Computable criteria for Schauder basis of dilated functions

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Abstract

Consider a periodic function $F$, such that its restriction to the unit segment lies in the Banach space $L^r = L^r(0, 1)$ for $r > 1$. Denote by $S$ the family of dilations $F(nx)$ for all $n$ positive integer.

The purpose of this talk is to discuss the following question: When does $S$ form a Schauder basis of $L^r$? To answer this question, I will present a new “Multi-term” criterion for determining the condition under which such families generate a Schauder basis of $L^r$ for $r \neq 2$.

The starting point is the criterion for Riesz basis in the case $r = 2$, originally adopted in [L. Boulton, G. J. Lord, Proc. R. Soc. A, 471 (2015)]. This will be followed by an illustration on how to extend this approach to the Banach space setting using polynomials in several complex variables.

We will then examine the application of this criterion in the case of $F$ being the $p$-trigonometric functions. These functions arise naturally in the context of the non-linear eigenvalue problem associated to the one-dimensional $p$-Laplacian in the unit segment. The “Two-term” criterion successfully provides two refined thresholds $p_0 < 2$ and $p_1 > 2$ such that a basis is guaranteed for all $p \in [p_0, p_1]$. These results sharpen those of [L. Boulton, H. Melkonian J. Math. Anal. Appl., 444 (2016)] and [D. E. Edmunds, P. Gurka, J. Lang, J. Math. Anal. Appl. 420 (2014)].