paper on Newton. In his eulogy for Guest, *The Mathematician in the Struggle*[Levy(1939)], Levy reported that the audience seemed nonplussed, but Guest spoke in support of Hessen. In the same article, Levy made explicit the link between the disputes in Marxism over politics and economics, and:

---

[13]‘reformist’ and ‘revolutionary’ again have strong negative and positive resonances, respectively, for Marxists.

> ... struggles and confusions of a highly theoretical and abstract nature. In this, David could bring to bear his very valuable knowledge of mathematical logic... (p157).

Subsequently, Guest seems to have largely abandoned mathematics research for wider educational and Marxist activity. His teaching notes on Marxist philosophy were published posthumously as *A Text Book On Dialectical Materialism*[Guest(1939d)]. In a short section on *Dialectics and Formal Logic*, Guest reiterated the line that formal logic, which he characterised as the 'logic of common sense' (p68), was based on absolute abstractions, and hence was:

> ... unable to grasp the inner process of change, to show its dialectical character. (p68)

To go beyond this, dialectical logic was required: attempts to further develop formal logic lead to metaphysical thinking. Here, Guest cited Engels' association of formal logic with lower mathematics, and dialectical logic with higher mathematics, from *Anti-Dühring*. As we shall see, Turing's work in this period laid the basis precisely for characterising 'the inner process of change', further shrinking the space for dialectics.

In 1934, a several prominent CPGB members contributed to the collection *Aspects of Dialectical Materialism* [Levy et al.(1934)Levy, MacMurray, Fox, Arnott, Bernal, and Carritt]. In his paper *A Scientific Worker Looks at Dialectical Materialism* [Levy(1934)], Levy summarised the orthodox account but ended it with an Aristotelian circumscription of dialectics:

> ... the so-called laws of the dialectic ... appear to add little or nothing to the detailed methods of analysis that scientific workers have produced ... In a sense, they cannot be expected to add anything to these, for they profess to stand above science ... For science, therefore, it [dialectical materialism] is an *interpretative* method rather than a method of investigation.[Levy(1934)] (p30)

In the same collection, the X-ray chrystallographer J. D. Bernal (1901-1971) also sought to distance dialectics from science, echoing Levy in his paper *Dialectical Materialism*[Bernal(1934)]:

> It [dialectical materialism] is not a critique of science; it does not claim to be a substitute either for experimental method or for the logical proof of laws or theories, but it does in a very important way supplement science by providing a definite method of coordinating the larger groups of special sciences and in pointing the way to new experiment and discovery (p98)

2.12 Summary

We have ended this chapter in the early 1930s, when formal logic seemed frustrated by self-referential paradoxes, and an explicit materialism became the orthodoxy within an ascendant Marxism, now the defining philosophy of the burgeoning Soviet Union. In the next chapter, we will explore the further crisis in formal logic brought on by new paradoxes of self reference within meta-mathematics, and how these yet offer a positive way forward for materialism, beyond the limitations alleged by pessimistic scientists and philosophers, and prescribed by dialecticians.

Chapter 3

The crisis in logic and the apotheosis of anti-formalism

3.1 Introduction

Hilbert's programme may be conveniently summarised as seeking to establish whether or not the formalisation of arithmetic used in Russell and Whitehead's *Principia Mathematica*:

- is *consistent*, ie it is not possible to prove that both a formula and its negation are theorems;
- is *complete*, ie there are no theorems which cannot be proved to be so;
- has a *decision procedure*, ie a terminating mechanical procedure or algorithm, to establish whether or not an arbitrary formula is a theorem.

Russell's paradox had already strongly suggested that mathematics could not be consistent. Subsequently, as we shall next explore, Gödel established that mathematics could not be complete, and Turing that mathematics could not have a decision procedure for theoremhood.

As we shall see, these results further convinced the dialectical materialists of the limitations of formal logic, and the rectitude of not studying it. However, Turing's work gave key insights into how to construct general purpose computers, and the formalisms that the meta-mathematicians deployed proved foundational for programming languages. The development of practical computers underpinned the mid to late 1940s Allied atomic bomb programmes. This led to the reversal of Soviet policy in an effort to catch up, and the dialectical materialist opposition to logic was conveniently elided.

T (true)	1
F (false)	2
$=$	3
$\wedge$ (and)	4
$\vee$ (or)	5
$\neg$ (not)	6
$\Rightarrow$ (implies)	7
(	8
)	9
a	10
b	11
c	12
...	

Figure 3.1: Codes for symbols

3.2 Encoding formulae

The key to meta-mathematics, where mathematics is used to quite literally talk about itself, lies in finding a mathematical representation, or *encoding*, of formulae, for manipulation by other formulae. In Chapter 1 we saw how the form of a language may be defined in terms of its symbols and syntax. If we assign numbers to symbols, we can turn a formula into a composite number, and then decompose it back into its symbols, using arithmetic.

For example suppose we give symbols the codes shown in Figure 3.1. Consider the formula for the syllogism Modus Ponems:

$$(a \wedge (a \Rightarrow b)) \Rightarrow b$$

that is 'a', and 'a' implies 'b', implies 'b'. Writing down the codes gives:

```
8 10 4 8 10 7 11 9 9 7 11
```

Then, we can turn this into a single number by starting with 0, and repeatedly multiplying by 100 and adding the code for each symbol. Our example gives:

```
0*100 + 8 = 8
8*100 + 10 = 810
810*11 + 4 = 81004
81004*100 + 8 = 8100408
8100408*100 + 10 = 810040810
810040810*100 + 7 = 81004081007
81004081007*100 + 11 = 8100408100711
8100408100711*100 + 9 = 810040810071109
810040810071109*100 + 9 = 81004081007110909
```

```
81004081007110907*100 + 7 = 8100408100711090907
8100408100711090907*100 + 11 = 810040810071109090711
```

We can then get back to the original symbols by repeatedly dividing by 100 and taking the remainder:

```
810040810071109090711/100 = 8100408100711090907 remainder 11
8100408100711090907/100 = 81004081007110909 remainder 7
81004081007110909/100 = 810040810071109 remainder 9
810040810071109/100 = 8100408100711 remainder 9
8100408100711/100 = 8100408107 remainder 11
81004081007/100 = 810040810 remainder 7
810040810/100 = 8100408 remainder 10
8100408/100 = 81004 remainder 8
81004/100 = 810 remainder 4
810/100 = 8 remainder 10
8/100 = 0 remainder 8
```

Given the number for a formula, arithmetic can be used to check if is is well formed. For example, if we see a '(', we expect there to be a matching ')'. To check this, we look for an 8 followed by a matching 9, surrounding a well formed sequence of other symbols[1], .

Note that this encoding is limited to 99 symbols. Suppose we had a symbol with number 101. Then repeating it - 101 101 - would give 101*100+101 which is 10201. Dividing by 100 then gives 102 remainder 1, so the encoding is not unique.

3.3 Gödel and completeness

In a seminal 1931 paper, Kurt Gödel (1906-1978) used an encoding of number theoretic predicate calculus to demonstrate that it was incomplete, that is there are theorems which cannot be proved[Gödel(1967)]. Kleene[Kleene(1952)] gives a thorough account.

Rather than using a simple multiplication technique, Gödel assigned a prime number to each position in a symbol sequence. He then accumulated the codes for symbols by multiplying together the prime numbers, with each raised to the power of the code at its position. For example,

$$(a \wedge (a \Rightarrow b)) \Rightarrow b == \texttt{8 10 4 8 10 7 11 9 9 7 11}$$

would be encoded as:

$$2^8 * 3^{10} * 5^4 * 7^8 * 11^{10} * 13^7 * 17^{11} * 19^9 * 23^9 * 29^7 * 31^{11}$$

[1]that might include other matching '('s and ')'s.

This encoding works for an arbitrary number of symbols. Decoding then involves a technique called *prime factorisation* which enables the exponents of all the prime factors in a number to be found. Of course, this encoding gives unimaginably large numbers, but it was not Gödel's intention to work directly with them.

Gödel wanted to encode proofs, that is sequences of formulae where each is an axiom or a theorem, or follows from an axiom or theorem by application of a rule of inference. He showed how to construct functions that would pull these *Gödel numbers* apart to check, not just that they were well formed, but that they corresponded to formula for valid proofs. This works for establishing whether or a sequence of formula is or is not a proof, but is quite different to checking whether or not a proof exists for an arbitrary formula.

On the assumption that it was possible to write a function to tell whether or not a formula was a theorem, Gödel constructed a paradox using self reference, reminiscent of Russell's paradox. He showed how to make a formula with, say, Gödel number *N*, that said, in effect:

> *the formula with Gödel number* N *is not a theorem.*

Now, if this formula is a theorem, then what it asserts is true, so it can't be a theorem. And if it isn't a theorem, then what it asserts is false, so it must be a theorem. That is, assuming that there could be a function to check whether or not a formula was a theorem rendered the system inconsistent. And omitting the assumption rendered it incomplete.

Logicians who accept the LEM are agreed that preferring consistency to completeness is the safest course. For an incomplete system, there will be arbitrary formula whose status as theorems we cannot guarantee to determine. If they are simply added to the system as axioms, then it might possible to use them to prove contradictions. However, for a consistent system, once we prove that a formula is a theorem, we know that we cannot prove its contrary.

3.4 Turing and termination

Five years later, in a further seminal paper, Alan Turing (1912-1954) showed that the third requirement of Hilbert's programme could not be satisfied[Turing(1937a)]. That is, it is not possible to construct a a terminating mechanical procedure for deciding whether or not a formula is a theorem.

Turing's approach was very different to that of the mathematical logicians we have considered above. Rather than using a formal system derived, say, from predicate calculus applied to set or number theory, Turing considered how people solve problems by hand. He speculated about someone using a pencil and squared paper, writing down a problem in the squares, letter by letter, and then working backwards and forwards, changing squares, and writing in new ones.

Turing generalised this conception to what is now known as a *Turing machine (TM)*. A TM has a finite linear tape of cells. Each cell may hold a symbol. There

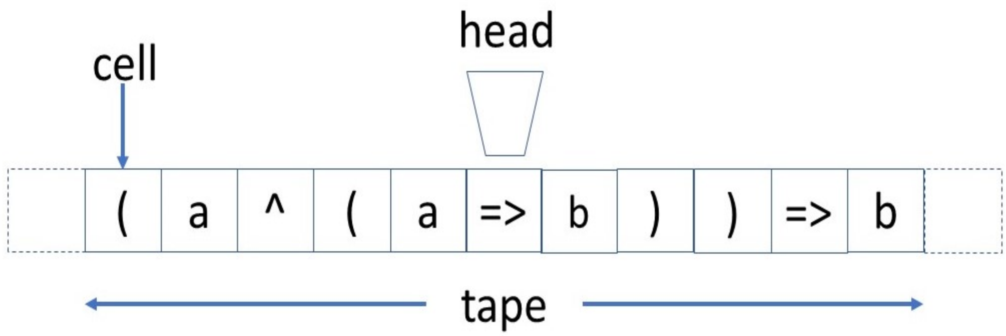

Figure 3.2: Turing machine

is a reading head that can inspect and change cells. The head is positioned over the 'current' cell. For example, Figure 3.2 shows a stylised TM with Modus Ponems on the tape, symbol by symbol.

The tape may be moved to the left or the right, one cell at a time. New empty cells are added when either end of the tape is reached. Thus, the tape may grow to be arbitrarily long, but, at any stage, it is always bounded, that is, it is always of finite length.

A TM executes by repeatedly inspecting and modifying the tape, one cell at a time. The propensity of the machine, that is how the current symbol determines what it will do next, in the light of what it has done previously, is called the current *state*.

A TM is controlled by a set of instructions with five components, known as quintuplets. Each instruction says:

- $state_{old}$: if the machine is in this old state,
- $symbol_{old}$: and this old symbol is under the head,
- $state_{new}$: then change to this new state,
- $symbol_{new}$: change the cell under the head to this new symbol,
- *direction*: and move the tape one cell in this direction, ie left or right, or halt,

The machine is set up with the instructions in a control unit. This repeatedly looks for an instruction whose old_{state} and old_{symbol} match the current state and symbol under the head, which is then carried out. If there is no such instruction, the computation fails.

For example, consider a TM that negates a binary number. It takes a tape of 0s and 1s between * and *, and turns 0s to 1s, and 1s to 0s. See Figure 3.3. This says:

1. in state 0 reading a *, change to state 1, write a * and move right;

	$state_{old}$	$symb_{old}$	$state_{new}$	$symb_{new}$	dir
1.	0	*	1	*	R
2.	1	0	1	1	R
3.	1	1	1	0	R
4.	1	*	1	*	H

Figure 3.3: Binary inversion TM

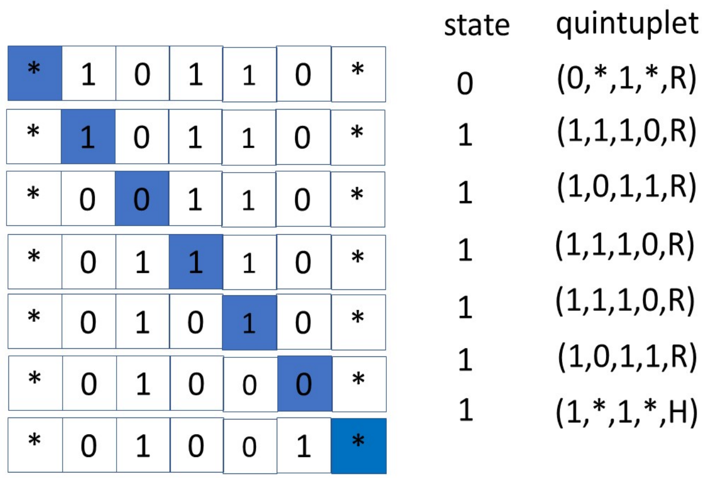

Figure 3.4: Binary inversion TM sequence

2. in state 1 reading a 0, stay in state 1, write a 1 and move right;

3. in state 1 reading a 1, stay in state 1, write a 0 and move right;

4. in state 1 reading a *, stay in state 1, write a * and halt.

For example, Figure 3.4 shows the stages in inverting 10110 to 01001. The current cell is coloured blue.

Elaborate TMs may be constructed from the basic operations of searching for, changing, and copying sequences of symbols.

Turing's key insight was that a TM tape could hold any symbol sequence, including that for a set of TM instructions. Indeed, it is possible to construct a *Universal* TM (UTM) that will execute an arbitrary TM, held on its tape, symbol by symbol, with appropriate data.

Figure 3.5 shows one way to represent an arbitrary TM and its data on a tape for execution by a UTM[2]:

- Q_1 to Q_M are the quintuplets for the TM we wish to simulate, where each quintuplet is in a standard layout as above;

[2]Note that the blocks for old state and quintuplets represent more than one cell

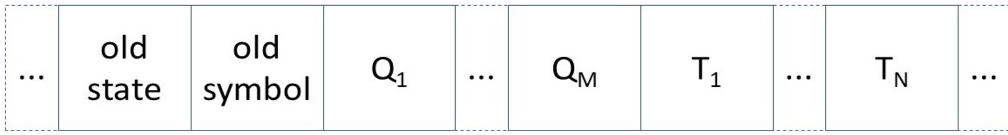

Figure 3.5: Universal Turing machine tape

- T_1 to T_N is the initial tape;
- *old state* is the current state of the simulated TM;
- *old symbol* is the symbol under the head of the simulated TM. It has been replaced in T_1 to T_N by some unique symbol.

The UTM proceeds as follows. It searches the quintuplets Q_i for one that starts with the old state and old symbol. If it can't find one, it stops. Otherwise, it copies the new state over the old state, finds the current symbol market in the tape cells T_i, overwrites the marker with the new symbol, moves left or right by one cell depending on the direction in the quintuplet, remembers the current symbol, overwrites it with the marker, moves left to the old symbol and overwrites it with the current symbol. If the simulated TM goes left of the left most tape cell T_1, then all the tape cells are shifted one cell to the right, in effect adding a new leftmost cell.

Asking whether or not an arbitrary TM will ever terminate, Turing constructed a paradox as follows. Suppose we have the symbols for a 'test' TM and its data on a tape. Let's assume that we can build a 'halting' TM that will inspect the tape, and stop in one state if the 'test' TM halts on its data, and in another state if the 'test' TM doesn't halt on its data.

It is easy to write a TM that doesn't halt. It starts on a blank cell, writes a 0 and moves one cell to the right. If there isn't a cell to the right, a new one is added. In effect, this TM will 'loop' forever, extending the tape to infinity, without ever reaching it.

We can modify the 'halting' TM, to halt in one state if the 'test' TM doesn't halt on its data, and to execute the loop instruction if the 'test' TM does halt on its data.

Suppose we apply the modified 'halting' TM to itself and some tape. If it doesn't halt on itself, then it halts, and if it does halt on itself, then it goes into the loop, and doesn't halt. Turing concluded that it wasn't possible to construct a TM to tell if an arbitrary TM halts, that is, the TM *halting problem* is *undecidable.*

This result falls out of Turing's more elaborate argument using Cantor diagonalisation in considering TMs that generate real numbers. While above we critiqued the whole notion of diagonalisation, as involving completed infinities, this does not invalidate the paradox at the heart of the halting problem..

3.5 The Church-Turing thesis

Several other formalisms for meta-mathematics were explored in the same period that Gödel and Turing were working. Along traditional lines, Stephen Kleene (1909-1994) developed recursive function theory, which inverted Peano's induction into a form for defining functions over number or sets [Kleene(1936a)].

In contrast, Moses Schönfinkel (1888-1942) had developed combinatory logic in 1924[Schönfinkel(1967)]. This was refined by Haskell Curry (1900-1982) in 1929 [Curry(2017)], and led to the highly influential work by Alonzo Church (1903-1995) on λ calculus, from 1932 [Church(1932)].

These systems seem particularly problematic for adherents of dialectical materialism. Schönfinkel's and Curry's combinatory logics are built from operators whose properties are defined by how they interact with other operators, by eliminating or duplicating them. And Church's λ calculus is one of pure abstraction, with rules for combining abstractions through substitution. Neither makes any reference to concrete values.

Nonetheless, a key property of these systems is that they are all capable of representing logic and arithmetic, so they can all capture the notions of theorem and proof at the heart of Hilbert's programme. This is the root of the *Church-Turing thesis*, that all systems for performing calculations on numbers are equivalent, in the sense that any may be translated into any other. That is, there is nothing that can be expressed in one system that cannot be expressed in any other system.

Recursive function theory, λ calculus, and Turing machines were all demonstrated to be equivalent soon after they were developed[Kleene(1936b), Turing(1937b)]. Subsequently, they were shown to be equivalent to von Neuman machines, that is generalised digital computers, and, hence, to arbitrary, programming languages. Formalisations of analogue computing, and of quantum computing [Deutch(1985)], have also been shown to satisfy the Church-Turing thesis.

Note that this is a speculative *thesis* not a provable *theory*. We do not know how many different equivalent systems there might be. Nonetheless, it is a falsifiable thesis, as it may yet be proved that some new system has different properties to those that are currently known, and so there is no mutual translation. That is, the Church-Turing thesis is subject to experimental investigation, as part of empirical normal science. Indeed, we will argue subsequently that it is central to a wider understanding of reality.

At heart, all these systems enable computation. This is explicit in Turing machines, where the machine manipulates the data on the tape. For combinators and λ calculus, where there is no obvious separation of instructions and data, rules are applied to formula until they cannot be simplified any further.

Herein lies the fundamental difference between Turing machines and other formalisms. A set of TM instructions is a blueprint for building an actual machine which, like a digital computer, will run autonomously on its data. In contrast, other formal systems require a human being, or an interpreting device, to carry out their rules. We think that this is a strong argument for the primacy of Turing machines in elaborating a materialism, and will return to this subsequently.

3.6 British dialectical materialist responses

For the dialectical materialists, Gödel's and Turing's results were further demonstrations of the limitations of logic without dialectics. Writing in 1938, the Communist mathematician Alister Watson (1908-1982)[3] thought that all the paradoxes:

> *... merely express in one way or another the well-known difficulties which arise when we attempt to treat an infinite process as completed.* [Watson(1938)] (p450)

Watson is referring to Aristotle's distinction between potential and actualised infinity. We agree with Watson that the assumption of actualised infinities make, for example, Cantor diagonalisation problematic. However, Watson would be wrong to identify Turing machine tapes as completed infinities. In any case, Turing's result of the undecidability of the halting problem may be established without diagonalisation.

Watson's scepticism about meta-mathematics is clear:

> The attempts which have been made in the subject of the Foundations of Mathematics to justify or condemn mathematical arguments taken in the abstract, have given rise to a host of confusions, from which it has taken the most immense labour to escape. (p451)

He does not directly deploy the dialectical materialist critique of formal logic. However, in a footnote to this passage, he says that he was writing in opposition to Dedekind's claim that the foundations of mathematics did not require any mention of 'measurable quantities" This is reminiscent of the Marxist objection to variables that are not derived from known quantities.

In 1938, in *The Marxist Philosophy and the Sciences* [Haldane(1938)], the evolutionary biologist J. B. S. Haldane (1892-1964) observed:

> On the whole we may take it that Marxists are rather sceptical of the more ambitious logical theories. For example, the system of Russell and Whitehead, in the *Principia Mathematica* is doubtless true, or largely true, if sufficiently sharp classification is possible.

That is, the truth or falsity of this logical system depends on the ability to elaborate concrete 'existents', 'relations', and 'propositions' arranged in classes, such that further classes may be abstracted. However, Haldane doubts the possibility of setting up such ontologies:

[3] Watson introduced Turing to Ludwig Wittgenstein, and was alleged to have been a Soviet spy: `https://en.wikipedia.org/wiki/Alister_Watson`.

> ...it is probable that too great an emphasis has been attached to logical systems which will only work for material that has certain highly abstract properties, which are rather less frequently and much less completely exemplified in the real world than logicians would like to think. [Haldane(1938)]

In 1939, in the Preface to the English translation of *Dialectics of Nature*, Haldane wrote, of the mathematicians' claims to have removed contradictions in mathematics, noted by Engels sixty years earlier:

> Actually they have only pushed the contradictions into the background, where they remain in the field of mathematical logic. Not only has every effort to deduce all mathematics from a set of axioms, and rules for applying them, failed, but Gödel has proved that they must. [Haldane(1939)]

This is missing the key point that, while Gödel's results circumscribe what can be proved, they tell us nothing about establishing properties of individual formula. Indeed, automated techniques based on axiomatic systems are proving increasingly applicable to substantial real world problems, like digital computer design.

Like Watson, Haldane does not use use the language of dialectical materialism directly. Nonetheless, his implication, as for his comments on Gödel's results, is that logic is not adequate for concrete reality, for which dialectics is required.

3.7 Soviet logic after Menshevising Idealism

The fortunes of Soviet mathematics, from the 1930s onwards, have been widely documented, though not always dispassionately, for example in Vucinich[Vucinich(1999)], Lorentz[Lorentz(2002)], Seneta[Seneta(2004)], and Kutateldze[Kutateladze(2011)]. Mathias's egregiously titled 'Logic and Terror' [Mathias(1987)] discusses Soviet logic to 1950. Anellis [Anellis(1994)] criticises Mathias for relying on the 'polemical and prejudiced account' in Philipov[Philipov(1952)]. Nonetheless, Mathias contains telling, if poorly referenced, quotes from period publications:

> 'Formal logic is always a most trustworthy weapon in the hands of the predominant exploiting classes. a bastion of religion and obscurantism' (from a 1934 work on Dialectical and Historical Materialism)
>
> 'the laws of formal logic are opposed to the law of dialectical logic. formal logic is empty, poor, abstract, for the laws and categories which it sets up do not correspond to objective reality'. (Concise philosophical Dictionary, 1940) (p7)

including the 1936 *Large Soviet Encyclopedia* (1936):

> 'formal logic is a metaphysical form of thinking . . .the lowest stage in the development of human knowledge, replaced by dialectic as the highest form of thinking'.
>
> 'Formal logic, as we have seen, is not included in dialectic, but is displaced refuted and overcome by it'.
>
> '...the anti-Leninist deviations in the All-Union Communist Party (VKP(b)). Formal logic thinking is a characteristic trait of Menshevism frequently noted by Lenin who levelled devastating dialectical criticisms at the Menshevik formal-logical deductions of syllogisms and sophisms'. (p7)

While the broad study of logic was curtailed, 'Red experts' like Kolmogorov and Yanovskaya were still able to explore and teach logic, and had access to western research. For example, Yanovskaya started to teach mathematical logic in 1936 at Moscow State University, and, in 1943, was appointed Director of the Seminar in Mathematical Logic[Kilakos(2019), p52].

The fortunes of logic in the Soviet Union were restored after World War Two, though not without continued fierce dispute over its relationship to dialectics, as we shall discuss below.

3.8 Digital computers

The key development of the 20th century may well prove to have been that of digital computers. Certainly, early 21st century life would be pretty well unthinkable without them. The history of computers is again thoroughly documented. Here, we will focus on salient aspects, albeit very briefly.

The first modern mechanical calculators are commonly attributed to Blaise Pascal (1623-1662), whose mid 17th century 'Pascaline' adding machine was based on meshed cogwheels. For this device, addends were dialed onto wheels, digit by digit.

Gottfried Leibniz (1646-1716) then developed a mechanical multiplier at the end of the 18th century, based again on cogwheels, augmented with a toothed drum to change the multiplier. The Leibniz device was the basis of mass use mechanical calculators, until the development of transistorised machines in the early 1960s.

In the 1820s, Charles Babbage (1791-1871) designed and built a mechanical 'Difference Engine', which could generate tables of functions. This was a substantial cogwheel based device, and few were subsequently constructed. Babbage also designed, but never completed, a general purpose mechanical 'Analytic Engine', which is recognised as equivalent to a modern stored program computer. The design was near to the limits of contemporary mechanical engineering, and it is still not clear whether its construction is feasible.

After Herman Hollerith (1860-1929) developed electro-mechanical tabulating machines based on punch cards, in the late 19th century, these became standard data processing equipment for governments and large corporations, until well after the development of computers. In particular, *International Business Machines (IBM)* became an internationally dominant card punch equipment manufacturer, offering increasingly sophisticated devices. For example, the IBM 801, from 1931 could multiply two numbers from, and record the answer on, a single punch card.

The Second World War, unprecedented in mass cruelty and immiseration, hastened the development of modern computers[Dyson(2012)]. The Harvard Mark 1, built by IBM as the Automatic Sequence Controlled Calculator (ASCC), was designed by Howard Aitken (1900-1973) in 1937, and first ran in 1944. It was electromechanical, and could store data, but lacked the capability to store programs[4]. Instructions were encoded on punched paper tape to control a linear sequence of operations. Looping programs were accomplished by repeating the tape, and branching by changing tapes. The Mark 1 was used by John von Neumann (1903-1957) to perform calculations for the Manhattan Project, developing the first atomic bombs.

The Electronic Numerical Integrator and Computer (ENIAC), amongst the first all electronic general purpose computers, was completed in 1945. Much faster than the Harvard Mark 1, it was programmed by plugging components together in appropriate configurations. ENIAC was used, amongst other things, in the calculations for the first hydrogen bombs.

von Neumann had worked on set theory for his 1925 PhD, and had communicated with Gödel in the early 1930s about the incompleteness results. Strongly influenced by Turing's ideas about the Universal TM, where data and instructions share the same memory, von Neumann included this design, now known as the von Neumann architecture, in the highly influential *First Draft of a Report on the EDVAC*[5] [Von Neumann(1945)]. The report was circulated freely, influencing early digital computer development worldwide. Mid-century, there were numerous firms developing and selling von Neumann architecture computers, particularly in the USA and the UK.

Thus, while dialectical materialists, and Western European philosophers alike, saw the failure of Hilbert's programme as limiting the reach of mathematics, nonetheless it had profound and very long lasting influences. Furthermore, the 1930s formalisms developed by mathematical logicians, especially recursive function theory and λ calculus, have long been the basis of the semantics, and the design, of practical programming languages.

3.9 Analogue computers

It is important to contrast digital computers, which work with discrete, integer representations, with analogue computers, which are intended to work with continuous, real representations[Cockshott et al.(2012)Cockshott, Mackenzie, and Michaelson]. The latter depend on inverting the modelling of physical processes by equations. Like dig-

[4] The separation of program and data memory is still termed the Harvard architecture.

[5] Electronic Discrete Variable Automatic Computer

ital computers, analogue computers may be mechanical, based on wheels, pulleys and sliders, or electronic, based on operational amplifiers. These are set up to reflect an equation to be solved. As the inputs are continuously varied, the outputs may be measured.

Analogue computers are far faster than digital computers, and may even be faster than the systems they simulate. However, they have major drawbacks. To provide an input to an analogue computer, either a dial must be set, or a digital input of fixed precision converted to analogue form. Similarly, to to gain information from an analogue computer, its output must be measured. This is only achievable to the precision of the measuring device, typically a dial, meter or visual trace. Gaining a more precise output requires analogue to digital conversion. Further, while analogue computers will solve the general form of some equation, they are restricted to that equation, and multiple units are required for more complex problems based on multiple equations.

Major uses of mechanical analogue computers included gunnery control systems for battleships, from the early 20th century onwards, which had to take take account in real time of a ship's, and its target's, motion. Large general purpose mechanical 'differential analysers' were built in the late 1920s to solve differential equations by integration, to designs by Vanever Bush (1890-1974).

Haldane was aware of the potential of these devices. Discussing Hartree's work on substantial industrial problems, including steering an aircraft blind, he wrote:

> ... a stage is reached in the process where this particular machine becomes necessary. That, I think, is a development of considerable interest. It suggests that possibly we are the beginning of a new epoch of mathematics, based on a much more extensive use of practical methods than is yet considered respectable in most universities. [Haldane(1938), p53]

Since the mid 20th century, analogue computers have been entirely supplanted by digital computers. We will return to questions of real numbers and measurement below.

3.10 Cybernetics

Cybernetics, the study of 'control and communication in the animal and the machine', was an area of major activity after WW2, alongside the development of computers. Norbert Wiener's (1894-1964) highly influential book [Wiener(1948)] set out the key principles, drawing on information theory, statistical mechanics, and Pavlovian behavioural psychology, to elaborate how machines might learn through feedback, to optimise activities against observed outcomes.

Like Turing's, Wiener's work was based on abstracting from human behaviour, but at the level of the nervous system rather than higher cognition. During WW2, investigating how to automate fire control systems for anti-aircraft artillery, he noted the central requirement to:

> ... usurp a specifically human function - in the first case, the execution of a complicated pattern on computation, and in the second, the forecasting of the future. (p6)

Wiener observed the importance of feedback in governing activity that involved continuously predicting future behaviour, for example in steering a craft, or following a moving target, and that this was carried out autonomously (pp6-8). He, and his collaborators, enunciated the:

> ... essential unity of problems centring about communication, control, and statistical mechanics, whether in the machine or living tissue. (p11).

Wiener also acknowledged the strong influence of mathematical logic on cybernetics. He observes that, for both formalists and intuitionists:

> ... the development of a mathematico-logical theory is subject to the same sort of restrictions as those that limit the performance of a computing machine. As we shall see later, it is even possible to interpret in this way the paradoxes of Cantor and of Russell. (p13)

That is, Wiener saw the paradoxes as limiting human reasoning in general, not just mathematics, because brains are machines. Citing Turing, he wrote:

> ... the study of logic must reduce to the study of the logical machine, whether nervous or mechanical, with all its non-removable limitations and imperfections. (p125)

Nonetheless, Wiener saw information as neither matter nor energy. Hence, we strongly contest his claim that:

> No materialism which does not admit this can survive at the present day. (p132)

because information must be embodied.

Though Wiener was not a Marxist, he had close connections with Levy, who he knew from when he had studied with Russell before WW2, and with Haldane and Bernal. On a visit to Haldane in 1947, he spent time with Turing, then working on the ACE computer at the National Physical Laboratory, and the Cambridge team developing the EDSAC computer (p23). Clearly, British Marxists knew of computers in this period.

Wiener was well aware of the social implications of cybernetics and computing. He thought that:

> The modern industrial revolution is ... bound to devalue the human brain, at least in its simpler and more routine decisions. (p27)

However, ideological objections to cybernetics proved a major barrier to the development of Soviet computers, as Gerovitch[Gerovitch(2002)] recounts. To clarify this, we need to make an apparent segue sideways, and consider Soviet thinking about linguistics

3.11 Linguistics

For much of the 19th century, philology dominated linguistics. This sought to trace languages back to their origins, by identifying common roots in words from different languages. Much of this work was distorted by concerns with establishing the historical primacy of contemporary national groupings.

Modern linguistics was founded by Ferdinand de Saussure (1857-1913), whose posthumous *Cours de linguistique générale* [de Saussure(1959)] was published in 1916. Saussure distinguished between language (*langue*), and speaking (*parole*):

> separating: (1) what is social from what is individual; and (2) what is essential from what is accessory and more or less accidental. (p14)

Saussure further distinguished the use of a relatively unchanging language by contemporary speakers from how language changes in time, as users and usages changes:

> Everything that relates to the static side of our science is synchronic; everything that has to do with evolution is diachronic. Similarly, *synchrony* and *diachrony* designate respectively a language-state and an evolutionary phase. (p81)

Unlike the philologists, Saussure thought diachrony as of little use for understanding language:

> No society, in fact, knows or has ever known language other than as a product inherited from preceding generations, and one to be accepted as such. That is why the question of the origin of speech is not so important as it is generally assumed to be. The question is not even worth asking; the only real object of linguistics is the normal, regular life of an existing-idioim. (p71-2)

Saussure saw linguistics as a component of a wider semiology:

> *A science that studies the life of signs within society* is conceivable; it would be a part of social psychology and consequently of general psychology; I shall call it *semiology* (from Greek *sēmeîon* 'sign'). (p16)

Sussure was particularly concerned with how *signs* combine a *signifier*, that which points, and the *signified*, that which is pointed at, ie a concept or idea:

> Ambiguity would disappear if the three notions involved here were designated by three names, each suggesting and opposing the others. I propose to retain the word *sign* [*signe*] to designate the whole and to replace *concept* and *sound-image* respectively by *signified* [*signifé*] and *signifier* [*signifiant*]; the last two terms have the advantage of indicating the opposition that separates them from each other and from the whole of which they are parts. As regards sign, if I am satisfied with it, this is simply because I do not know of any word to replace it, the ordinary language suggesting no other. (p67)

Saussure prefers the term ‘sign’ to ‘symbol’, as signs are arbitrary, where symbols are chosen for what they suggest (p68). Note that Saussure was primarily concerned with relations between signs in systems, not with semantics in itself, seeing changes in meaning as involving ‘*a shift in the relationship between the signified and the signifier*’ (p75)

Finally, Saussure hints at the relationship between linguistic activity and computation, discussed in Chapter 4:

> The mechanism of language, which consists of the interplay of successive terms, resembles the operation of a machine in which the parts have a reciprocating function even though they are arranged in a single dimension. (p128)

The Soviet linguist Valentin Vološinov (1895-1936) sought to develop a dialectical approach to language in *Marxism and the Philosophy of Language* [Vološinov(1986)] from 1929. Vološinov counterpoised the *individualistic subjectivism* of the Humboldt school[6], to the *abstract objectivism* of Saussure (p48). Vološinov rejected Saussure's abstraction, identifying language as entirely sociological, that is produced by interacting speakers (p98). Thus, Vološinov saw *themes*, that is semantics, as central to understanding (p99ff).

For Vološinov, language was central to disentangling the Marxist problematic of the relationship between base, ie material conditions, and superstructure, ie social forms (pp18ff). Ideology is determined by the base, with the word mediating between base and superstructure.

Vološinov saw language materialised in speech as primary. Themes bear ideology, and signs have ‘social *multiaccentuality*’ (p23). Words have *evaluative accent* determined by *expressive intonation* (p103). The intonation, and how it is interpreted, also reflect the class orientations of speakers and listeners. Thus, ‘Sign becomes an arena of class struggle’ (p23), where:

> The ruling class strives to impart a supraclass, eternal character to the ideological sign, to extinguish or drive inward the struggle between social value judgements which occurs in it, to make the sign uniaccentual. (p23)

Vološinov approvingly cites his contemporary Nikolai Marr (1864-1934) in asserting that ‘linguistics is the child of philology’. Both criticised traditional philology for focusing on utterances as monologues separated from dynamic verbal interactions (p72), but both thought it possible to derive the origins of languages in the contexts of material cultures.

However, unlike Vološinov, Marr thought that no language could be classless. Matejka [Matejka(1986)] attributes Vološiinov's fall from favour to this difference (p173).

Marr also rejected formal logic as class based:

[6] We won't discuss this further.

> Formal logic, a product of class thinking, together with the class that created it, is swept away by the dialectical materialist thinking of the proletariat in which thought gains ascendancy over language. (cited in Mathias[Mathias(1987), p7])

The preferred Soviet linguistic until 1950, Marr enunciated a now largely discredited *Japhetic* theory, that Indo-European languages were preceded by related languages from the Caucasus and Middle East. In a dispassionate 1948 paper [Matthews(1948)], Matthews refers to Marr's paleontological method which, latterly, had:

> a concentration on semantics rather than the emphasis on phonetics which characterised his earliest and earlier approach, and the typical neglect of morphology, presumably because of its newness and perhaps also because of its exaggerated significance in formalist Indo-European scholarship. (p188)

3.12 The revival of Soviet logic

The detonation of three atomic bombs by the USA in 1945 radically changed Soviet priorities in science, and weakened the dominance of ideology in policy, in particular anti-formalism.

Two Soviet agencies were set up in August 1945 to manage Soviet atomic bomb development [Gerovitch(2002)] (p131). The first Soviet computer project began in Kiev in 1946, directed by Sergey Lebedev (1902-1974), and the MESM became operational in 1950. However, after the publication of Wiener's book[Wiener(1948)], a campaign against cybernetics was mounted, on the grounds that it was a capitalist innovation to weaken working class organisation and, ultimately, entirely supplant workers who would be left destitute [Gerovitch(2002)] (p128). This suggests that the ideologues had little grasp of Marxist economics, and the central role of living labour in the production of surplus value under capitalism. In Marx's scheme, profits derive from human activity, not machines. And, as Usdin [Usdin(2005)] notes, the Soviet military were quick to deploy cybernetics and 'push ideological considerations aside' (p312).

Nonetheless, computers had to be presented as 'mathematical machines' to evade cybernetic scrutiny [Gerovitch(2002), p131ff]. A clear distinction was made between the unacceptable use of cybernetics to model human behaviour, and of computers for calculations and automation. Analogies between computers and human brains were deemed 'absurd' [Gerovitch(2002)] (p142-3). This stance seems entirely retrogressive for materialism. If humans are more than machines, then their additional qualities must derive from non-corporeal properties.

Returning to logic, change came quickly in Soviet education. A 1946 CPSU(B) Central Committee directive [CPSU(B) CC(1952)], cited by Campbell[Campbell(1952)], noted that it was:

> quite improper that logic and psychology are not taught in secondary schools. (p343)

and set out plans, with resources, for their introduction. Thus, Mathias [Mathias(1987)] recounts how a 1918 edition of Chelapanov's *Textbook of Logic* was republished in 1946, followed by Strogovich's *Logic*, a new textbook by Asmus, and a further edition of Strogovich (p8).

Lorentz [Lorentz(2002)] observes that, following Lysenko's alleged achievements in genetics, there were moves in 1948 to systematically align wider Soviet science with the notion that science had a class character. However, the plans for mathematics and physics were halted by Beria, the chief of Atomic Missile Projects, after 'influential physicists explained to him that this may damage these projects' (p217-8).

In 1952, Campbell [Campbell(1952)] noted the problems for Soviet logic education of squaring 'bourgoise' logic with dialectical logic. He summarises a 1950 article by Osmakov[7] [Osmakov(1950)] as arguing that:

> Unlike a world outlook, the logic of thinking is a *classless* phenomenon. (p281)

Further, in a formulation strongly reminiscent of Aristotle:

> The concepts on the basis of which the logic of thinking proceeds reflect objective reality well or badly according to the ideology or world view of the thinker (p282)

Osmakov explicitly distinguishes the logic of thinking and the science of logic (p283), and discusses three false views of their relationship:

> (a) That there is but one scientific logic, viz. Formal Logic. (b) That Dialectical Materialism incorporates the science of logic within itself. (c) That there are 'two' valid sciences of logic dealing with two different aspects of phenomena; viz. Formal Logic and Dialectical Logic. (p283)

In another Aristotelian formulation, Osmakov concludes of the second that:

> All this would be avoided if Soviet logic were recognised as an independent science which investigates the laws and forms of human thinking with due attention from the outset to the fundamental phenomenon in their dialectical development. (p284)

Bazhanov [Bazhanov(2001)] and Kilakos [Kilakos(2019)] recount how Yanovskya was central to the revival of Soviet formal logic. In 1947, she had translated *Grundzüge der theoretische Logik* by the formalists Hilbert and Ackermann, and, in 1948, Tarski's *Introduction to logic and the methodology of deductive sciences* [Bazhanov(2001),

[7] Head of the Philosophy Department in the Directorate for the Teaching of Social Sciences, USSR Ministry for Higher Education.

p132]. In 1947, Yanovskya was arguing that 'methodological formalism of mathematical logic' should be distinguished from the idealist philosophy underlying it, because[8] mathematical logic:

> can be considered not only as logic of mathematics but also as mathematics of logic, for it is in large part the result of the application of mathematical methods to the problems of logic [Kilakos(2019), p57]

This argument returns to Boole's mathematicisation of logic, neatly inverting the stated objective of meta-mathematics.

Yanovskaya promoted formal logic throughout the rest of her life. Held in high esteem, she was the Chair of Mathematical Logic at Moscow State University, from 1959 until her death in 1966 [Bazhanov(2001), p132].

As we shall explore below, an enduring achievement of revived Soviet formal logic was the elaboration of constructivism, which accepted the finitist premises of intuitionism, while shearing it free of idealism.

3.13 Stalin on linguistics

In 1950, the public reversal of anti-formalism was heralded by Stalin's repudiation of Marr. As Lorentz [Lorentz(2002), p217] observes, Stalin paid close attention to developments in exact science, and, while endorsing Lysenko, had expressed scepticism about science's general class character[9]:

> Ha, ha, ha. And what about mathematics? And about Darwinism? (p217).

Marxism and Problems of Linguistics [Stalin(1950)] was published in *Pravda* in 1950, followed by four further clarificatory exchanges. In the original article, Stalin roundly rejects two key tenets of Marr's linguistics, arguing that language is neither base nor superstructure, and is not class marked. These criticisms also apply to Volos̑inov.

In a subsequent reply to Krasheninnikova, Stalin chided Marr for overemphasising semantics, while acknowledging its importance for linguistics. From the context, Stalin appears to again be criticising Marr for attributing different meanings to words and expressions, depending on a speaker's class. He suggests that such differences are very few, and lie in individual words, not in grammar, which is common to all speakers.

Most significantly, Stalin signaled the end of the attribution of formalism as a decisive critique:

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[8]Quoting S. A. Yanovskya, 'Michel Rolle as a critic of the analysis of the infinitely small (in Russian)', *Trudy Instituta Istorija Estestvoznanija*, Volume 1, pp327-46 (p341).

[9]Quoting V. J. Birstein, 'The Perversion of Knowledge: The True Story of Soviet Science', Westview, 2001, p249-251

> N. Y. Marr considered that grammar is an empty 'formality', and that people who regard the grammatical system as the foundation of language are formalists. This is altogether foolish.
>
> I think that 'formalism' was invented by the authors of the 'new doctrine' to facilitate their struggle against their opponents in linguistics.

Mathias[Mathias(1987)] quotes Strogovich as welcoming this judgement:

> J.V. Stalin's works on linguistics provide the key to the solution of all the questions of logic which have been the subject of lengthy debates. (p12)

Incidentally, Stalin's articles seem to have caused some disquiet in lay UK Communist circles. The Nobel prize winner Doris Lessing (1919-2013), who was a CPGB member from 1952 to 1956, reports a writers group discussing the pamphlet, in her 1962 feminist novel: *The Golden Notebook* [Lessing(1973)]. The five members have difficulty understanding Stalin's intent. One hazards that 'Perhaps the translation is bad', and goes on to say:

> Look, I'm not equipped to criticise it philosophically, but surely this sentence here is a key sentence, the phrase 'neither superstructure nor base'- surely that is either completely out of the Marxist canon, a new thought completely, or it's an evasion. Or simply arrogance. (p300)

Other group members adopt a 'rough-and-ready attitude, a sort of comfortable philistinism'. As one says:

> All this theoretical stuff is just over my head. (p300)

Thereafter, Soviet logic flourished, though debate about the relationship between dialectics and logic continued [Mathias(1987)]. Seventeen years later, in 1967, Kopnine [Kopnine(1967)] was still citing Engels[10] in defending the Hegelian verities, writing:

> Formal logic and dialectic, as methods of knowing reality, occupy in relation to each other the same positions as elementary and advanced mathematics.(p101)

Kopnine ends his exegesis with the Aristotelian formulation:

> Dialectic logic does not deny the value of formal logic. After the appearance of dialectic logic, formal logic loses its prime importance as a theory of thought. ... The experience of the development of contemporary scientific thought has shown that the two logical systems, dialectic and formal logic, achieve fruitful results in the acquisition of new knowledge. Science needs strict rules of deduction and systems of categories in order to provide a firm basis for the fertility of the imagination and for the creative activity of thought when it takes in new objects from reality (p102)

[10]and Lenin.

But Soviet scientists, and the wider public, paid far less heed to ideological rectitude well before then. The distinguished Soviet Computer Scientist Andrei Ershov (1931-1988), in a 1976 lecture series to the British Computer Society [Ershov(1980)], provided a history of Soviet Computing, showing how rapidly it had caught up with the West.

He mentioned how in 1954 he was advised to miss out a section on programming from his diploma thesis, and instead submitted material on operator algorithms, because that was more 'mathematical' (p27). However, the period 1956-60 was:

> ...accompanied by the overcoming of the dogmatically negative approach to the ideas of cybernetics and of the unity of laws of control and information processing in machines and human beings, with the first experiments in computer applications to simulation of human activity. (p17).

A 1959 report[Carr III et al.(1959)Carr III, Perlis, Robertson, and Scott] by US Computer Scientists visiting the Soviet Union showed that they were very impressed by progress in logic and computing. They note that:

> There are obviously more and better logicians and mathematicians connected directly or indirectly with computers at Moscow, Kiev, and Leningrad Universities than at any universities in Western Europe or the United States. (p17)

They further note that, outside of MIT, the USA lacked programs in 'computer oriented' logic comparable to that at Moscow State University (p17), and warned that the production of:

> ... qualified computer oriented mathematicians, not just computer programmers - may, soon surpass that of the United States. (p17)

Indeed:

> ... Soviet use of computers may be expected to surpass in quality and quantity that of the United States... (p17)

Finally, the visitors observe that:

> The continuing interest of the entire Soviet population in Cybernetics (illustrated by giant sales of Wiener's book *The Human Use of Human Beings*') is penetrating the society. (p17)

The bibliography (p18-20) lists an impressive range of recent Soviet publications on all aspects of computing. Bazhanov [Bazhanov(2001)] notes that in 1956 there were 75,000 copies of Yanovskaya's book on logic in circulation (p132).

3.14 Constructivism

The Soviet school of *constructive* logic was founded by Andrey Markov Jr.[Markov(1954)] (1903-1979)[11] [Markov(1954)]. Constructivism had its roots in intuitionism, but with a materialist orientation, as in Kolmogorov's approach.

Given the idealism underlying intuitionism, its embrace by Soviet logicians may seem surprising. However, Markov's collaborator Ngorny suggests that, before the 'thaw', set theory was seen as materialist rather than idealist, despite its Platonic roots [Nagorny(1994), p469]. Further, Vandoulakis [Vandoulakis(2015), para 46] notes that Brouwer's intuitionism was seen as consistent with dialectical materialism, once its logical framework was separated from his philosophy. As Ŝanin wrote in 1962[Ŝanin(1968)]:

> ...outside of the surface of intuitionistic philosophy one finds in many cases very valuable concrete observations and profound concrete analysis of the fundamental problems relating to the processes of forming mathematical abstractions and logical foundations of mathematics. (p7)

Markov's student Kushner identified four central characteristics of Markov's constructivism [Kushner(1999), pp268-9]. First of all, the objects under investigation are finite and generated by finite constructive processes, from a finite alphabet according to definite rules of algorithm formation. Ŝanin rightly saw this as key to the success of constructivism:

> The enrichment of mathematics with the precise concept of arithmetic algorithm served as a starting point for fruitful investigation in a new direction by many authors. (p6)

Secondly, Kushner notes that, as with intuitionism, both the law of the excluded middle and double negation are rejected, and proofs of existence must be based on construction. Third, potential infinities are accepted, but not actualised infinities. And, finally, computability is associated with algorithms, and the Church-Turing thesis of the equivalence of all models of computability, demonstrated empirically, is accepted.

Like Turing, Markov started from mathematics as a material process using pen and paper. As an example, he considers drawing a row of vertical pen strokes on the paper forming his book manuscript. Discussing the how this is subsequently reproduced by printing, he observes that:

> The constructive object [ie the original drawing] is a material body consisting of paper and dried ink, and the drawing given above is a copy of this constructive object, consisting of paper and dried typographic paint. It, too, is a constructive object, since the preparation of a copy may be considered a constructive act. [Markov and Nagorny(1988), p1].

[11]The son of Andrey Markov (1856-1922), known for foundational work on stochastic processes.

Markov explicitly acknowledges the limits of material reality:

> Carrying out constructive processes, we often come up against obstacles connected with a lack of time, space and material. One usually succeeds in somehow by-passing these obstacles. However, our constructive possibilities really are limited, and there are no grounds for supposing that the obstacles caused by their restrictedness can always be obviated. Rather to the contrary, it seems that modern physics and cosmology testify to the impossibility in principle of surmounting such obstacles. [Markov and Nagorny(1988), p10]

Nonetheless, in an Aristotelian formulation, the *abstraction of potential feasibility*:

> allows us to consider arbitrarily long constructive processes and arbitrarily large constructive objects. Their feasibility is potential: they would be feasible in practice, had we available sufficient space, time and material. [Markov and Nagorny(1988), p10]

Markov explicitly contrasts this approach with classical mathematics based on set theory, which allows abstractions of actual infinity [Markov and Nagorny(1988), p10] and for which existence is a consequence of refuting non-existence, where 'a method for constructing the desired object may even be unknown' [Markov and Nagorny(1988), p12].

Markov's notation was based on rules for rewriting symbol sequences, similar to Chomsky's subsequent characterisation of classes of formal grammars, which we will discuss in Chapter 4. The general form of a rule is that a symbol sequence may be replaced by some other sequence.

For example[Markov and Nagorny(1988), p142], suppose an integer is represented by a sequence of vertical strokes. Then, the difference between two integers N and M, written $N * M$ is found by:

— * — → *
* → .

Thus, $4 - 2$ evaluates as:

————*—— → ———*— → ——* → ——.

that is 2.

A *scheme* of such rules is called an *algorithm*, which Markov characterised as:

> a) the precision of the prescription, leaving no place to arbitrariness, and its universal comprehensibility - the definiteness of the algorithm;
> b) the possibility of starting out with initial data, which may vary within given limits - the generality of the algorithm;
> c) the orientation of the algorithm towards obtaining some desired result, which is indeed obtained in the end with proper initial data, the conclusivenes of the algorithm.[Markov(1954), p1]

Using this notation, Markov systematically reconstructs a considerable portion of Peano arithmetic, using induction.

Markov suggests that his approach satisfies what he calls Church's Thesis, of the equivalence of models of algorithm. He further argues that Turing machines are 'extremely convincing' as:

> *in the essentials*, a Turing machine's performance adequately simulates the behaviour of a computing mathematician[Markov and Nagorny(1988), p109]

going from one state of mind to another.

The notation was intended for practical experimentation with formal systems. For example, the 'meta-algorithmic' programming language Refal [Turchin(1967)] incorporated Markov rules within a more conventional framework, as an aid to exploring formal semantics. However, Kushner[Kushner(2006), p562] observes that, while Markov's work was known in the west, wider take up was hampered by notational complexity.

As with intuitionism, accepting constructivist limitations on infinitary reasoning removed the foundations of much essential classical mathematics. Thus, as with intuitionism, considerable research was undertaken to refound such mathematics on a finitary base.

In particular, constructivism accepted integers and rational numbers, but not real numbers, which are necessarily infinitary. Hence, a vital step was the reformulation of real numbers as constructive functions. In the constructivist approach, drawing on work by both Turing and Weyl, a real numbers is defined by an algorithm that generates rational numbers of increasing precision, determined by another algorithm. As Ŝanin put it:

> ...the actual use of concrete real numbers in the natural sciences and engineering is essentially based on the possibility of extracting from an individual representation of a real number an algorithm giving a sequence of rational approximating values for it. [Ŝanin(1968), p13]

This recognised explicitly that, in practical applications, rational numbers are used in manipulating physical reality, because measurement is bounded.

Despite Markov's materialist approach, he shared the view that the undecidability results were limitations on mathematics but not human beings:

> Therefore the conative, research enterprise in mathematics (as well as any other branch of learning) will never be transferred to machines, capable only of assisting man but not replacing him.[Markov(1954), p441]

3.15 Conclusion

We have explored how the deep paradoxes of self-reference challenged Hilbert's programme of reconstructing mathematics through logic. We have also seen how these gave further succour to Soviet materialists seeking to assert the primacy of dialectics in science. And we have discussed how pragmatic considerations at the end of World War Two led to the re-invigoration of Soviet logic through its tacit separation from dialectics.

Nonetheless, despite rapid progress once anti-formalism dogmatism was abandoned, the Soviet Union never became a leading centre of computing. We will not explore this further here.

In the next chapter, we will look at how automata, derived from Turing machines, can offer a principled and systematic account of language as reality transforming interaction.

Chapter 4

Language, automata and meaning

4.1 Language and meaning

There is a longstanding distinction between the meaning of a statement as its value, compared with what it is about. These may be termed reference and sense, denotation and connotation, and extension and intension.

Consider the descriptions of a number:

one more than two

The extensional meaning results from evaluating it to a value: three. Another way of expressing this is that the extensional meaning is all the things that can replace *number* in:

the number that is one more than two

and retain the statement's truth. That is, we treat *number* as a variable whose sole valid value is 'three'.

Now consider the description of a number:

half of six

which also has the extensional meaning of three. These statements' intensional meanings are quite different. The first involves counting, and the second dividing. We could[1] use Peano arithmetic to formalise them, and prove that they are equivalent. They then have the property of substitutive synonymy. That is, they can be used interchangeably, in formal expressions about numbers, without changing the meaning of the expression that uses them.

Now, suppose that Chris can count but can't do division. Then:

Chris knows that three is one more than two.

[1]But won't.

is true. But we cannot replace *one more than two* with '*half of six*:

> *Chris knows that three is half of six.*

and retain truth.

Bertrand Russell sought to analyse and reformulate intensional constructs to make them extensional [Russell(1905)]. In Russell's approach, we might write:

> *Chris knows that 'there is a number that is one more than two'.*

to make the context of 'knows' clear. If we replace the quoted phrase with an apparent equivalent:

> *Chris knows that 'there is a number that is half of six'.*

then we cannot replace *there is a number that* with *three* and retain truth.

Rudolf Carnap (1891-1970) was a logical positivist, who sought fact in positivist science for manipulation by logic. Originally, Carnap considered all statements to be either extensional or meaningless. In his 1928 *The logical structure of the world; pseudoproblems in philosophy* [Carnap(1967)] he enunciated a totalising approach of denying meaning to any linguistic constructs that could not be grounded in fact. In particular, he disputed that either realism (i.e. materialism) or idealism was meaningful:

> *neither the thesis of realism that the external world is real, nor that of idealism that the external world is not real can be considered scientifically meaningful.*[2] (p334)

That is, Carnap saw knowledge as grounded in verification via empiricism. Nonetheless, Carnap termed himself a 'physicalist' who saw physics as exemplifying how to establish facts.

Willard Quine (1908-2000), critiqued logical positivism in his 1953 *From a Logical Point of View* [Quine(1963)]. In the essay *Two Dogmas of Empiricism* (pp20-46), he first disputes that there is a deep distinction between analytic truths, based on meanings without considering facts, and synthetic truths, based on facts (p20). For example, we might contrast properties of the number three, which are internal to formalised arithmetic, with what someone knows about the number three, which is a reported fact. Secondly, Quine rejects the reductionism that only accepts as meaningful those statements that may be reconstituted as logical statements constructed from facts (p20).

Quine is left with an explicit pragmatism:

> Each man is given a scientific heritage plus a continuing barrage of sensory stimulation; and the considerations which guide him in warping his scientific heritage to fit his continuing sensory promptings are, where rational, pragmatic (p46)

[2]Italics in original.

As in our simple examples above, Quine was concerned with substitutivity as a criterion of synonymy of meaning, asking in what contexts substitution is legitimate, and pointing out that substitutions necessarily changes the form of statements (p56). His conclusion is again pragmatic:

> What matters rather is likeness in *relevant respects* ... a problem typical of empirical science. (p60)

For Quine, given that formal systems are constructed with variables marking points of abstraction:

> To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable. (p13)

Then:

> ... a theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be made true. (pp13-14)

This is a pleasingly generous position. It admits not just of manipulating facts in logical systems, but of anything a system can legitimately manipulate.

4.2 Model theory

In the 1921 *Tractatus Logico-Philosophicus* [Wittgenstein(1961)], Wittgenstein suggested that the world should be understood in terms of *states of affairs*, that is configurations of things, which he termed objects. He asserts that:

> 4.1 Propositions represent the existence and non-existence of states of affairs.
>
> 4.11 The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences). (p25)

Wittgenstein further identifies the *sense* of a proposition with:

> 4.2 ...agreement and disagreement with the possibilities of existence and non-existence of states of affairs. (p30)

He then proposed the use of truth tables, discussed above in Chapter 1, to explore these possibilities (p30), having argued that:

> 4.25 If an elementary proposition is true, then the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist.

In his 1947 book *Meaning and Necessity: a Study in Semantics and Modal Logic* [Carnap(1947)], Carnap, strongly influenced by Wittgenstein's states of affairs, systematically formalised the concept of extensional meaning through the idea of a *state description*:

> There is one and only one state-description which describes the actual state of the universe; it is that which contains all true atomic sentences and the negations of those which are false. Hence it contains only true sentences; therefore, we call it the true state-description. A sentence of any form is true if and only if it holds in the true state-description. (p10)

Note that, where Wittgenstein talks about determining the existence of a state of affairs, which is relative to what is known, Carnap asserts the possibility of describing 'the actual state of the universe'.

Carnap then systematically elaborated a formal *object language* for expressing extensional meanings through substitution of values from state-descriptions. Intensional statements were made in a constrained natural *meta-language*[3], for translation into the object language for verification against state-descriptions.

By 1961, Carnap no longer thought that:

> ... all statements about things can be translated into statements about sense data. [Carnap(1967), pviii]

Further, he was now cautious about how rigorously intensional statements might be converted to extensional:

> Hence I have later proposed a weaker version which claims that every nonextensional statement can be translated into a logically equivalent statement of an extensional language. It seems that this thesis holds for all hitherto known examples of nonextensional statements, but this has not yet been demonstrated; we can propose it only as a conjecture. (pix)

Nonetheless, the work of Wittgenstein and Carnap formed the basis of *model theory*, which now underpins the semantics of programming languages. And the development of Carnap's work continued, for example in Marian Przelecki's *Logic of Empirical Theories*[Przelelcki(1969)].

For Przelecki, a *model* is state of affairs or a state-description, formalised in set theory. Przelecki restricts models to what he calls *physical objects*, with predicative properties. He suggests that this is insufficient as a basis for a science like physics, and it needs to be extended, though he doesn't do so, with real numbers, and higher order logic constructs like relations and functions (p104). Given that science continuously develops, Przelecki further suggests that the current state of a discipline might be characterised by taking a 'cross section' to determine the changing balance between determinate analytic and indeterminate synthetic knowledge (p105-6).

[3] English rather than Hegel's preferred German.

4.3 Badiou, model theory and materialism

Alain Badiou's *The Concept of Model*, based on lectures given in 1968, sought to reconcile formal logic with the Marxist materialist tradition. Criticising both Carnap and Quine, he sees the distinction between empirical 'fact' and logical form as common to both perspectives (p7), and claims that this actually serves to bind together formal and empirical science: that is, there is a:

> dialectical complicity between logical neo-positivism and model theory. (p19)

The great strength of Badiou's analysis is that he characterises mathematics as a unified material practice, linking formal systems of rules to practical calculation:

> The philosophical category of effective procedure - of that which is explicitly calculable by a series of unambiguous scriptural manipulations - is truly at the centre of every epistemology of mathematics. (p26)

Badiou emphasises that while models are made to explore formal systems:

> *a model is the mathematically constructable concept of the differentiating power of a logico-mathematical system.*[4] (p40)

In turn, models are subject to experimental investigation:

> It is because it is itself a materialised theory, a mathematical result, that the formal apparatus can enter into the process of production of mathematical knowledge; and in this process, the concept of model does not designate an outside to be formalised, but a mathematical material to be tested. (p47)

Badiou further argues that a formal system is quite literally a 'machine for mathematical production', and notes that the:

> ... increasingly evident kinship between the theory of these systems and the theory of automata, or of calculating machines, strikingly illustrates the experimental vocation of formalism. (p43)

As discussed below, we see automata themselves as, not just experimental apparatuses, but providing a dynamic, physical basis for semantics, that goes beyond mathematical constructs, no matter how materialised.

Badiou prioritises set theory and integers for model making, but does not mention the Church-Turing thesis. This implies that all Turing complete (TC) systems may serve as formal model for each other: demonstrating that a new system is TC requires precisely the ability to translate between it and some known TC system. Given that computers are physical TC systems, they ground formal models in actual reality.

[4]Italics in original/

4.4 Automata

In chapter 3, we met the Turing machine, and saw how it has a bounded but arbitrarily extensible tape. We also saw that, according to the Church-Turing thesis, the TM is a model for all possible computations.

We will now explore how restricting TMs' tape properties changes their computational properties. We will then consider how this can be characterised in terms of grammars that describe structured data. [Hopcroft and Ullman(1969)] provides a succinct account.

We might consider a TM tape as a combined input and output, as well as a memory. The initial tape is the input and the final tape is the output, and the TM can arbitrarily inspect, modify, and extend the tape. We'll now consider three broad restrictions that we might place on TM behaviour, by changing properties of the tape.

First of all, we might insist that the tape cannot be extended, that is the output must be the same size as the input. This is called a *linear bounded automaton (LBA)*, and broadly corresponds to actual computers. Of course computers may be given streamed input of arbitrary length, but only a finite amount can be held in memory at any moment. In practice though, we treat computers as if they were Turing machines, and get grumpy when they run out of memory.

Secondly, we might give a FSA a tape which is extensible in one direction. The tape behaves like a *stack*, where items may be added to (*pushed*) or removed from (*popped*) the top, but only the top item can be inspected. This is known as a *push down automaton (PDA)*. PDAs are not a common form of practical machine, but are central to software for initial syntax analysis of computer programs.

Third, we might restrict access to a finite tape to only inspecting it from start to finish, without changing the tape or reversing the direction. Thus the machine can only chew through the tape, changing from state to state. If it is possible to enter a new state from more than one old state, then the machine has no memory of its computation history. This sort of machine is called a *finite state automaton (FSA)* or *Moore machine*.

A FSA which can also emit outputs is called a *Mealey machine*. This is equivalent to adding a single cell writable tape to a Moore machine. Mealey machines are very widely used for controlling processes that go through fixed sequences of actions, for example, sets of traffic lights, or different washing machine cycles.

4.5 Machines and semantics

We'll now consider a simple, yet rich, example of a FSA. In the UK a pedestrian crossing is triggered by someone pressing a button. Inside the crossing controller is a physical Mealey machine[5] connected to the button and a timer. See Figure 4.1.

[5] Once electro-mechanical, now digital.

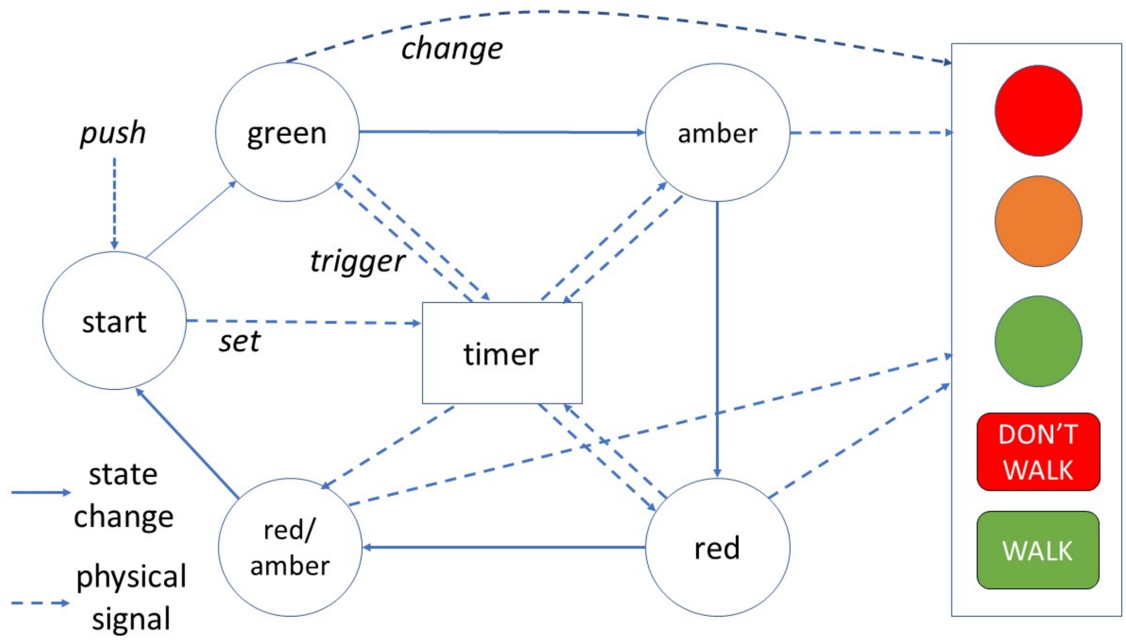

Figure 4.1: Traffic lights

The machine's behaviour is summarised in Figure 4.2. In the start state, the light is green, and displays 'DON'T WALK'. When the button is pushed, the machine sets a timer, and changes to the green state. When the timer triggers, the machine sets the light to amber, sets a timer, and changes to the amber state. When the timer triggers, the machine sets the light to red and the display to 'WALK', sets a timer, and changes to the red state. When the timer triggers, the machine sets the light to red/amber, sets a timer, and changes to the red/amber state. And when the timer triggers, the machine sets the light to green and the display to 'DON'T WALK', and returns to the start state.

This constitutes a language with just one sentence, composed of the word 'push',

Old state	Light setting	Input	Action	New state
start	green + DON'T WALK	push	set timer 1	green
green	green + DON'T WALK	timer 1	set amber	
			+ timer 2	amber
amber	amber+ DON'T WALK	timer 2	set red	
			+ timer 3	red
red	red + WALK	timer 3	set red/amber	
			+ timer 4	red/amber
red/amber	red/amber + WALK	timer 4	set green	start

Figure 4.2: Traffic lights state transitions

but a very rich extensional meaning: the sequence of states from 'start' back to 'start' again.

Note that timer is also an input to the machine. So the machine inputs are the button press followed by the timer sequences. If the button is pressed in any state except the first, nothing happens, because this is ungrammatical. An interesting aspect is that the timer sequence is internal to the machine: that is, the machine is autonomously changing its own states. Thus, the pedestrian 'speaks' to the machine by pressing the button, and the machine then 'talks' to itself.

Also, the machine 'talks' to the pedestrians and road users through the lights. So 'push' has a very rich, socially constructed intentional meaning, that depend on the light constructor embodying traffic law which both the pedestrian and road users understand.

4.6 Language games

We have elided the distinction between checking whether or not a symbol sequence has a required structure, and wider computing. The connection is that, given a grammar of some type, there is an algorithm to generate its checking machine. However, it's not yet clear how affirming structure relates to the meanings of symbol sequences.

The key lies in what else the machine does as it checks the sequence. Once we locate meaning in state changes in materialised systems, we can clarify wider interaction in terms of linguistic exchanges.

Just like Carnap, Wittgenstein retreated from his totalising vision of systematically elaborating states of affairs. He published little in his lifetime after *Tractatus*, and his later work is known through posthumous collections.

In *Philosophical Investigations* [Wittgenstein(1958)], published in 1953, Wittgenstein explores the idea of a *language game* as a way of exploring how language functions Like other games, language game involve players manipulating things according to rules. They proceed by players taking it in turn to interact through talking and doing things, according to rules they share.

For example:

> Where is the connection effected between the sense of the expression 'Let's play a game of chess' and all the rules of the game? - Well, in the list of rules of the game, in the teaching of it, in the day-to-day practice of playing. (p80)

Wittgenstein counterposes this to the Augustinian idea of ostensive meanings based the association of names with things, so children learn language by having things named as they're pointed at. Rather, Wittgenstein suggests, children learn language through training, which involves talking, that is interacting (pp2-4).

He introduces a language game of a builder and assistant, where the builder calls out the name of a component and the assistant provides it (p3). He then considers the builder asking the assistant how many slabs there are. Then, the assistant saying ‘five slabs’ in response is quite different to the builder saying ‘five slabs’ and expecting the assistant to hand over five slabs (p10). That is, rather than naming things, even broadly understood, the same linguistic constructs have different meanings in different contexts.

For Wittgenstein, we take part in a multiplicity of small language games, in diverse concrete circumstances:

> We remain unconscious of the prodigious diversity of all the everyday language games because the clothing of our language makes everything alike. (p224)

However, language games are not fixed, but develop in interaction, and may break down if players don’t act appropriately, or actions have consequences that weren’t foreseen. This may be resolved because we are immersed in language use:

> ... we lay down rules, a technique for a game, and then when we follow the rules, things do not turn out as we had assumed. That we are therefore as it were entangled in our own rule. (p50)

4.7 Mesolithic capitalism

We will now explore in some detail a language game based on Marx’s simple reproduction schema from Volume 1 of *Capital* [Marx(1970)].

Consider a world where shellfish grow on the strand, in between the sea and the land. A person needs to eat one shellfish a day to survive. There is a capitalist who has title to the shellfish. And there is a worker who will collect shellfish. The medium of exchange is rocks, which the capitalist also owns. The capitalist will pay one rock for one shellfish, and charge two rocks for one shellfish. It is entirely baffling how this state of affairs came about, or why the worker puts up with it, but, hey, this is a language game[6].

There are fours stages to the game. At the start, there are shellfish on the shore and the capitalist has two rocks. During the employment stage, the worker agrees to work for the capitalist, with the following dialogue:

> capitalist: *get me shellfish!*
> worker: *pay me rocks!*
> capitalist: *OK!*
> worker: *OK!*

[6] We call this game ‘Mesolithic capitalism’. The name is ironic. In Mesolithic times, no one owned title to anything, and shellfish were a poor source of sustenance for hunter-gatherers. There are vast shell mounds on coasts world wide from the Mesolithic, but none from the Neolithic, when pastoralism and farming developed. Now, of course, shellfish are luxury food, which those that gather them can rarely afford.

The worker goes to the shore and gathers two shellfish. Then, in the exchange stage, the dialogue is:

> capitalist: *give me shellfish!*
> worker: *give me rocks!*
> capitalist: *OK!*
> worker: *OK!*

The worker gives the capitalist two shellfish and the capitalist gives the worker two rocks.The worker is now hungry. In the sale stage, the dialogue is:

> worker: *sell me shellfish!*
> capitalist: *pay me rocks!*
> worker: *OK!*
> capitalist: *OK!*

And, in the consumption stage, the worker gives capitalist two rocks and the capitalist gives worker one shellfish. The worker and the capitalist each eat one shellfish.

At the end of the cycle, the capitalist has again two rock and the worker nothing. New shellfish grow, and the cycle repeats.

For this to function, the capitalist and workers must have the same model of the world in their brains. And that model includes each other, and that they share the model.

During the game, the world state, and the actors' internal models and dispositions, must remain mutually consistent. In particular, the language interpreting/generating mechanisms in the actors' brains must behave consistently.

The world is dynamic, so the models in the actors' brains must be dynamic, to reflect how it changes. Further, during interaction, as the world changes and the models in actors' brains change, the actors' dispositions change. This is all driven by what utterances are legitimate in the world's current state and the actors' current dispositions. Automata provide a unitary framework to account for all of this.

To interact, the actors must share the same language capacity, with consistent abilities to generate and interpret meaningful utterances. A generalised grammar[7] for the Mesolithic Capitalism language game is:

> *thing* → shellfish | rocks
> *imperative* → get me | give me | sell me | pay me
> *utterance* → *imperative thing* | OK
> *utterances* → *utterance* | *utterance utterances*

The meanings of utterances are to do with how world is and how it might be. They involve things, ie shellfish and rocks, and where they might be, ie on the sea shore, with the worker or the capitalist. Then there are the actors, the worker and capitalist,

[7] The notation is discussed below.

state	input	output	action
1a Employment	*get me shellfish!*	*pay me rocks!*	
1b	*OK!*	*OK*	collects shellfish
2a Exchange	*give me shellfish!*	*give me rocks!*	
2b	*OK!*	*OK!*	gives shellfish/gets rocks
3a Sale		*sell me shellfish!*	
3b	*pay me rocks!*	*OK!*	
3c	*OK!*		gives rocks/gets shellfish
4 Consumption			consumes shellfish

Figure 4.3: Worker states

state	input	output	action
1a Employment		*get me shellfish!*	
1b	*pay me rocks!*	*OK!*	
1c	*OK*		
2a Exchange		*give me shellfish!*	
2b	*give me rocks!*	*OK!*	
2c	*OK!*		gives rocks/gets shellfish
3a Sale	*sell me shellfish!*	*pay me rocks!*	
3b	*OK!*	*OK!*	gives shellfish/gets rocks
4 Consumption			consumes shellfish

Figure 4.4: Capitalist states

and their actions, ie getting, giving and taking things, and generating/saying and hearing/interpreting utterances.

We can account for what actors do by reference to the state of world, that is the dispositions of things and actors. We characterise things by where they are. Actors have more complex dispositions. What they hear, that is their inputs, and when they hear it, that is the state they're in, determines what they say, that is their outputs, and what they do, that is their actions. We can then tabulate their states in a complete cycle. See Figures 4.3 and 4.4.

We can also describe the world state in terms of how the things change against the actor utterances. See Figure 4.5.

The worker and capitalist both require consistent knowledge of who has what, which each can verify empirically. They have also learnt who needs what, and the rules of interaction.

These tables express the extensional meanings of the utterances for both actors. We could also attempt to make a vastly more complex model that express the intensional meanings. We might account for natural motivations, for example both actors need to eat. We might also account for social relations. For example, the capitalist doesn't want to work, the worker has to work, and the capitalist needs the worker to work.

state	shore	worker	capitalist	W speech	C speech
1.	2 shellfish	0 rocks	2 rocks	*pay me*	*get me*
		0 shellfish	0 shellfish	*rocks!/OK!*	*shellfish!/OK!*
2.	0 shellfish	0 rocks	2 rocks	*give me*	*give me*
		2 shellfish	0 shellfish	*rocks!/OK!*	*shellfish!/OK!*
3.	0 shellfish	2 rocks	0 rocks	*sell me*	*pay me*
		0 shellfish	2 shellfish	*shellfish!/OK!*	*rocks!/OK!*
4.	0 shellfish	0 rocks	2 rocks		
		1 shellfish	1 shellfish		

Figure 4.5: World states

Of course we should be concerned with how the social relations come about, and how they might change. And we should be concerned about how rules are maintained through custom, tradition, law and, ultimately, coercion. The core point here, though, is that social relations are required for, and reproduced in, language use, rather than being hardwired into language. And we can use automata to make rich models of very simple language games.

4.8 Grammars

In Chapter 1, we met the idea of a formalised logic, which enabled reasoning about meaningful, well formed symbol sequences. Noam Chomsky [Chomsky(1959)] explored different classes of well formed symbol sequences, without direct concern for their meanings. He identified four types of grammar, that have been shown to correspond to the automata we discussed above.

Let's consider a sentence made up of words. To check the sentence against a grammar, the rules of the grammar have to be applied systematically, checking that the words are in the right sequence.

For example, suppose the words are:

a ate cat mouse saw the

Then, for sentences like:

the mouse saw a cat
the cat ate the mouse

we might have the grammar:

article → a | the
noun → cat | mouse
verb → ate | saw
sentence → *article noun verb article noun*

This reads as:

- to find an *article*, find an ‘a’ or a ‘the’
- to find a *noun*, find a ‘cat’ or a ‘mouse’
- to find a *verb*. find an ‘ate’ or a ‘saw’
- to find a *sentence*, find an *article* followed by a *noun* followed by a *verb* followed by an *article* followed by a *noun*

Formally, the words are called *terminal symbols*, and the rule names *non-terminal* symbols. To generalise this notation we might use:

- a lower case letter for an arbitrary terminal symbol, ie a, b, c ...
- an upper case letter for an arbitrary non-terminal symbol, ie A, B, C ...
- a Greek letter for a *sentential form* consisting of a mixed sequence of terminal and non-terminal symbols, ie α, β, γ...

A Chomsky $Type3$ grammar has the form:

A → a
A → b B

That is, to find an ‘A’, find a terminal symbol ‘a’, or to find an ‘A’, find a terminal symbol ‘b’followed by a ‘B’. Type 3 checking corresponds to a finite state machine.

The above example, and the Mesolithic Capitalism grammar, are of Type 3, but finite word options have been grouped to simplify presentation.

A Chomsky *Type 2* grammar has the form:

B → β

That is, to find a ‘B’, find a sequence β. This is known as a *context free* grammar. Type 2 grammar checking corresponds to the actions of a pushdown automaton.

A Chomsky *Type 1* grammar has the form:

α B $\gamma \rightarrow \alpha\ \beta\ \gamma$

that is, to recognise a ‘B’ within the contextualising sequences α and γ, find a sequence β between an α and a γ. Note that the Type 2 form does not require the nesting context of α and γ in finding the β. A Type 1 grammar is known as *context sensitive*, and checking corresponds to the actions of a linear bounded automaton.

A Chomsky *Type 0* grammar has the form:

$\alpha \rightarrow \beta$

that is, to recognise a sequence α, find a sequence β. Checking such rules corresponds to the action of a Turing machine. A Type 0 grammar is very similar to Markov's notation.

These types form a *hierarchy*, where each type can define more than the next, just as the corresponding machines are ordered by computational power.

A Type 3, finite state grammar, can express addition, but not whether brackets are balanced, with every opening bracket having a closing bracket. Addition requires only enough states to capture the possible single digit carrys from each sum, but bracket matching would require arbitrary states to record arbitrary nesting of opening brackets.

Less restricted, a Type 2, context free grammar, checkable by a PDA, can express bracket matching. Opening brackets are pushed onto the stack, to be popped off when a closing bracket is found. But a Type 2 grammar can't express counting how many brackets there are, as it has no memory in which to count.

Even less restricted, a Type 1, context sensitive grammar, checkable by a LBA, can be used to express that, say, N 'a's are followed by N 'b's and N 'c's. Essentially, every time an 'a' is found, it and a corresponding 'b' and 'c' are eliminated. But, unlike a Type 0 grammar, a Type 1 grammar can't express arbitrary arithmetic, as only finite numbers can be represented in finite space.

The most powerful grammar, the Type 0, can capture any Turing complete computation. However, like Type 1 grammars, they are not used for practical computing, because it is really hard to think through problems in terms of bounding contexts, which may be arbitrarily large and complex. Rather, programming languages are used. These are like formal systems in their rigour, and natural languages in their expressiveness.

Using a programming language involves thinking in terms of the abstractions that it supports, in particular what sorts of things variables may represent. Psychologically, this elides the physical grounding of computations, in physical computers, manipulating physical instances of symbol encodings. Thus, what can be computed is determined, ultimately, by the physical nature of reality. This is particularly significant for computations that involve arbitrarily small constructs, like real numbers, or arbitrarily large ones, like databases that may grow open-endedly.

If reality is finite, then eventually we run out of stuff with which to do computations. That is, we may think we're programming in a Type 0 language with potentially infinite resources, but if we actually have bounded memory then our computation is necessarily Type 1. And, even if reality is infinite, it makes no sense to talk about finite computations that deploy infinite resources in a finite time [Cockshott et al.(2012)Cockshott, Mackenzie, and Michaelson].

4.9 Conclusion

Our account of the development of formal logic underpins our materialism. With Turing, we see mathematics as a mechanical activity grounded in physical machines. With Badiou, we see model making and logic as complementary experimental activities. With Wittgenstein, we see meanings created in interactions that change reality. And, with Markov, we see meaning characterised by algorithmic processes modeled as automata.

From the Church-Turing thesis, our materialism is reductionist. If computability is key to understanding reality, and all accounts may be demonstrated to have equivalent explanatory power, then their choice is a matter of pragmatism, not principle. In particular, we reject the Platonist primacy of pure mathematics, but recognise its power in abstraction.

Our materialism is fundamentally scientific. We reject the primacy given to dialectics in the Hegelian and Marxist traditions, but recognise its power in exposing and resolving contradictions in interrogating reality.

And our materialism is finitist. With the constructivists, we reject arguments based on actualised infinities.

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