

Multiannual Cycles in Field Vole Populations: Spatial Data and Spatiotemporal Models

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Xavier Lambin



Andy White



Mike Begon



Outline

- 1 Introduction: Field Voles in Kielder Forest
- 2 Why Do Field Vole Populations Cycle in Kielder?
- 3 Spatiotemporal Field Data
- 4 Mathematical Modelling of Periodic Travelling Waves

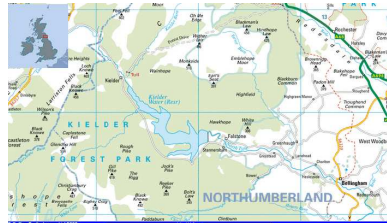
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Multiannual Rodent Cycles

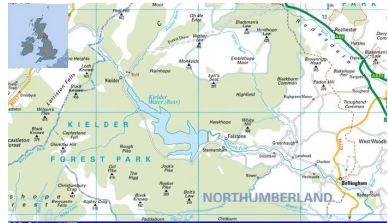
- Cyclic rodent populations are one of the most studied systems in ecology
- The multiannual fluctuations in abundance are not driven by the environment
- Despite more than 100 years of study, the underlying mechanisms remain unclear

Field Voles in Kielder Forest



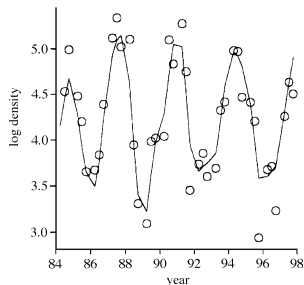
Field vole (*Microtus agrestis*)

Field Voles in Kielder Forest



Kielder Forest is a large commercial forest plantation (613km²).
Vole habitat is forest clear cuts (5-100 Ha) that last 12-15 years.

Field Voles in Kielder Forest



Field voles in Kielder Forest are cyclic (period 4 years)

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Hypothesis I: Predation by Weasels

- Weasels (*Mustela nivalis*) are the main terrestrial predator of voles in Kielder.



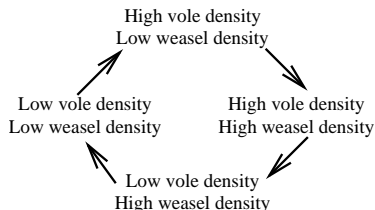
Hypothesis I: Predation by Weasels

- Weasels (*Mustela nivalis*) are the main terrestrial predator of voles in Kielder.
- In Finland, weasels drive multiannual vole cycles.
(*Evidence*: live-trapping and removal of weasels changes the vole dynamics to annual cycles).



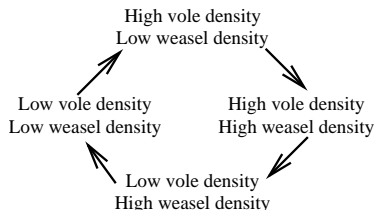
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Hypothesis I: Predation by Weasels

- Weasels (*Mustela nivalis*) are the main terrestrial predator of voles in Kielder.
- In Finland, weasels drive multiannual vole cycles. (*Evidence*: live-trapping and removal of weasels changes the vole dynamics to annual cycles).
- However, in Kielder, live-trapping and removal of weasels does not stop multiannual vole cycles.



Hypothesis II: Disease Effects on Reproduction

- Some rodent parasites affect host fecundity
e.g. cowpox in female bank voles and wood mice delays
reproduction until the following breeding season
(recent data from Manor Wood, UK)

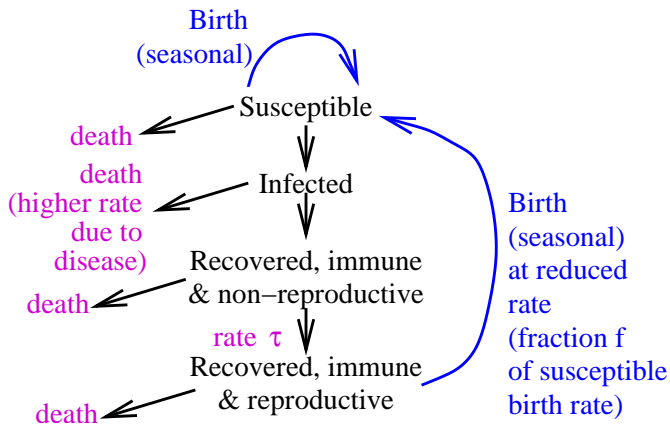
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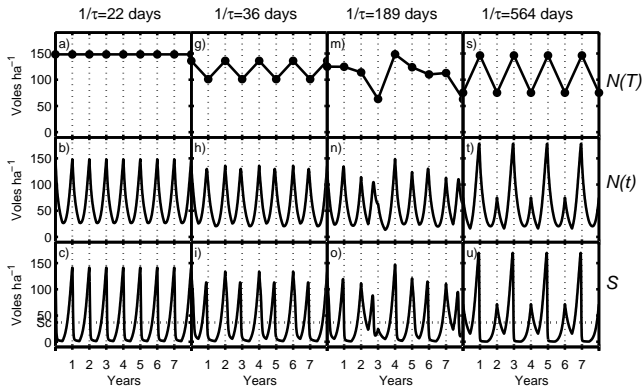
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- In Kielder, prevalences of field vole cowpox is significantly correlated with past population densities (also for TB)
- Therefore cowpox and other diseases may alter reproductive timing in a delayed density dependent manner – this could in turn lead to multiannual cycles

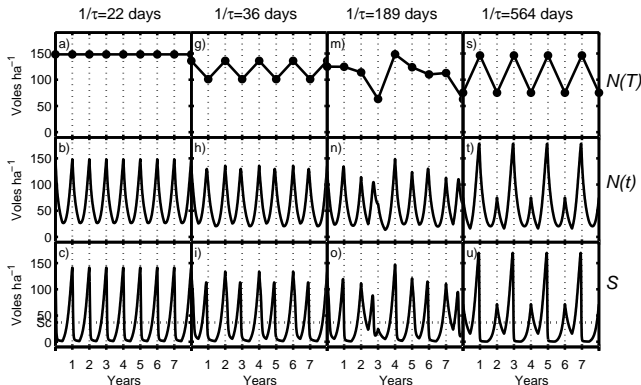
Hypothesis II: Schematic Illustration of the Model



Hypothesis II: Examples of Model Solutions

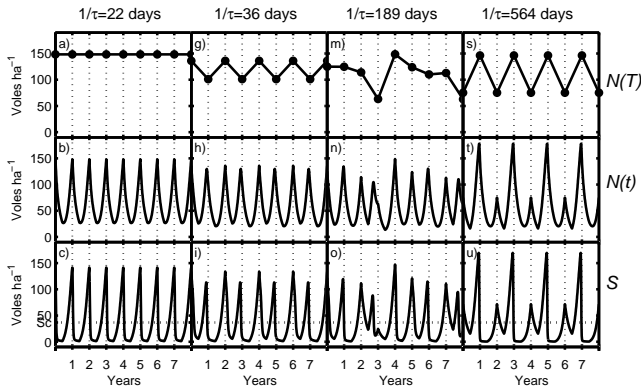


Hypothesis II: Examples of Model Solutions



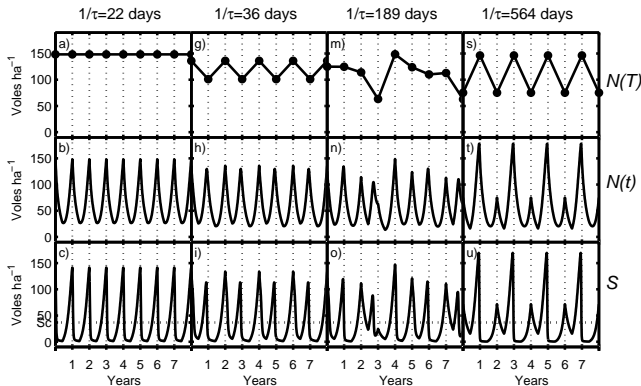
Basic dynamics: population crash in non-breeding season, recovery in breeding season.

Hypothesis II: Examples of Model Solutions



Large τ : recovery is fast enough for annual cycles.

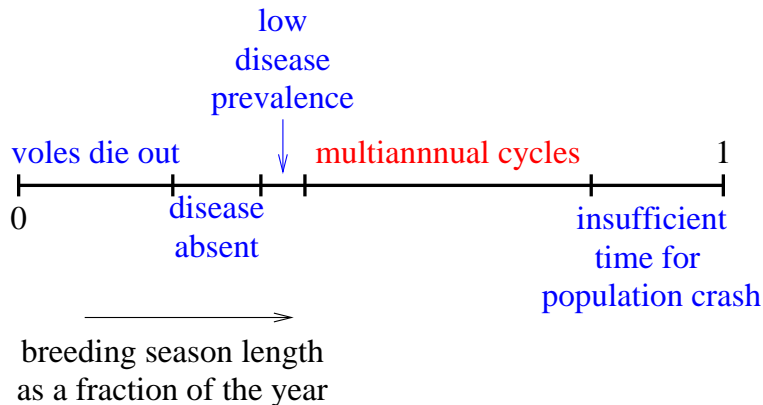
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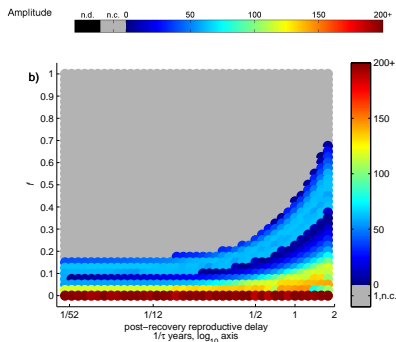
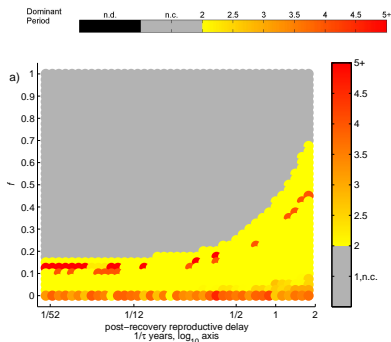
Smaller τ : recovery is too slow for annual cycles.

Hypothesis II: Seasonal Forcing is Important

Multiannual cycles do not occur if the reproductive season is either too short or too long



Hypothesis II: When are Disease-Induced Cycles Expected?



(other param estimates for field voles in Kielder forest and cowpox)

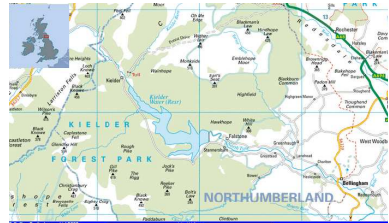
Hypothesis II: Conclusions

- Multiannual cycles can be caused by the combination of
 - 1 delayed reproduction following infection
 - 2 reduced fecundity after recovery
 - 3 annual forcing (seasonal reproduction)
- For cowpox in field voles in Kielder forest, the key requirement is a greatly reduced fecundity after infection (no data currently available)

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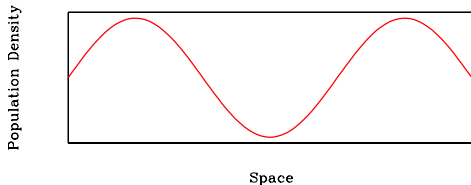
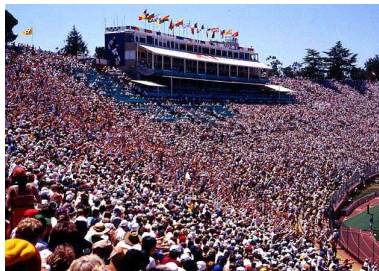


Field voles in Kielder Forest are cyclic (period 4 years)

Spatiotemporal field data shows that the cycles are spatially organised into a periodic travelling wave

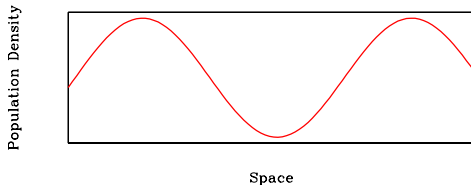
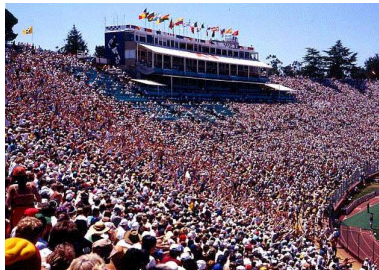
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A useful analogy
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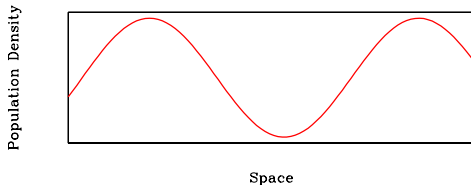
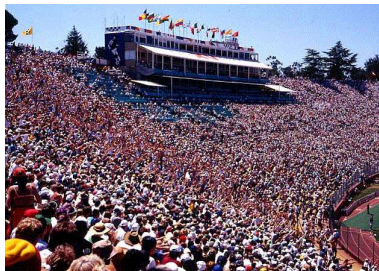
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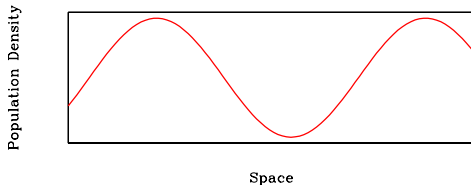
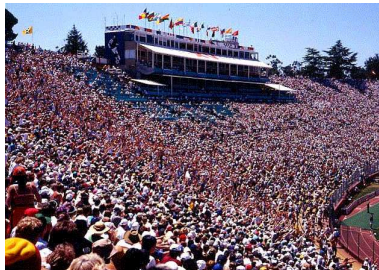
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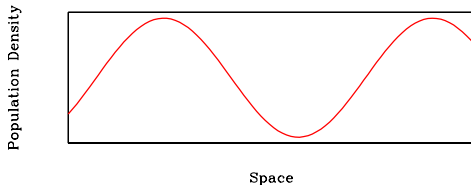
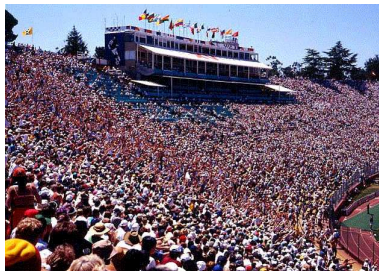
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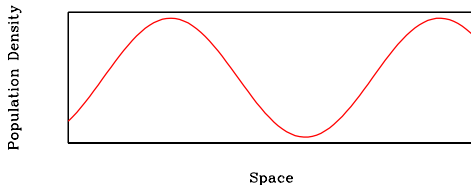
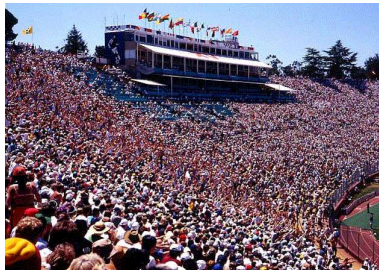
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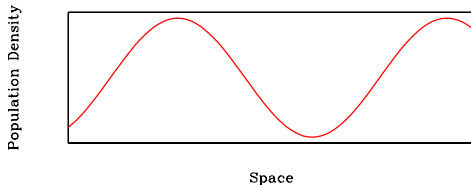
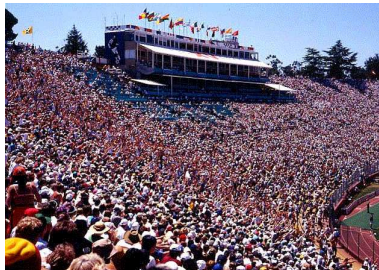
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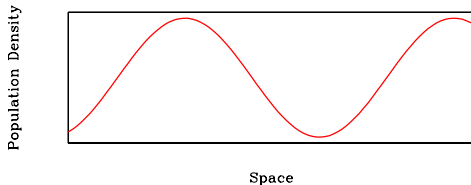
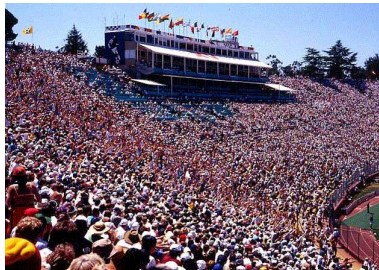
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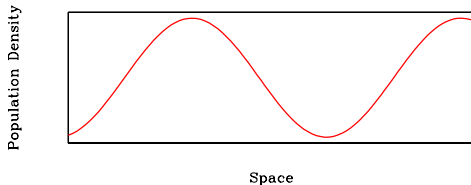
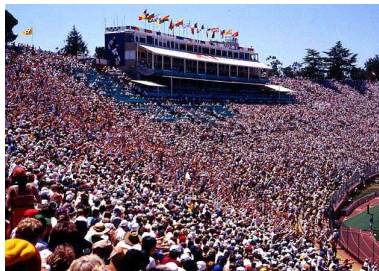
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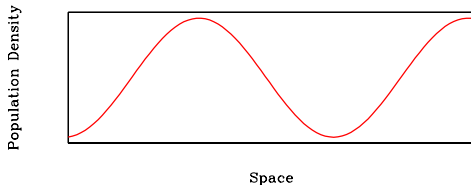
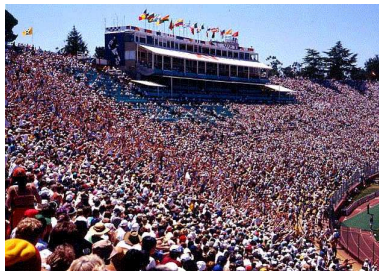
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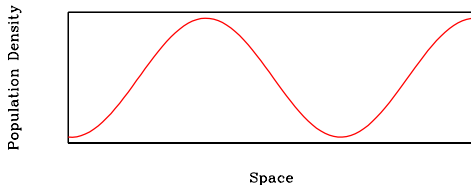
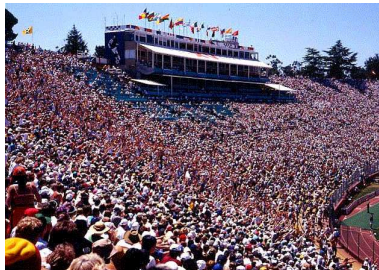
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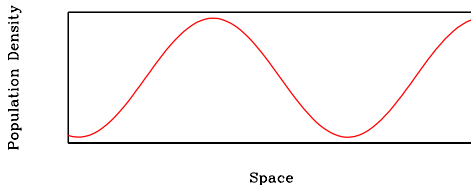
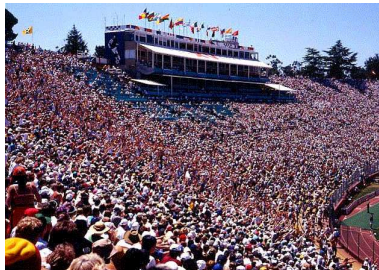
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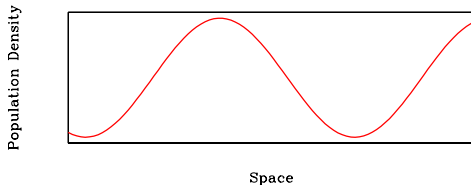
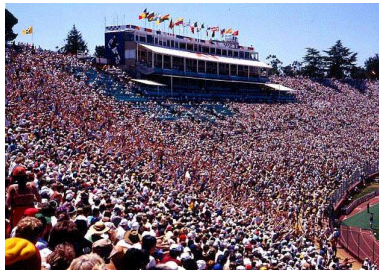
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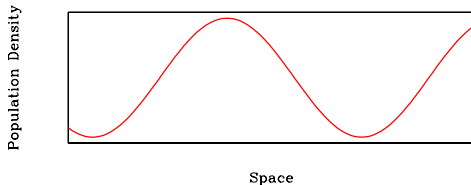
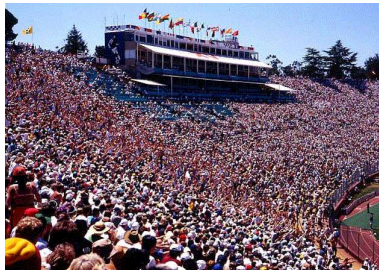
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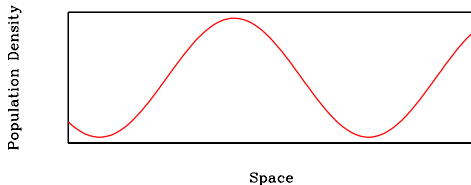
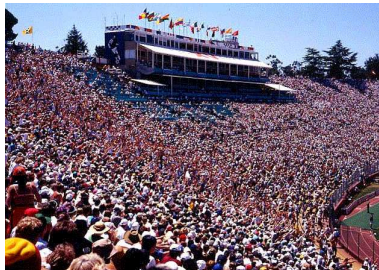
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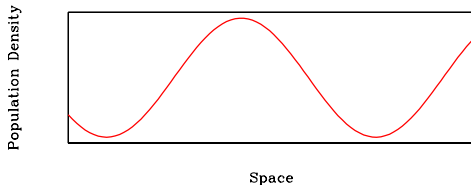
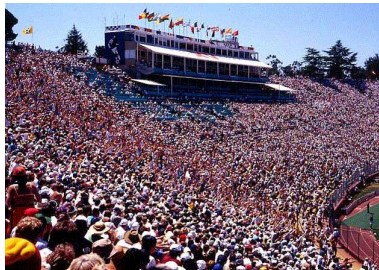
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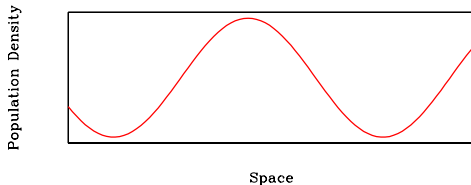
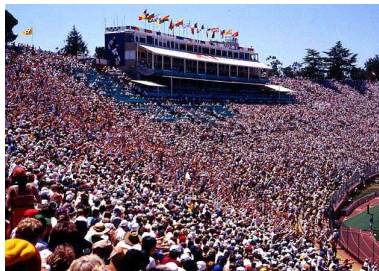
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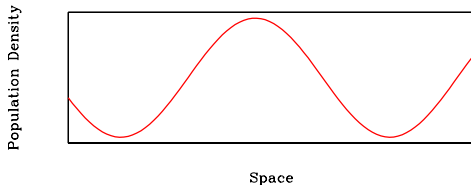
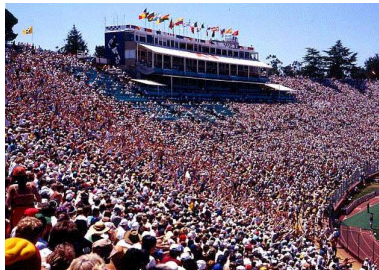
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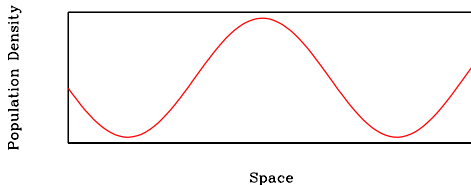
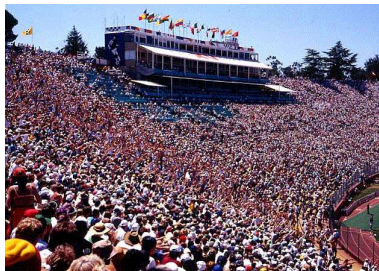
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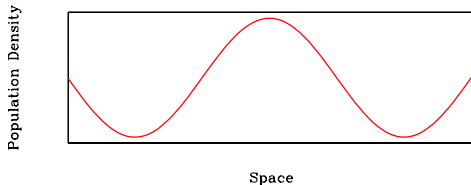
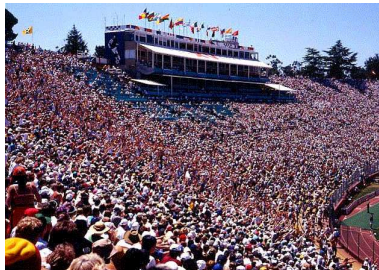
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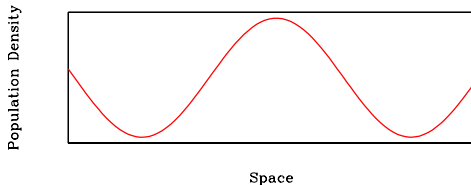
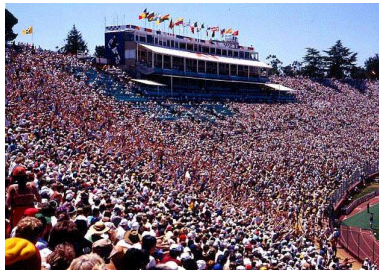
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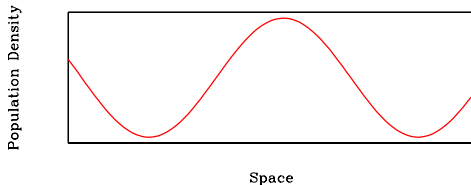
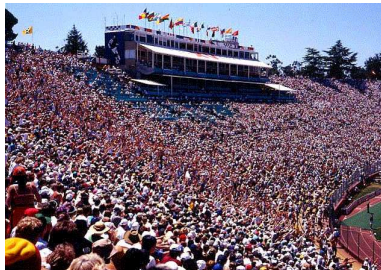
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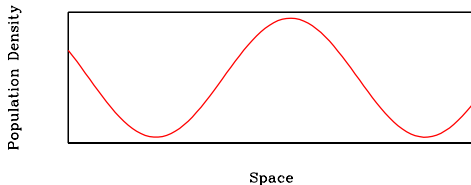
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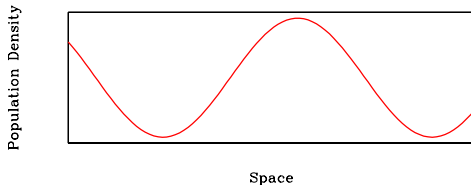
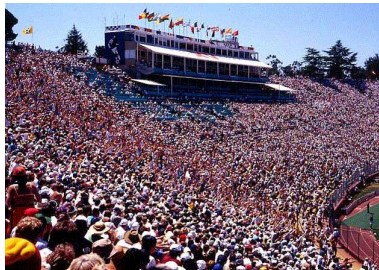
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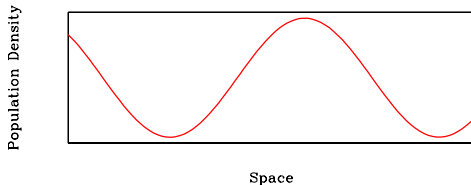
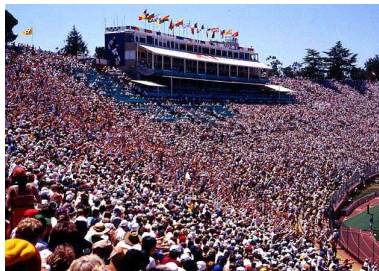
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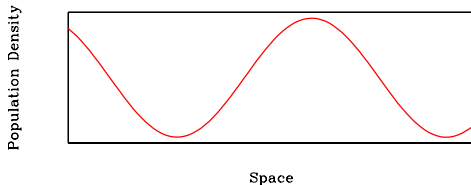
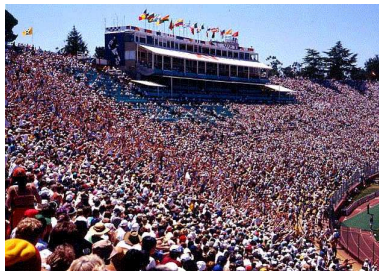
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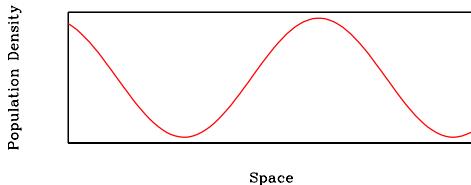
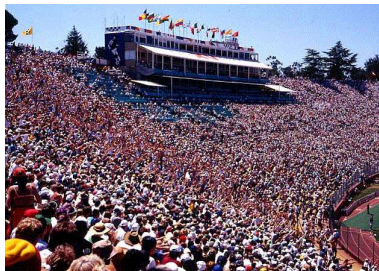
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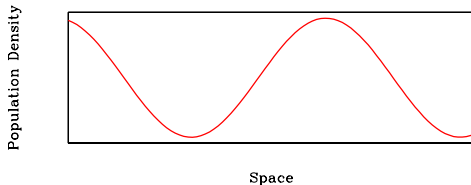
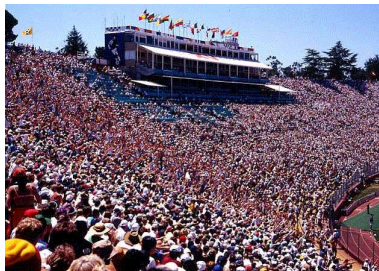
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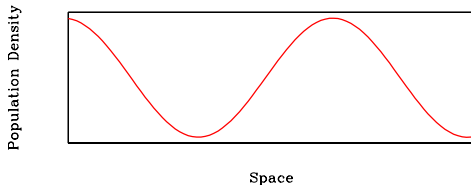
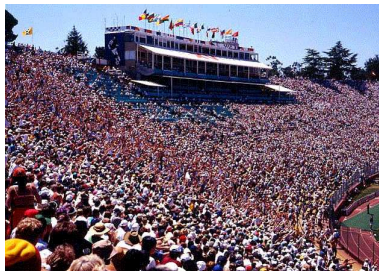
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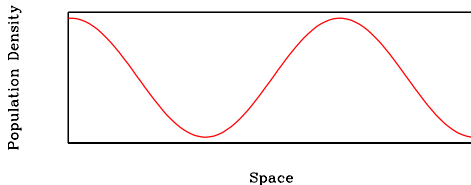
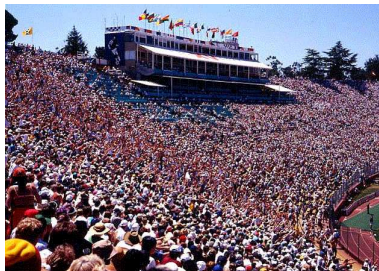
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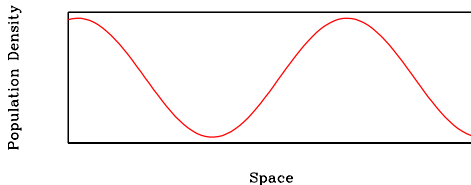
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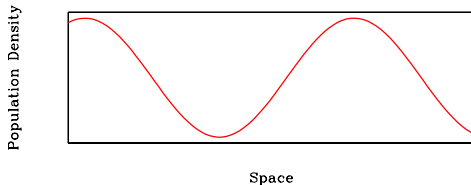
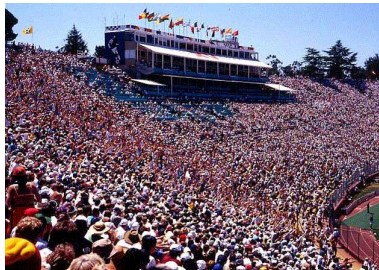
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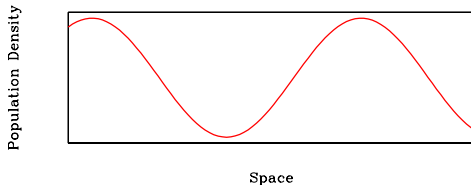
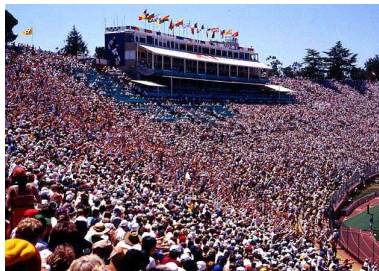
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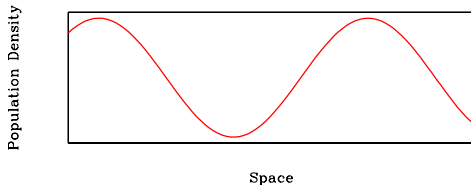
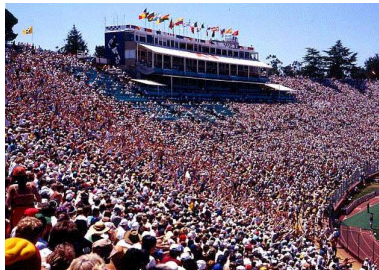
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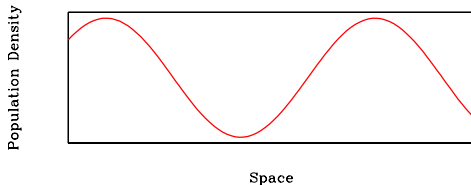
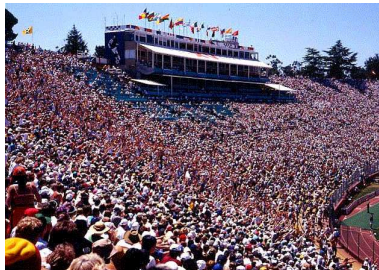
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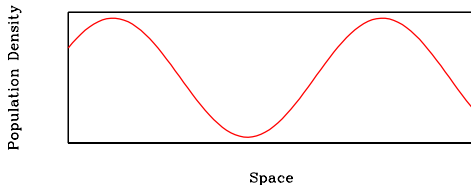
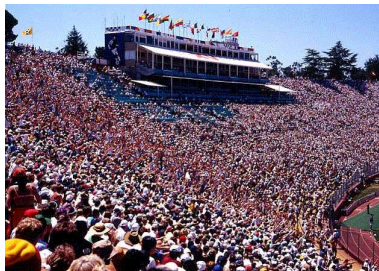
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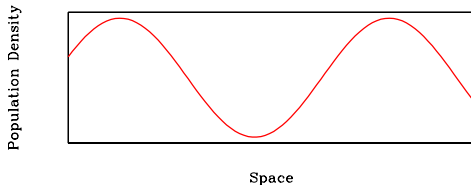
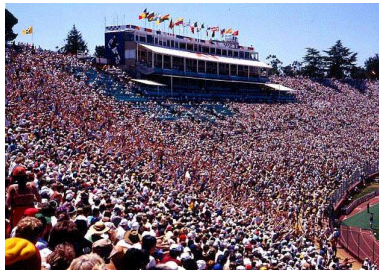
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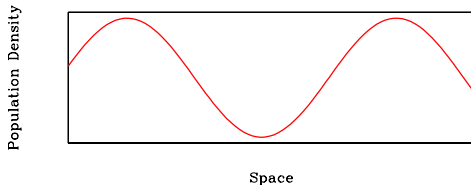
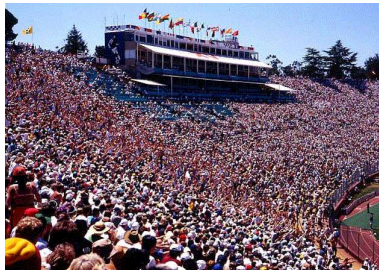
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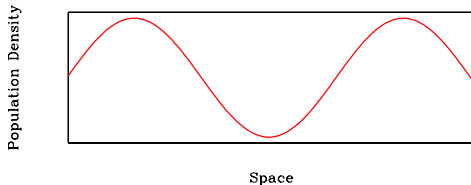
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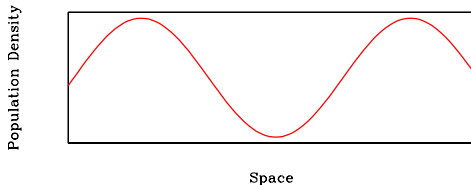
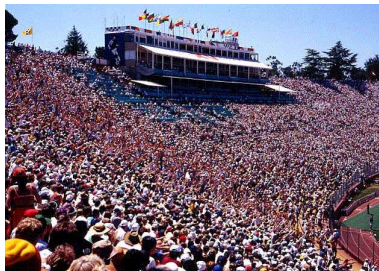
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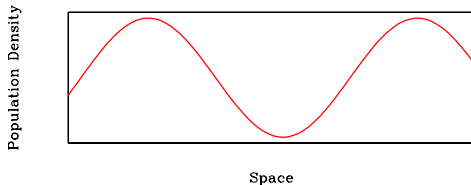
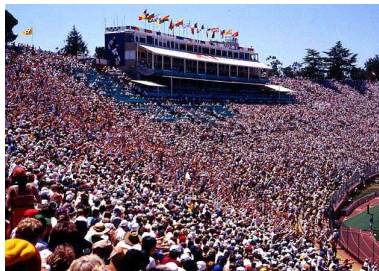
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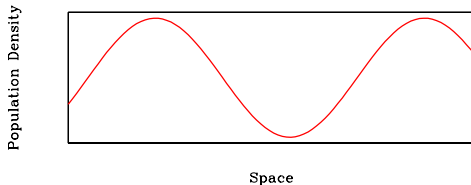
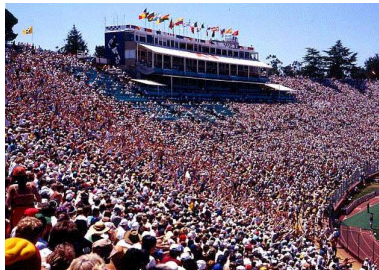
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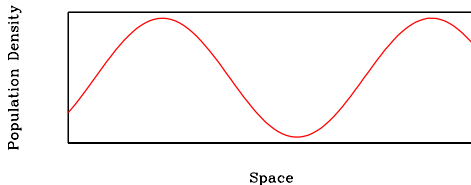
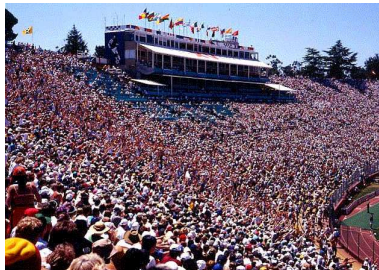
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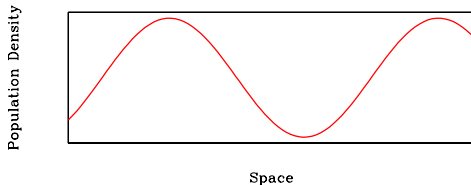
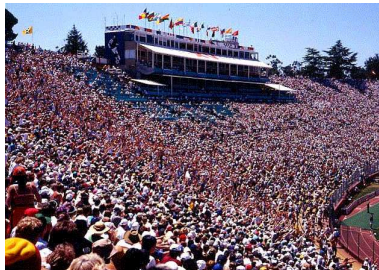
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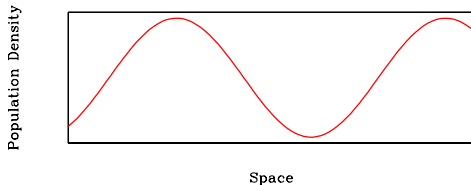
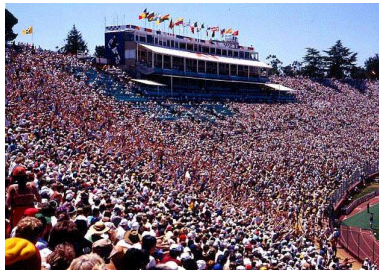
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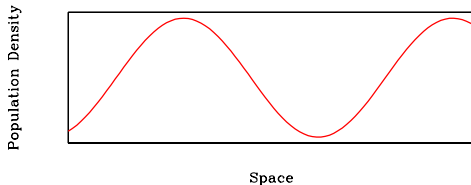
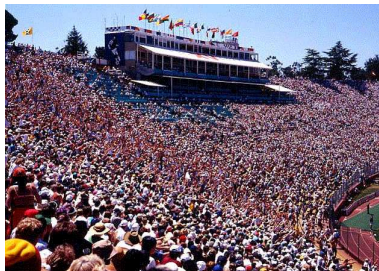
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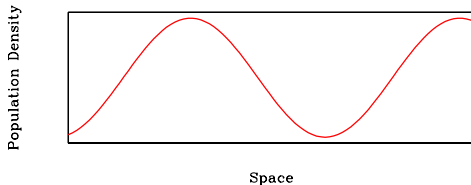
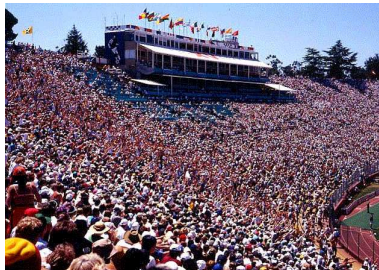
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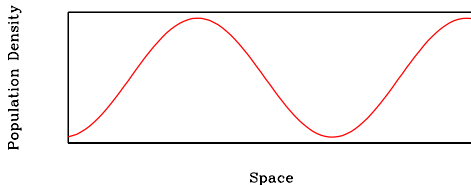
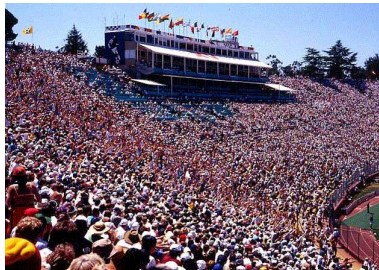
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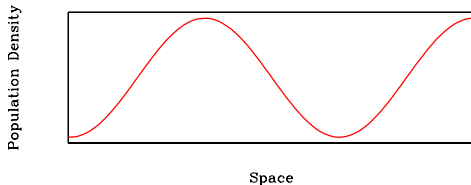
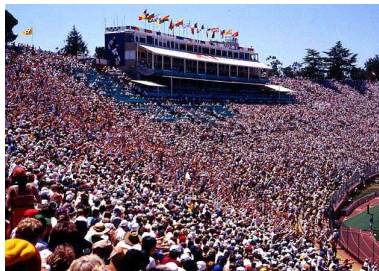
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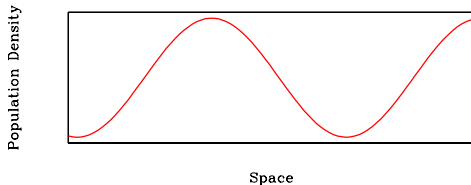
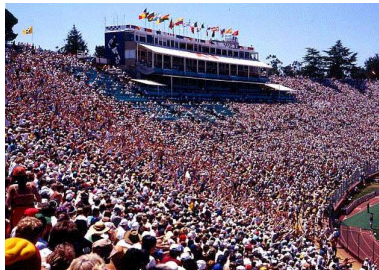
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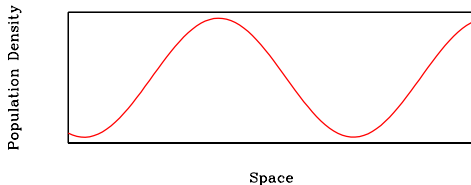
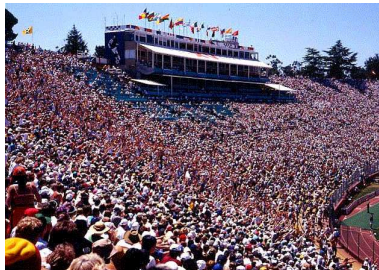
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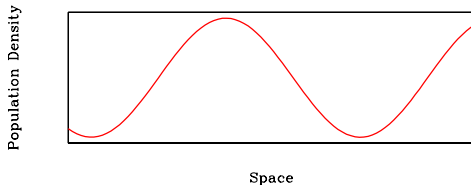
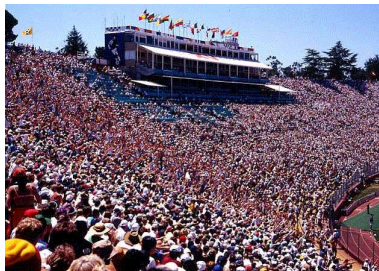
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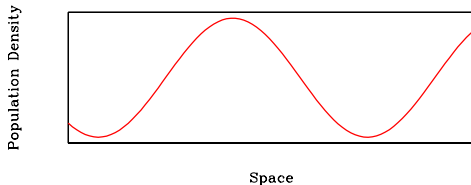
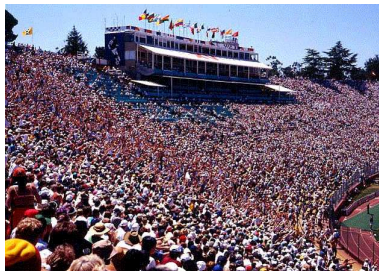
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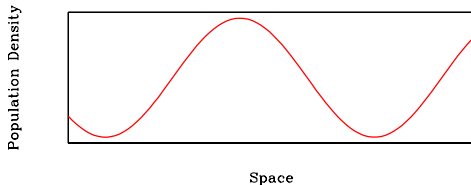
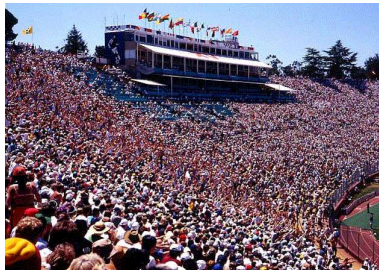
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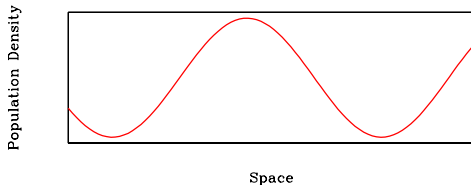
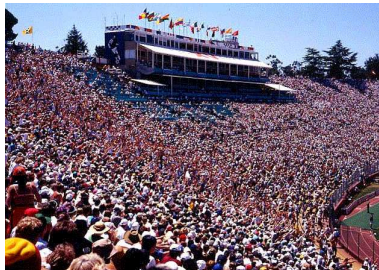
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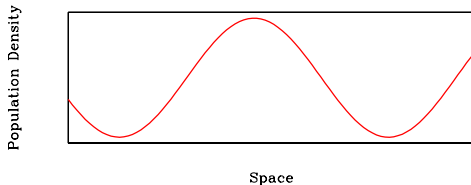
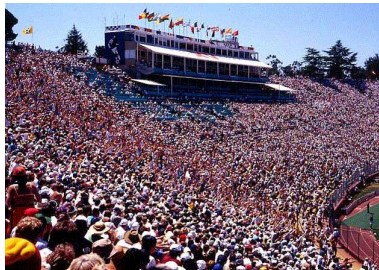
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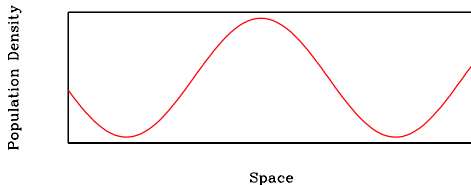
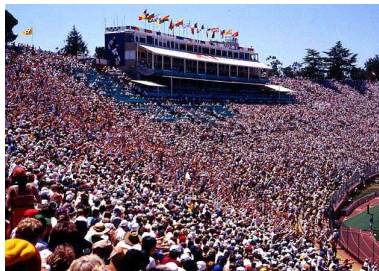
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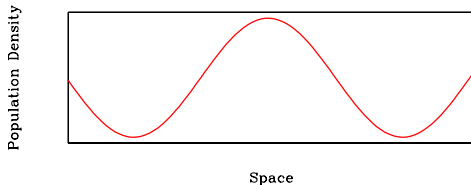
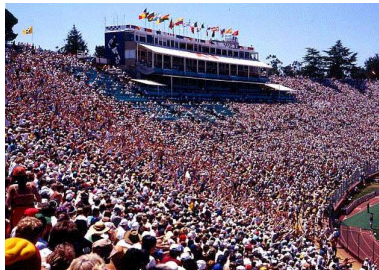
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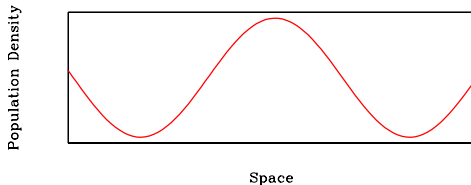
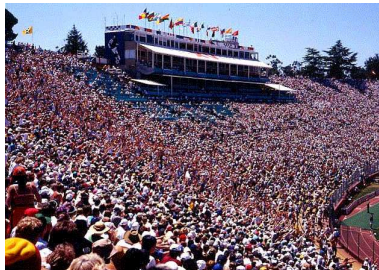
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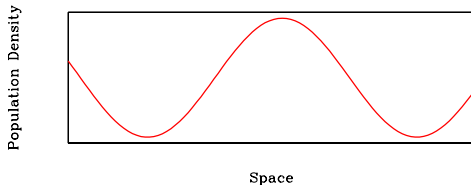
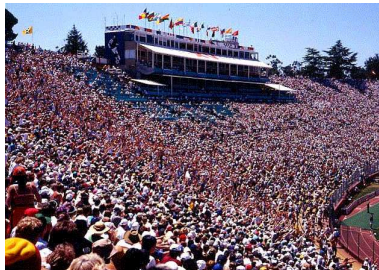
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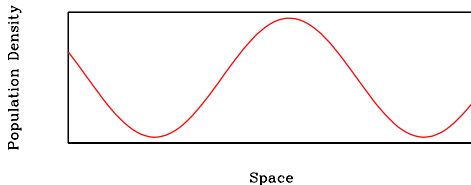
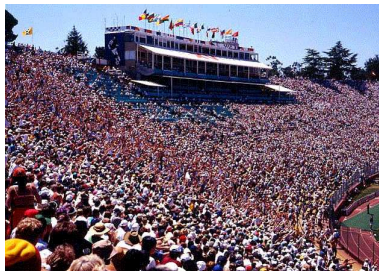
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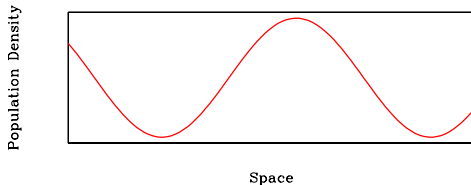
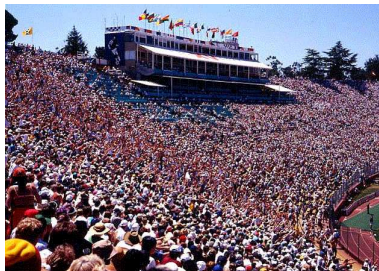
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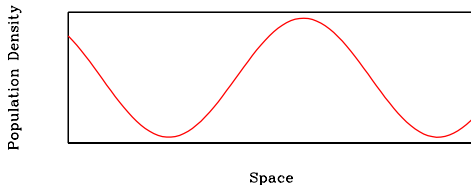
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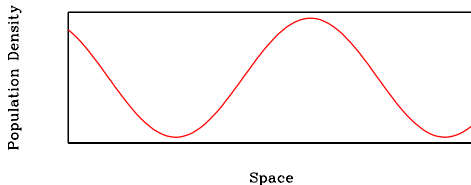
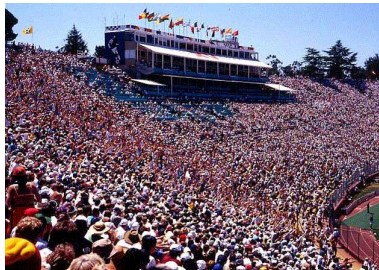
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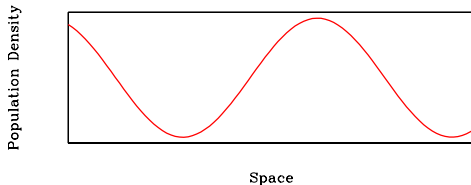
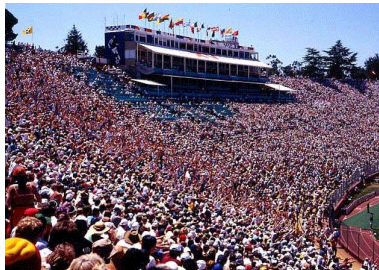
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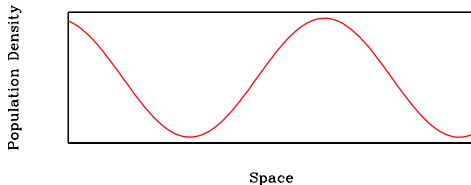
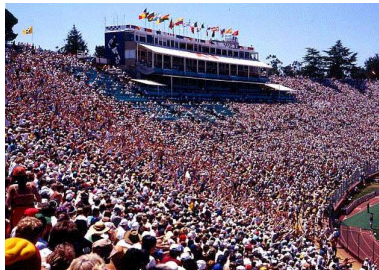
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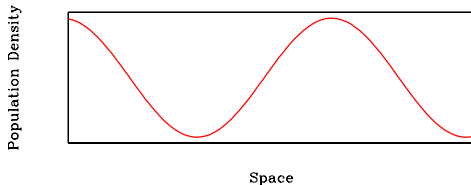
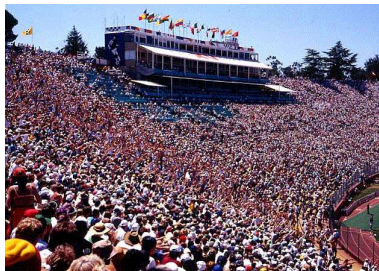
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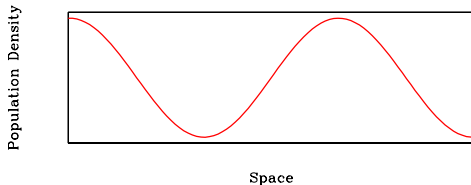
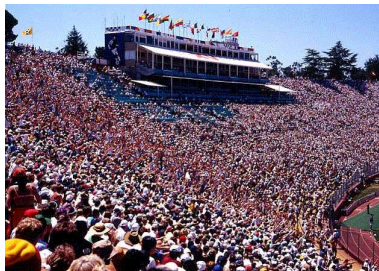
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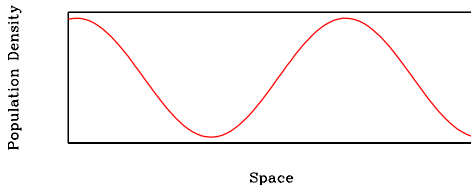
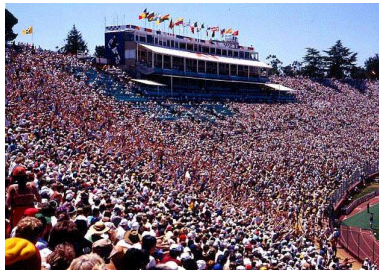
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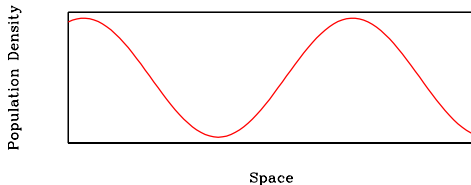
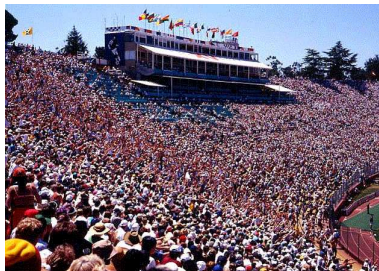
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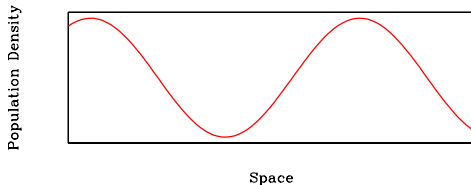
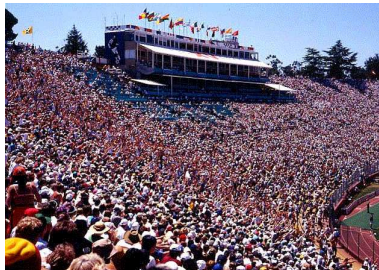
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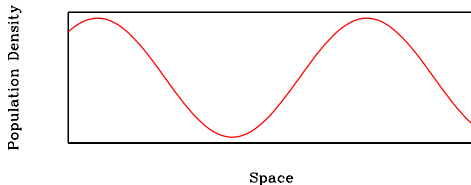
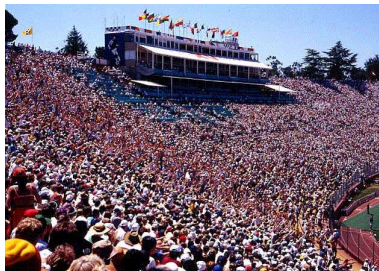
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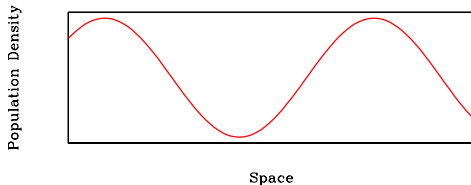
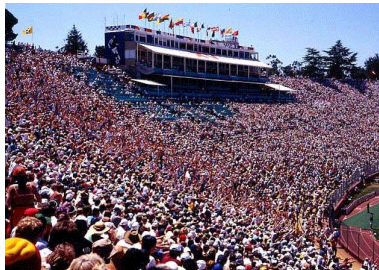
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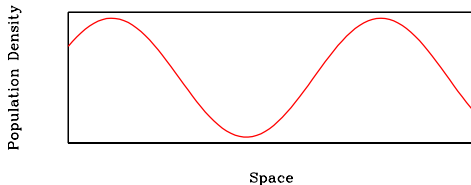
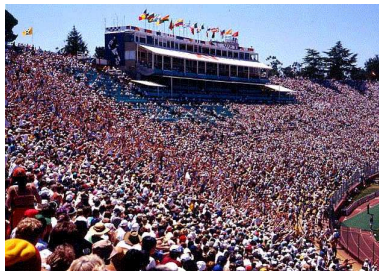
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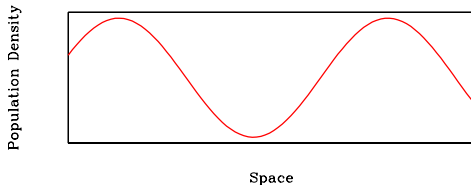
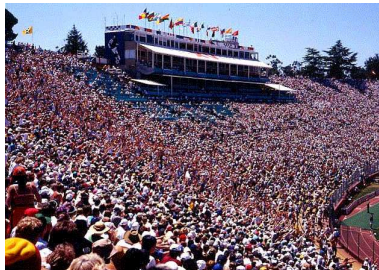
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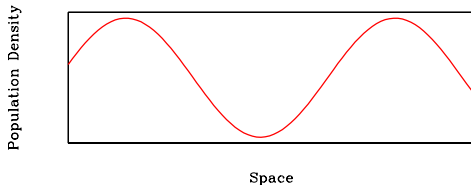
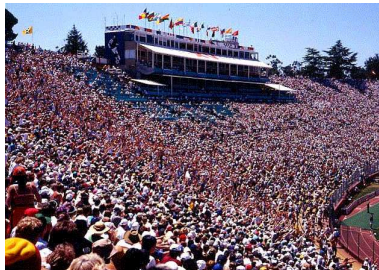
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A useful analogy
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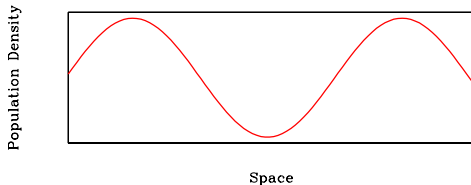
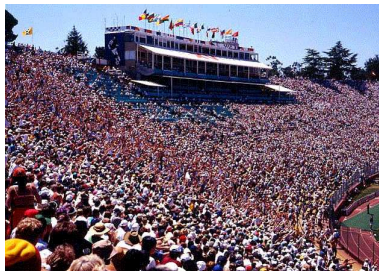
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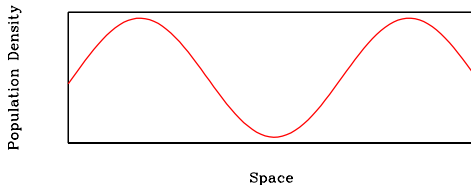
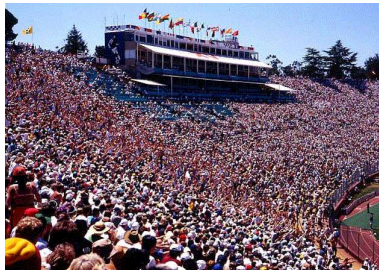
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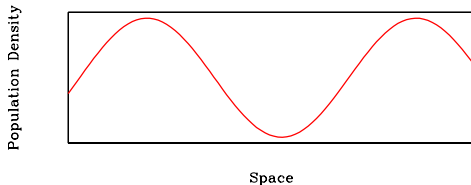
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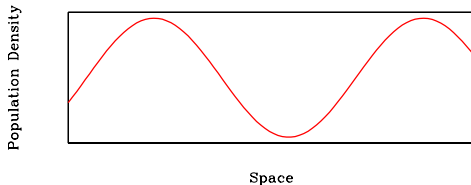
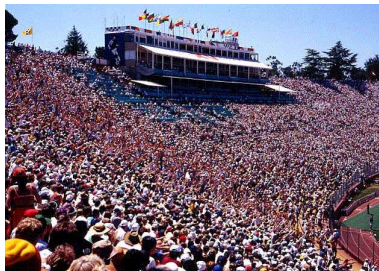
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Detection of a Periodic Wave from Field Data

Step 1: At each sampling site i , calculate a time series of growth rates $G_t^i = \log N_t^i - \log N_{t-1}^i$

Step 2: Calculate the cross-correlation coefficient of the growth rates for each pair of sites

$$\rho_{i,j} = \text{cov} \left(G_t^i, G_t^j \right) / \sqrt{\text{var}(G_t^i), \text{var}(G_t^j)}$$

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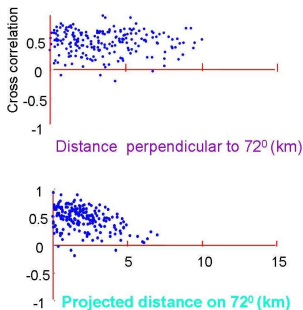
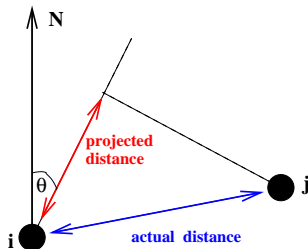
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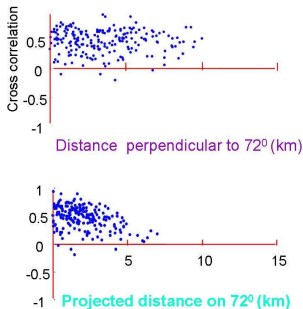
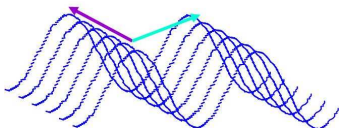
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Detection of a Periodic Wave from Field Data



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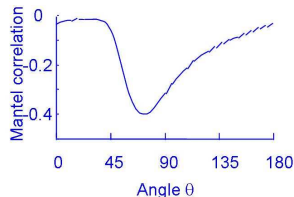
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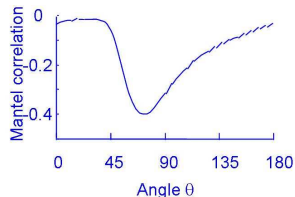
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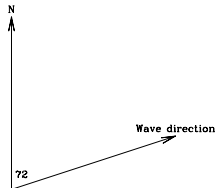
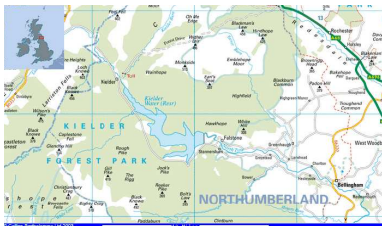
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Step 4: For each direction θ , use the Mantel correlogram to calculate the Mantel correlation, and plot this vs θ . A significant reduction in correlation in one direction suggests a travelling wave.



Step 5: Detailed fitting of statistical models, using projected distances, gives robust parameter estimates and significance levels. This implies a wave speed of 19km/year.

What Causes the Spatial Component of the Oscillations?



Hypothesis: the periodic travelling waves are caused by the large central reservoir

Outline

- 1 Introduction: Field Voles in Kielder Forest
- 2 Why Do Field Vole Populations Cycle in Kielder?
- 3 Spatiotemporal Field Data
- 4 Mathematical Modelling of Periodic Travelling Waves

Predator-Prey Equations

predators

$$\frac{\partial p}{\partial t} = \underbrace{D_p \nabla^2 p}_{\text{dispersal}} + \underbrace{akph/(1 + kh)}_{\text{benefit from predation}} - \underbrace{bp}_{\text{death}}$$

prey

$$\frac{\partial h}{\partial t} = \underbrace{D_h \nabla^2 h}_{\text{dispersal}} + \underbrace{rh(1 - h/h_0)}_{\text{intrinsic birth \& death}} - \underbrace{ckph/(1 + kh)}_{\text{predation}}$$

Boundary Condition at the Reservoir Edge

- Voles are an important prey species for owls and kestrels
- The open expanse of Kielder Water will greatly facilitate hunting at its edge



Short eared owl



Common kestrel

Boundary Condition at the Reservoir Edge

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- Therefore we expect very high vole loss at the reservoir edge, implying a Robin boundary condition

$$\frac{d}{dx} \left(\text{vole density} \right) = - \left(\text{large constant} \right) \cdot \left(\text{vole density} \right)$$

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- To a first approximation, this boundary condition is just

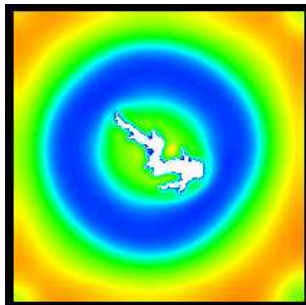
$$\left(\begin{array}{c} \text{vole} \\ \text{density} \end{array} \right) = 0$$

Field Vole Wave Generation Question

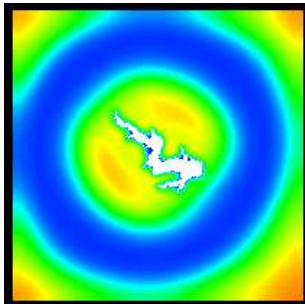
Question

Could the boundary condition at the reservoir edge play a role in generating the periodic travelling waves?

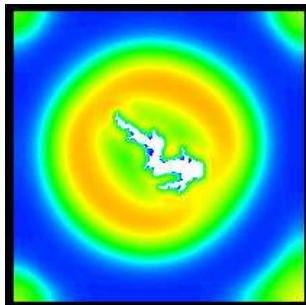
Typical Model Solution



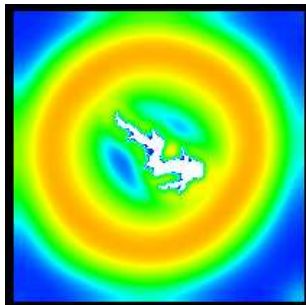
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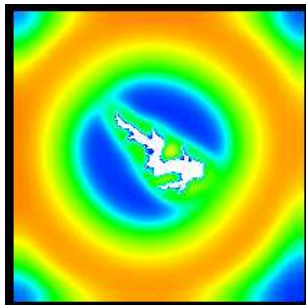
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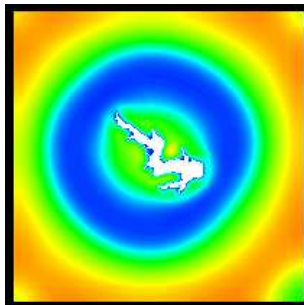
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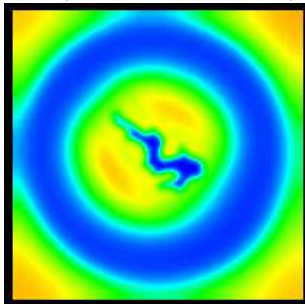


Movie of Typical Model Solution

Click here to
play the movie

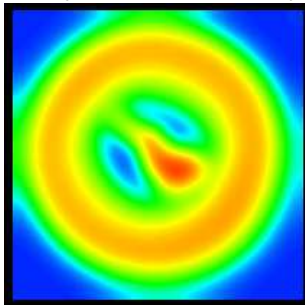
Removing the Reservoir

The periodic waves are driven by the reservoir. This is most easily demonstrated by simulating removal of the reservoir.



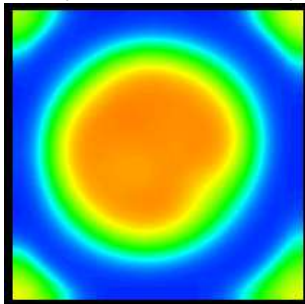
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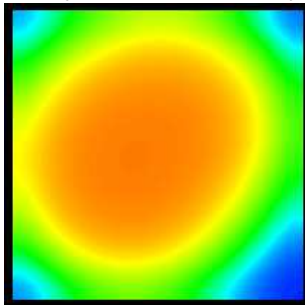
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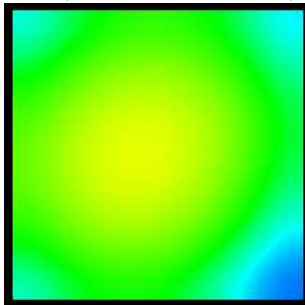
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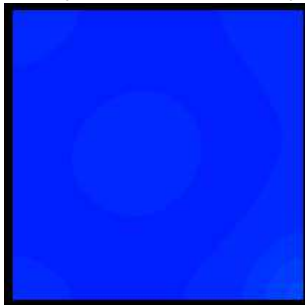
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Conclusion of Periodic Travelling Wave Study

The expected behaviour at the edge of Kielder Water provides a possible explanation for the observed periodic travelling waves

Future Work

- Spatiotemporal dynamics of the disease model
- More realistic modelling of the Kielder Forest habitat

List of Frames

1

Introduction: Field Voles in Kielder Forest

- Multiannual Rodent Cycles
- Field Voles in Kielder Forest

2

Why Do Field Vole Populations Cycle in Kielder?

- Hypothesis I: Predation by Weasels
- Hypothesis II: Disease Effects on Reproduction
- Hypothesis II: Seasonal Forcing is Important
- Hypothesis II: When are Disease-Induced Cycles Expected?

3

Spatiotemporal Field Data

- Field Voles in Kielder Forest
- What is a Periodic Travelling Wave?
- Detection of a Periodic Wave from Field Data
- What Causes the Spatial Component of the Oscillations?

4

Mathematical Modelling of Periodic Travelling Waves

- Predator-Prey Equations
- Boundary Condition at the Reservoir Edge
- Typical Model Solution
- Removing the Reservoir
- Conclusion of Periodic Travelling Wave Study