

Modelling Vegetation Pattern Formation in Semi-Deserts

Jonathan A. Sherratt

School of Mathematical & Computer Sciences
and
Maxwell Institute for Mathematical Sciences
Heriot-Watt University

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This talk can be downloaded from my web site

`www.ma.hw.ac.uk/~jas`

In collaboration with Gabriel Lord



Outline

- 1 Ecological Background
- 2 The Mathematical Model
- 3 Model Predictions: When Do Patterns Occur?
- 4 Model Predictions: Which Pattern Forms?
- 5 Conclusions

Outline

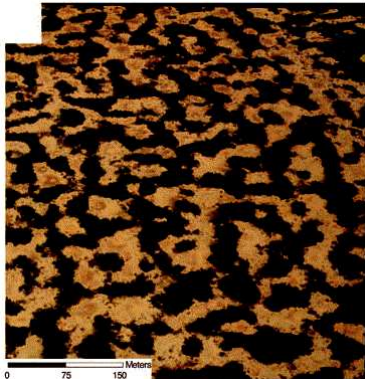
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Vegetation Pattern Formation



- Vegetation patterns are found in semi-arid areas of Africa, Australia and Mexico (rainfall 100-700 mm/year)
- First identified by aerial photos in 1950s
- Plants vary from grasses to shrubs and trees

More Pictures of Vegetation Patterns



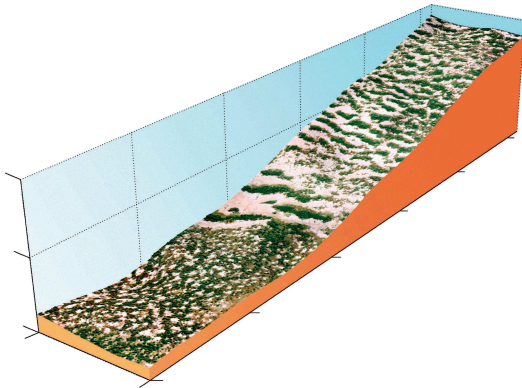
Labyrinth of bushy
vegetation in Niger

More Pictures of Vegetation Patterns



Striped pattern of
bushy vegetation
in Niger

Vegetation Pattern Formation (contd)



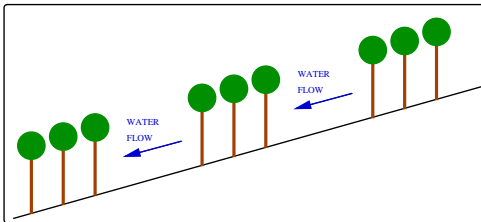
- On flat ground, irregular mosaics of vegetation are typical
- On slopes, the patterns are stripes, parallel to contours (“Tiger bush”)

Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water

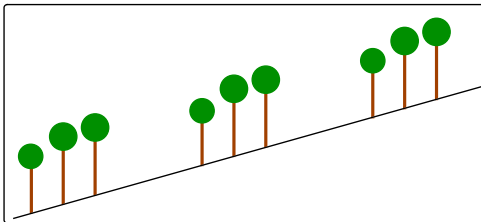
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- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



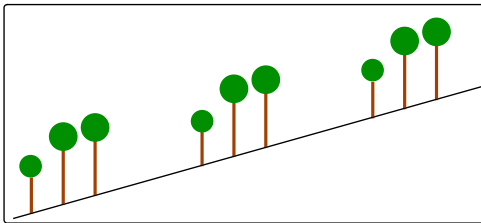
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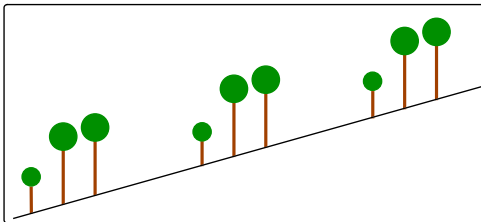
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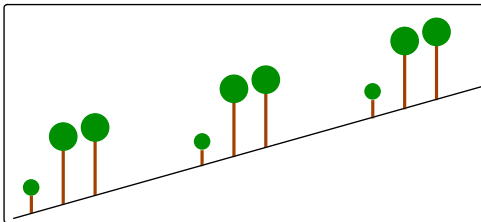
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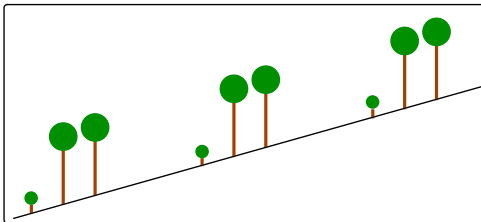
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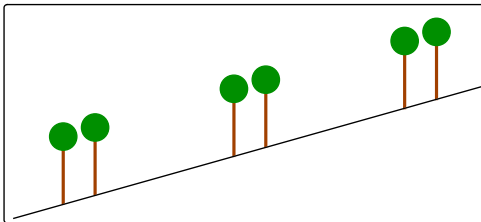
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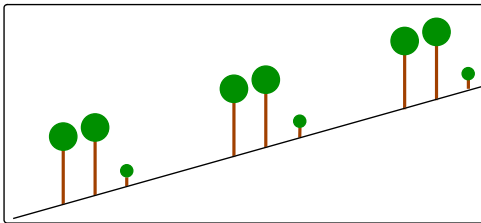
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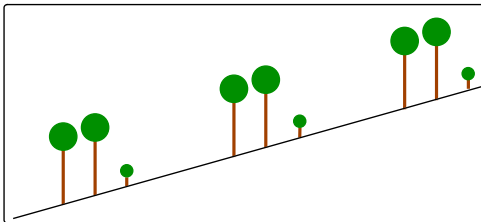
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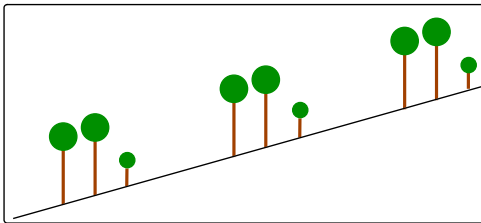
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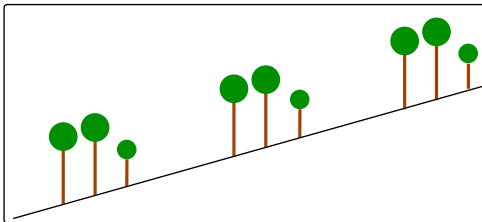
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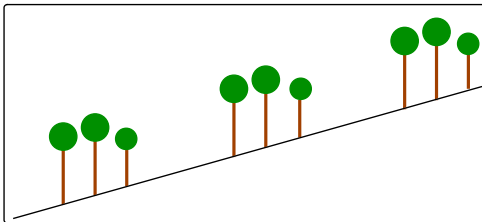
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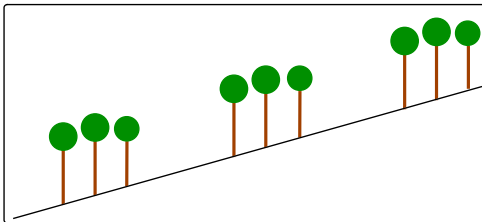
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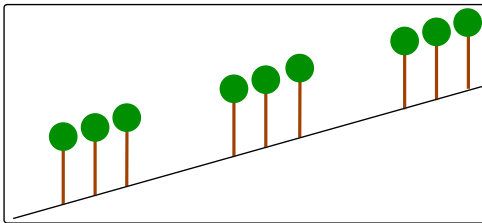
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Mechanisms for Vegetation Patterning

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- This mechanism suggests that the stripes would move uphill; this remains controversial.

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Mathematical Model of Klausmeier

Rate of change = Rainfall – Evaporation – Uptake by + Flow
of water plants downhill

Rate of change = Growth, proportional – Mortality + Random
plant biomass to water uptake dispersal

$$\partial w / \partial t = A - w - wu^2 + \nu \partial w / \partial x$$

$$\partial u / \partial t = wu^2 - Bu + \partial^2 u / \partial x^2$$

Mathematical Model of Klausmeier

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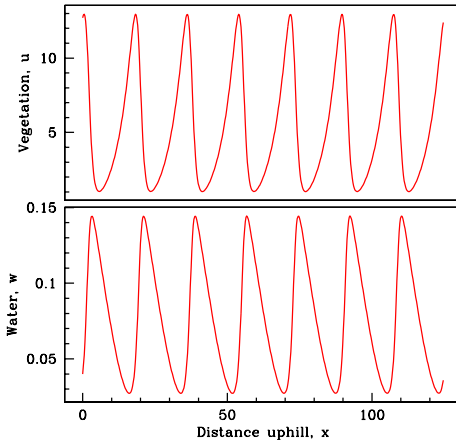
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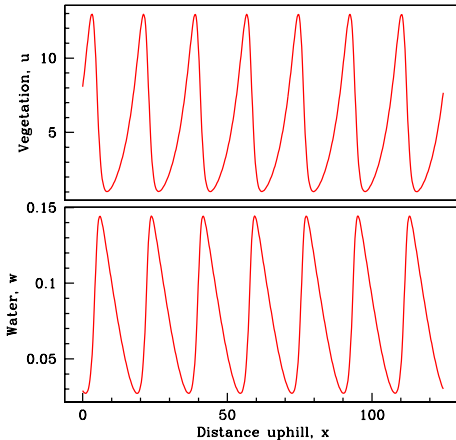
$$\partial u / \partial t = wu^2 - Bu + \partial^2 u / \partial x^2$$

The nonlinearity in water uptake by plants arises because the presence of roots increases water infiltration into the soil.

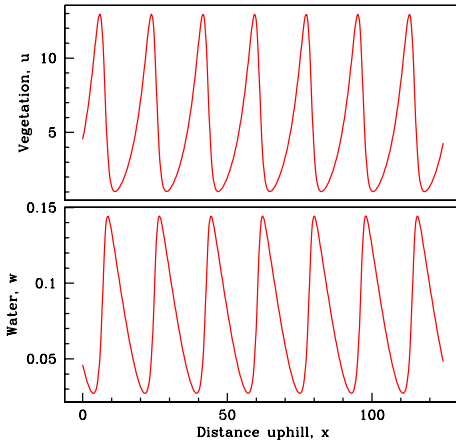
Typical Solution of the Model



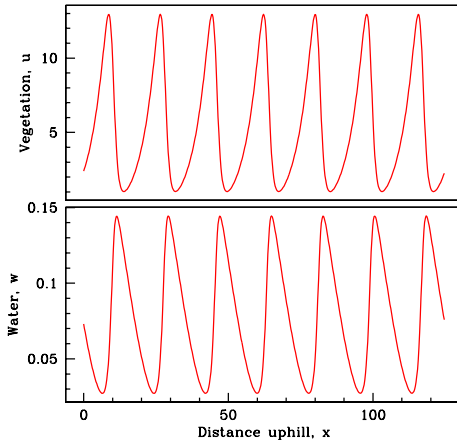
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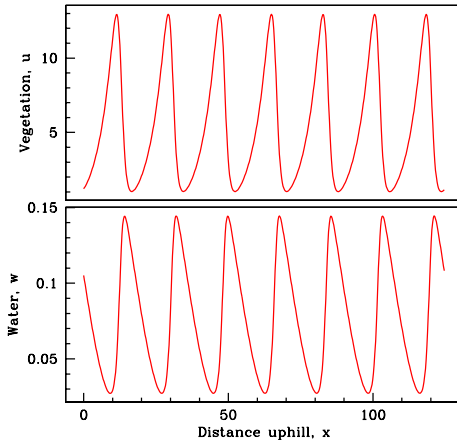
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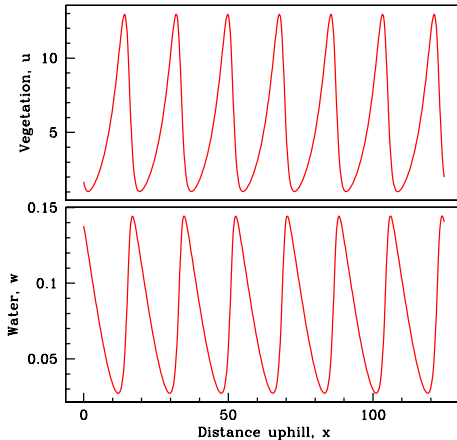
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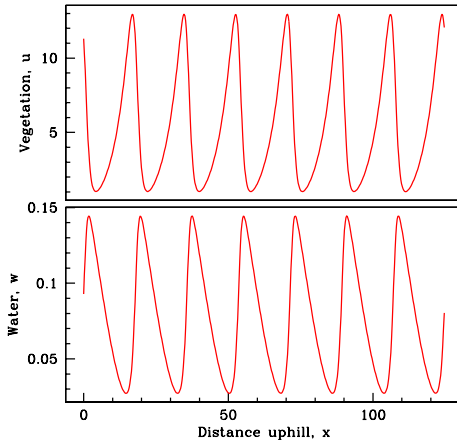
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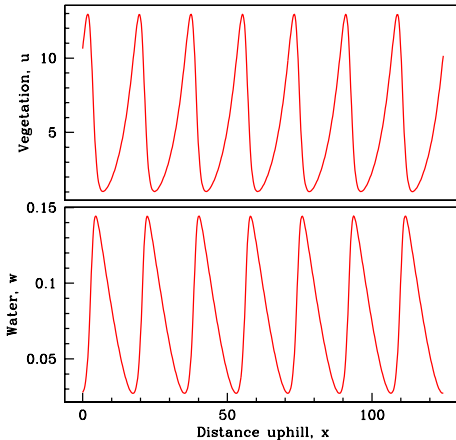
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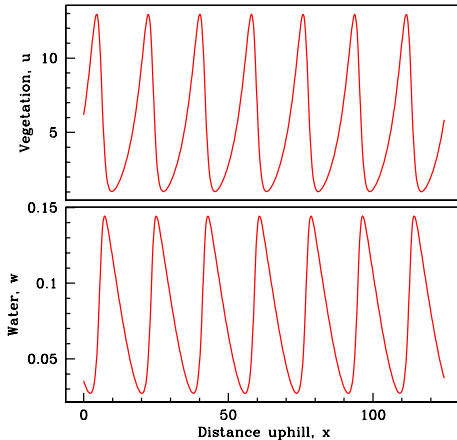
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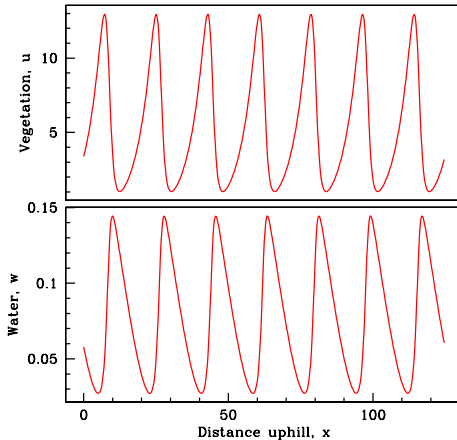
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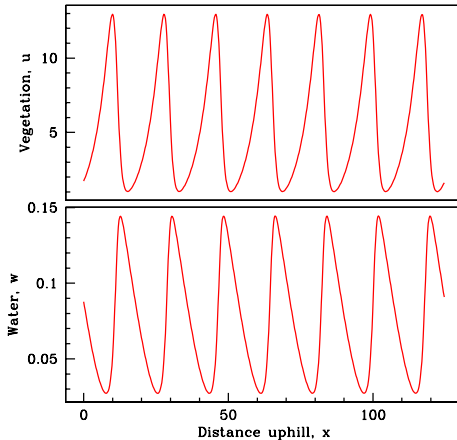
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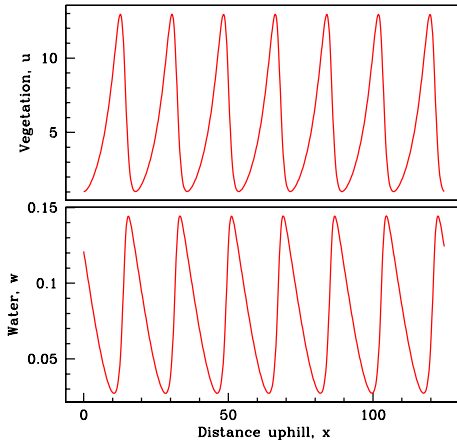
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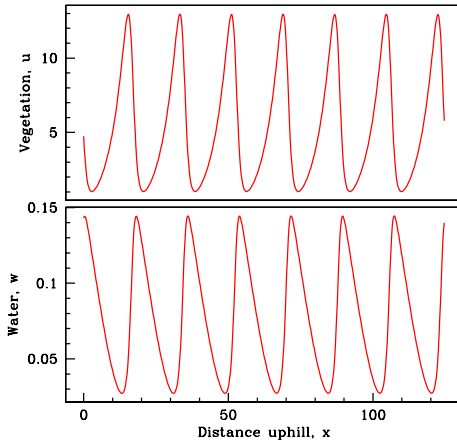
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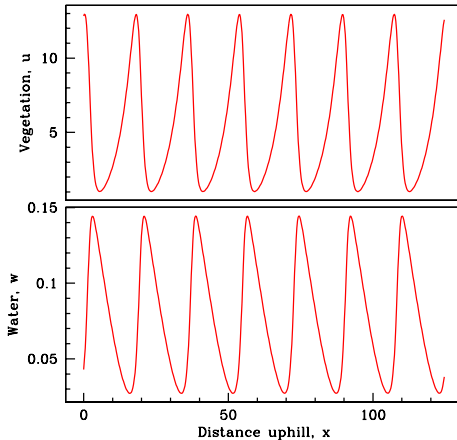
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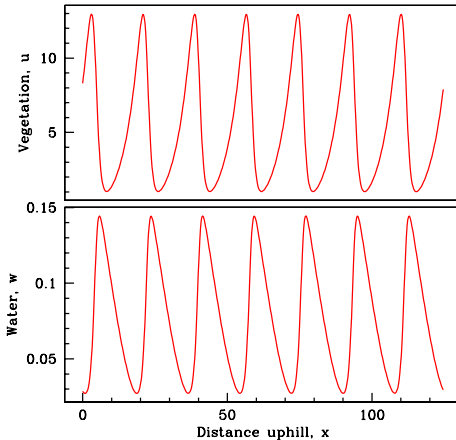
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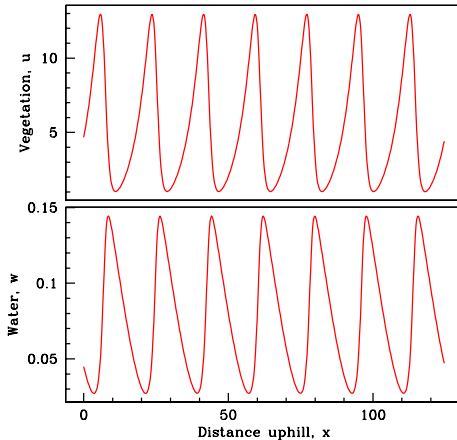
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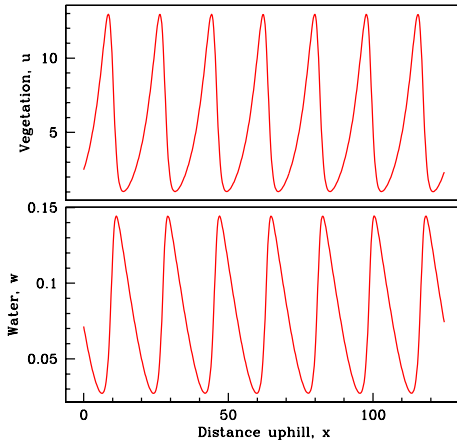
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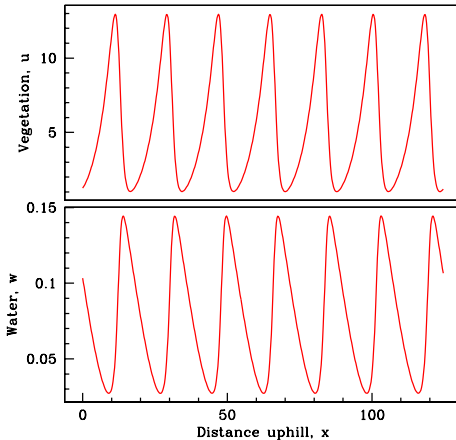
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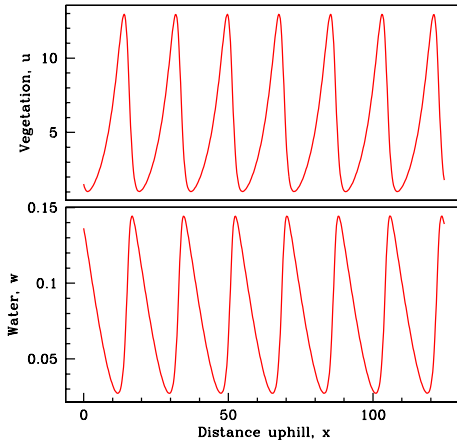
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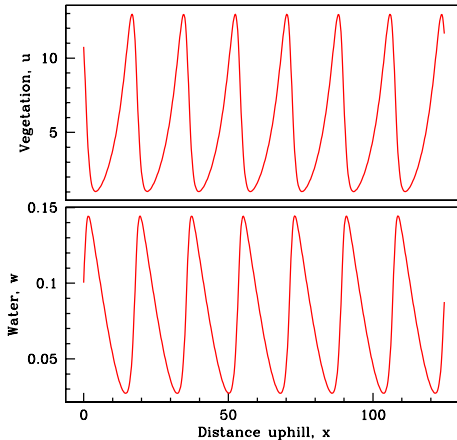
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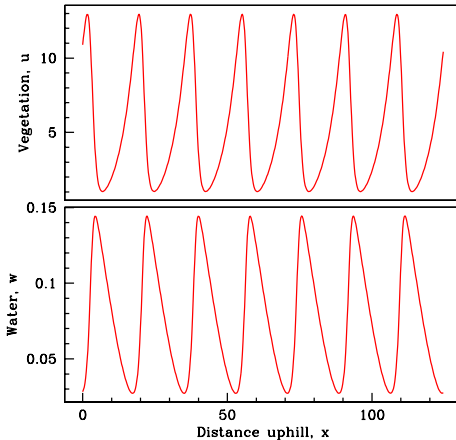
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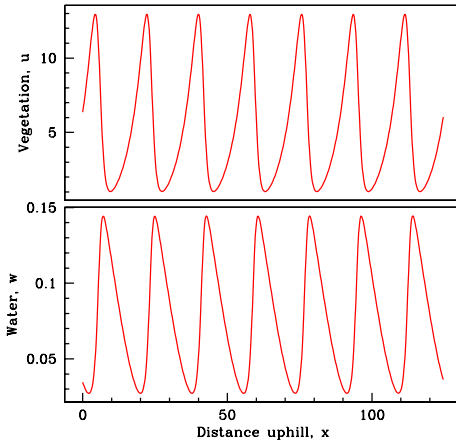
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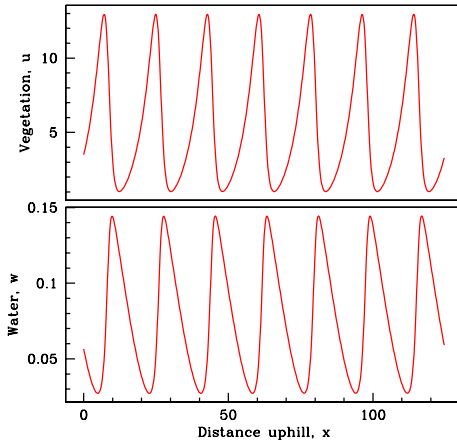
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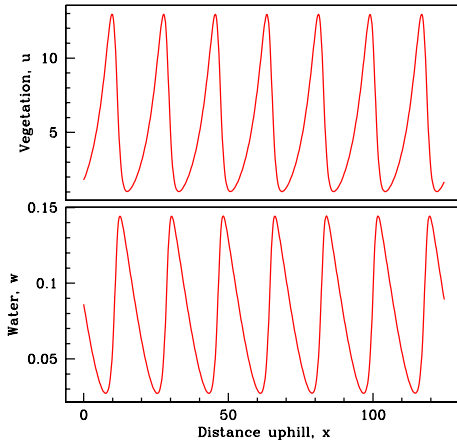
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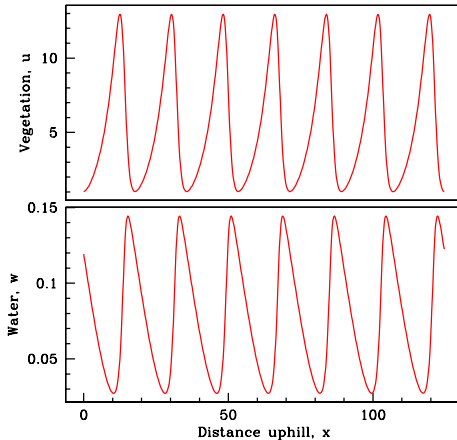
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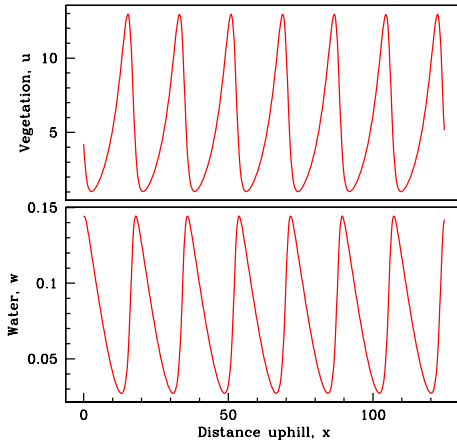
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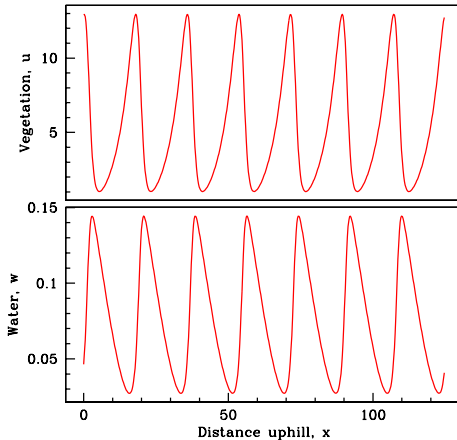
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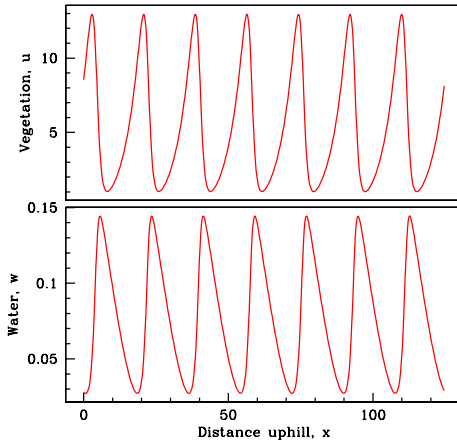
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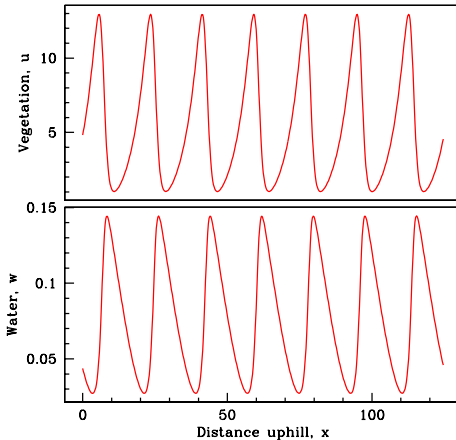
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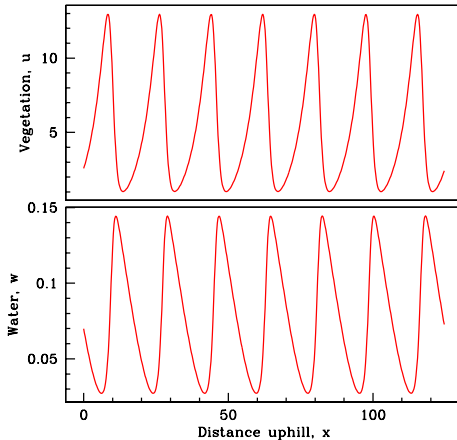
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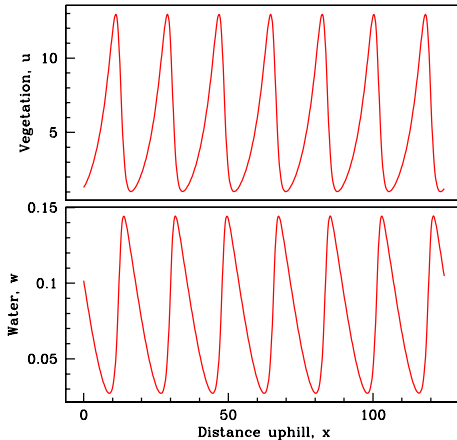
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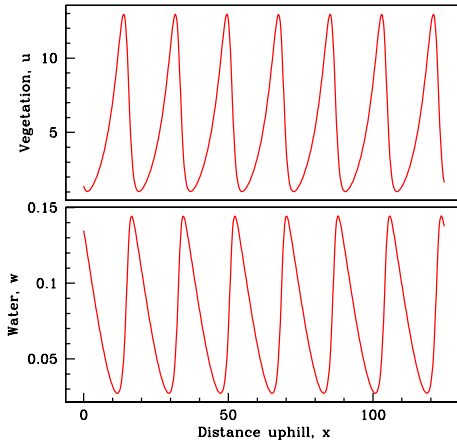
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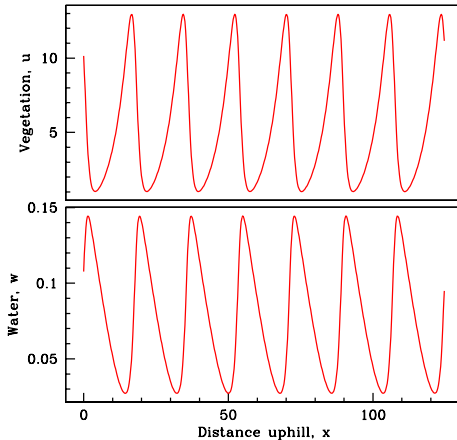
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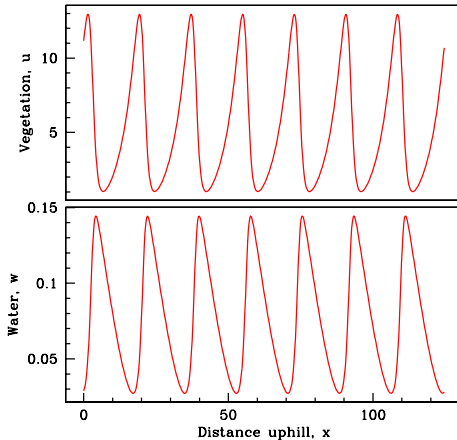
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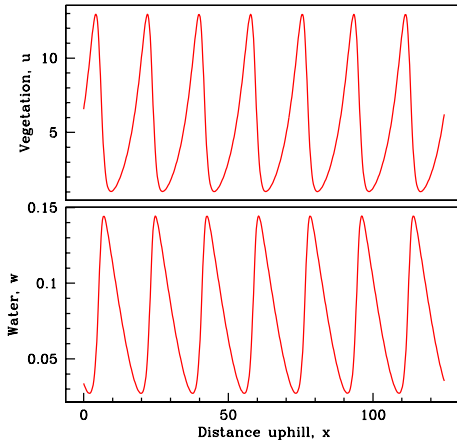
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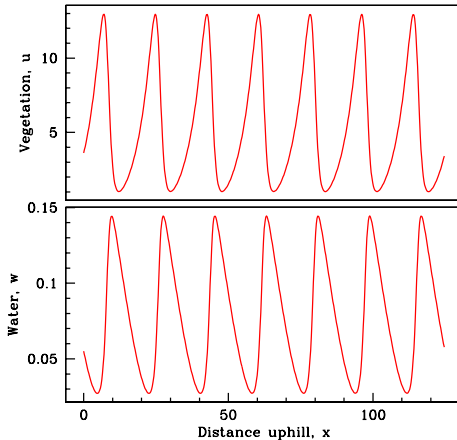
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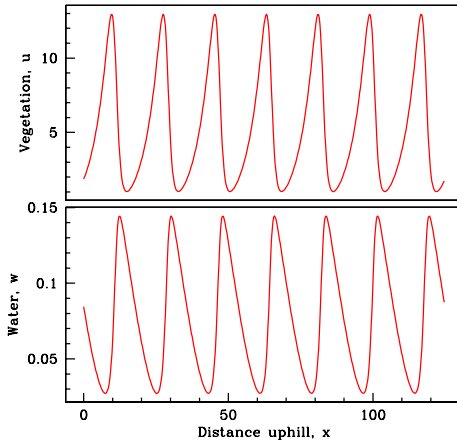
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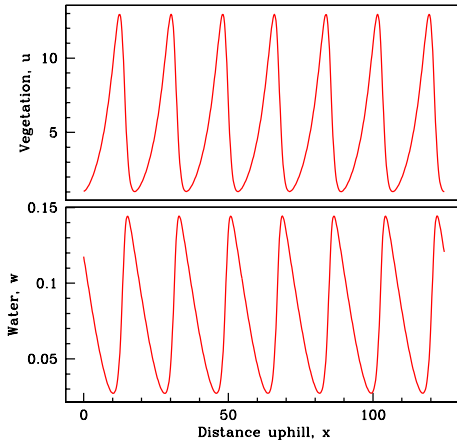
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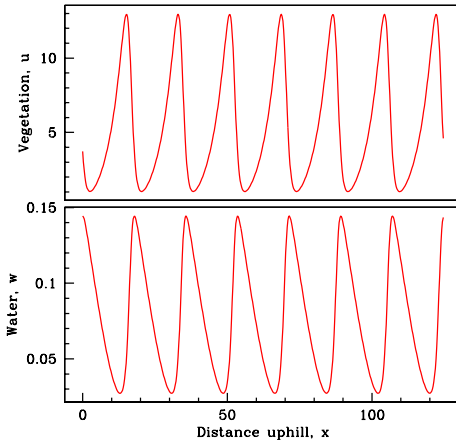
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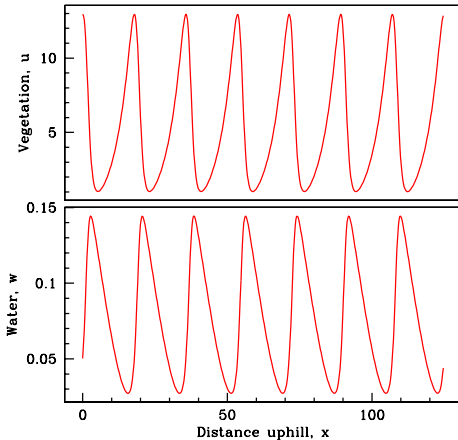
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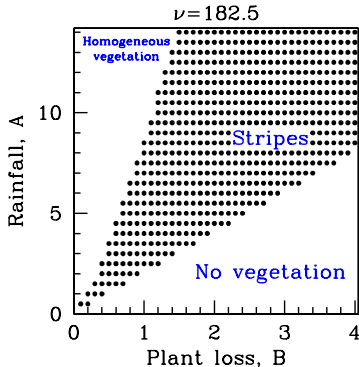
Homogeneous Steady States

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Homogeneous Steady States

- For all parameter values, there is a stable “desert” state (plant density zero).
- When rainfall is sufficiently high, there is also a non-trivial homogeneous state. This steady state can destabilise, giving patterns.

An Illustration of Conditions for Patterning



Predicting Pattern Wavelength

Pattern wavelength is the most accessible property of vegetation stripes in the field, via aerial photography.

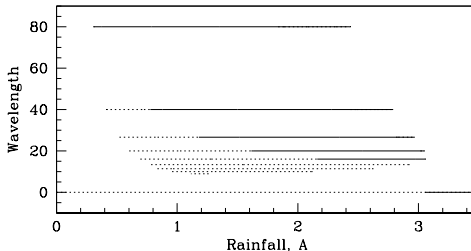
Mathematical prediction of wavelength as a function of parameters (rainfall, plant loss, slope) is difficult because there are **multiple pattern solutions**.

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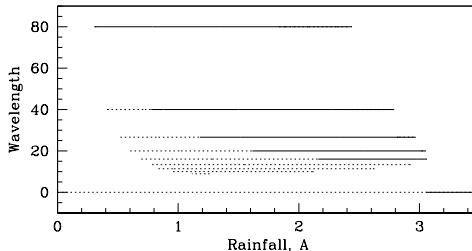
Multiple Pattern Solutions

We determine **pattern existence** via numerical bifurcation analysis of the pattern ordinary differential equations, and **pattern stability** via numerical bifurcation analysis of the discretized model partial differential equations.



Multiple Pattern Solutions

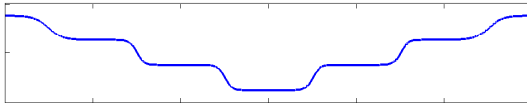
We determine **pattern existence** via numerical bifurcation analysis of the pattern ordinary differential equations, and **pattern stability** via numerical bifurcation analysis of the discretized model partial differential equations.



Conclusion: for a range of rainfall levels, there is more than one stable pattern.

Hysteresis

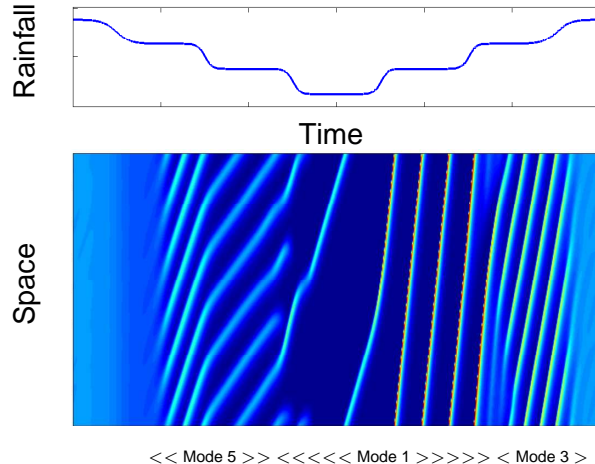
Rainfall



Time

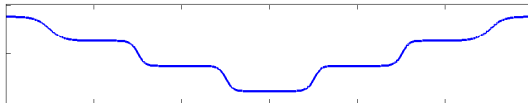
- The existence of multiple stable patterns raises the possibility of hysteresis
- We consider slow variations in the rainfall parameter
- Parameters correspond to grass, and the rainfall range corresponds to 130–930 mm/year

Hysteresis



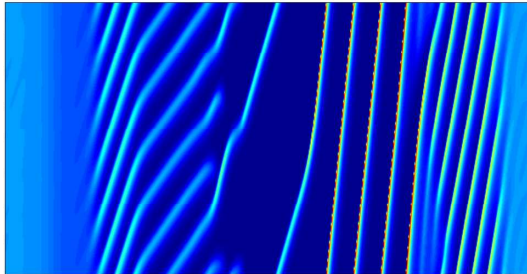
Hysteresis

Rainfall

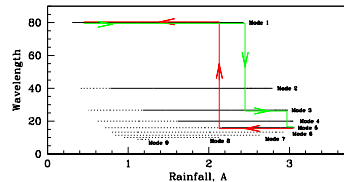


Time

Space



<< Mode 5 >> <<<< Mode 1 >>>> < Mode 3 >



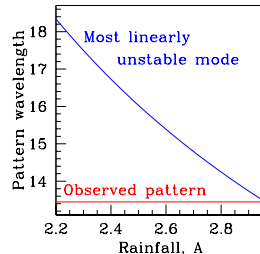
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Predictions of Pattern Wavelength

- In general, pattern wavelength depends on initial conditions
- When vegetation stripes arise from homogeneous vegetation via a decrease in rainfall, pattern wavelength will remain at a constant value.

$$\text{Wavelength} = \sqrt{\frac{8\pi^2}{B_V}}$$



Other Potential Mechanisms for Vegetation Patterns

Rietkirk Klausmeier model with diffusion of water in the soil

van de Koppel Klausmeier model with grazing

Maron two variable model (plant density and water in the soil) with water transport based on porous media theory

Lejeune short range activation (shading) and long range inhibition (competition for water)

All of these models predict patterns. To discriminate between them requires a detailed understanding of each model.

List of Frames

1

Ecological Background

- Vegetation Pattern Formation
- More Pictures of Vegetation Patterns
- Vegetation Pattern Formation (contd)
- Mechanisms for Vegetation Patterning

2

The Mathematical Model

- Mathematical Model of Klausmeier
- Typical Solution of the Model

3

Model Predictions: When Do Patterns Occur?

- Homogeneous Steady States
- An Illustration of Conditions for Patterning
- Predicting Pattern Wavelength

4

Model Predictions: Which Pattern Forms?

- Multiple Pattern Solutions
- Hysteresis

5

Conclusions

- Predictions of Pattern Wavelength
- Other Potential Mechanisms for Vegetation Patterns