Modelling Vegetation Patterns in Semi-Arid Environments

Jonathan A. Sherratt

Department of Mathematics
Heriot-Watt University

University of Leeds, 27 March 2008

This talk can be downloaded from my web site
`www.ma.hw.ac.uk/~jas`

In collaboration with
Gabriel Lord

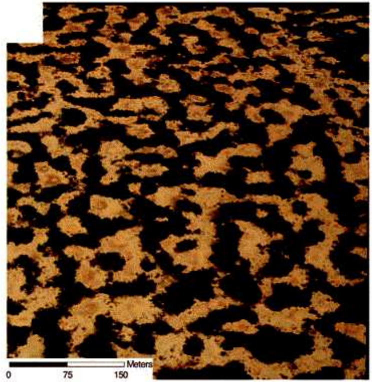
Outline

Outline

Vegetation Pattern Formation

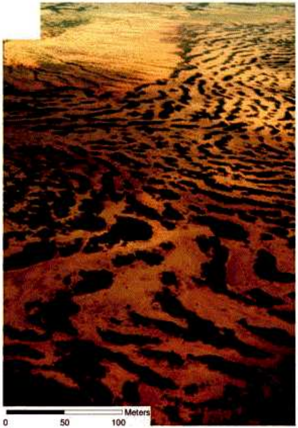
- Vegetation patterns are found in semi-arid areas of Africa, Australia and Mexico (rainfall 100-700 mm/year)
- First identified by aerial photos in 1950s
- Plants vary from grasses to shrubs and trees

More Pictures of Vegetation Patterns



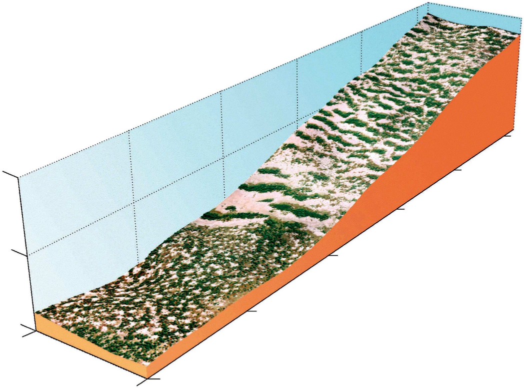
Labyrinth of bushy vegetation in Niger

More Pictures of Vegetation Patterns



Striped pattern of bushy vegetation in Niger

Vegetation Pattern Formation (contd)



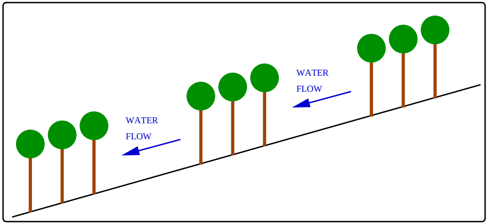
- On flat ground, irregular mosaics of vegetation are typical
- On slopes, the patterns are stripes, parallel to contours ("Tiger bush")

Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water

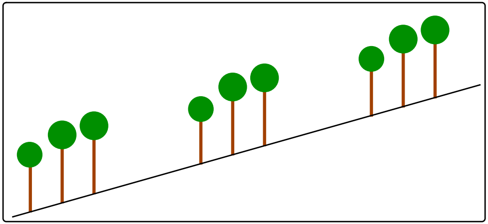
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



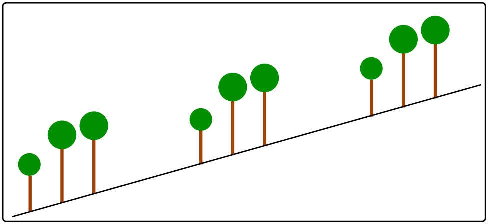
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



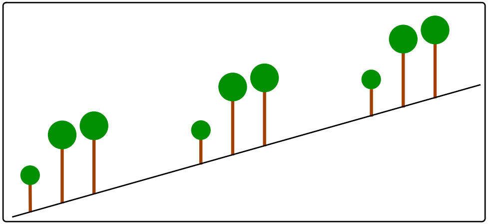
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



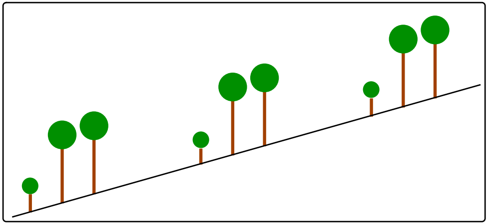
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



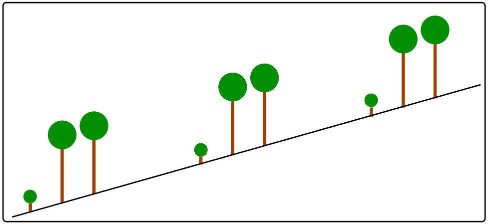
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



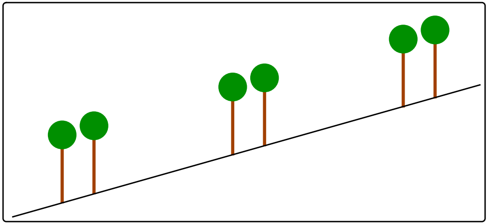
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



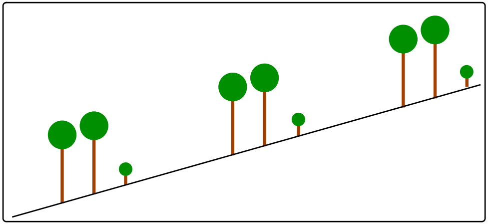
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



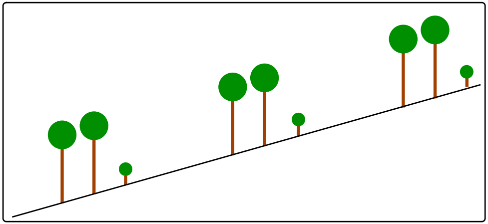
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



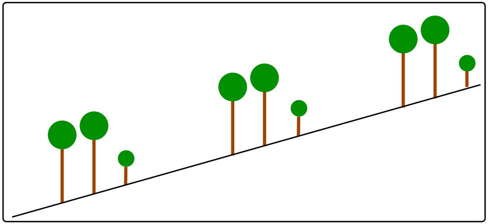
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



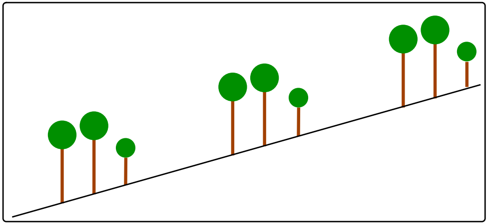
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



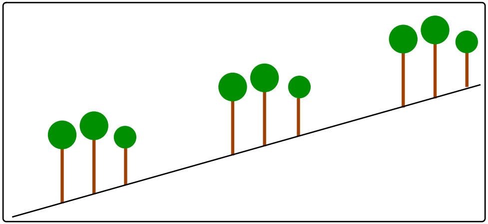
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



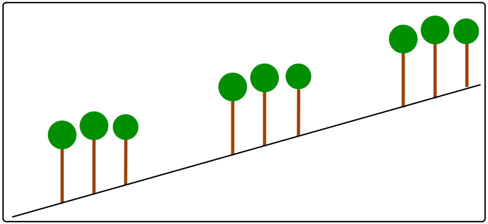
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



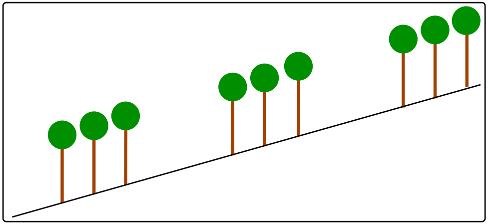
Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



Mechanisms for Vegetation Patterning

- Basic mechanism: competition for water
- Possible detailed mechanism: water flow downhill causes stripes



- This mechanism suggests that the stripes would move uphill; this remains controversial.

Outline

Mathematical Model of Klausmeier

Rate of change of water = Rainfall – Evaporation – Uptake by plants + Flow downhill

Rate of change plant biomass = Growth, proportional to water uptake – Mortality + Random dispersal

$$\partial w/\partial t = A - w - wu^2 + \nu \partial w/\partial x$$

$$\partial u/\partial t = wu^2 - Bu + \partial^2 u/\partial x^2$$

Mathematical Model of Klausmeier

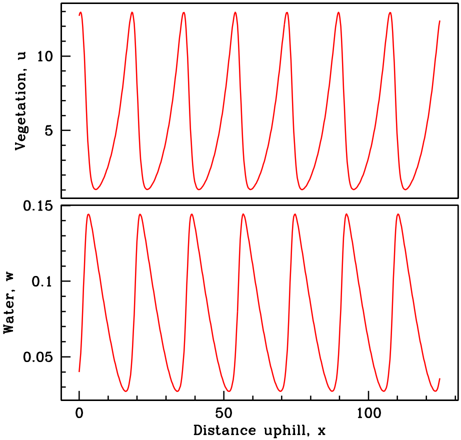
Rate of change of water	=	Rainfall – Evaporation	–	Uptake by plants	+	Flow downhill
Rate of change plant biomass	=	Growth, proportional to water uptake	–	Mortality	+	Random dispersal

$$\partial w/\partial t = A - w - wu^2 + \nu \partial w/\partial x$$

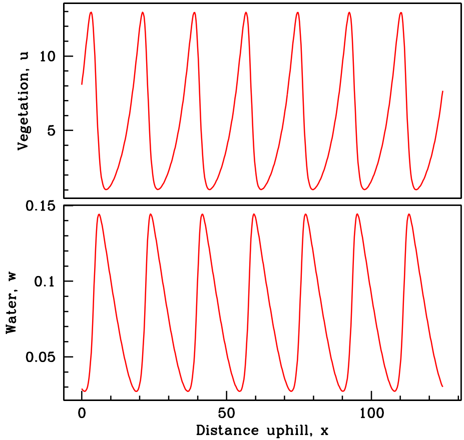
$$\partial u/\partial t = wu^2 - Bu + \partial^2 u/\partial x^2$$

The nonlinearity in wu^2 arises because the presence of roots increases water infiltration into the soil.

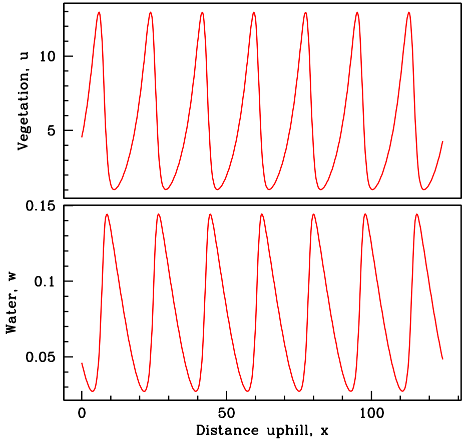
Typical Solution of the Model



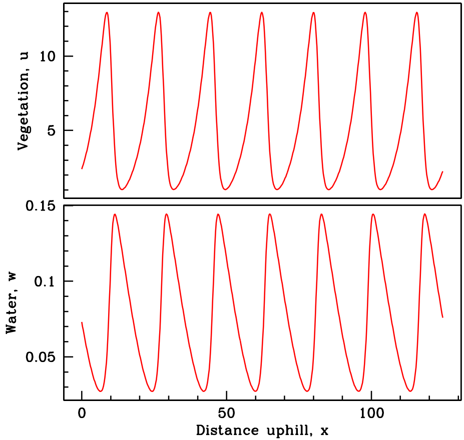
Typical Solution of the Model



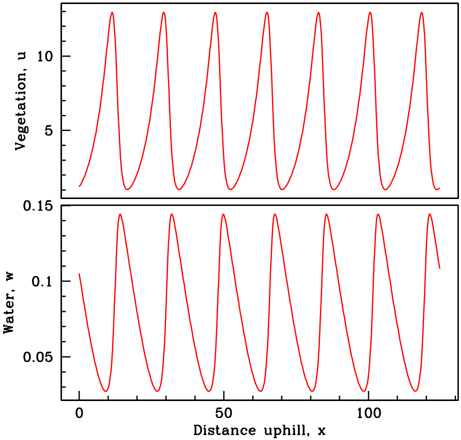
Typical Solution of the Model



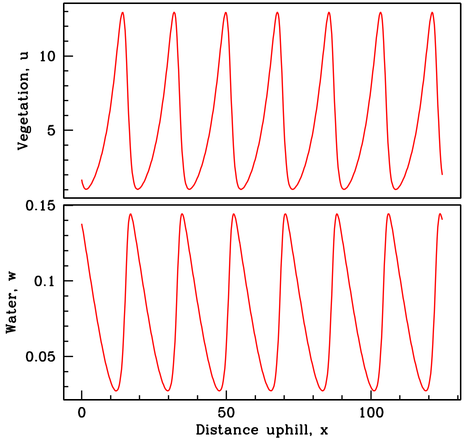
Typical Solution of the Model



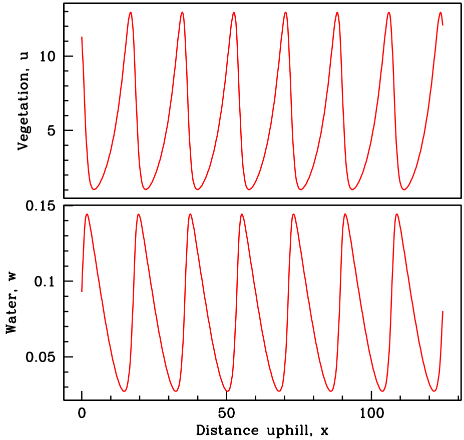
Typical Solution of the Model



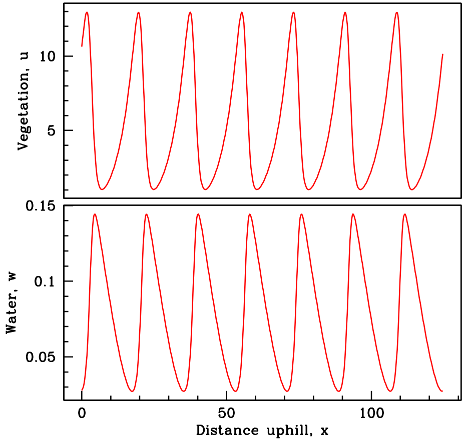
Typical Solution of the Model



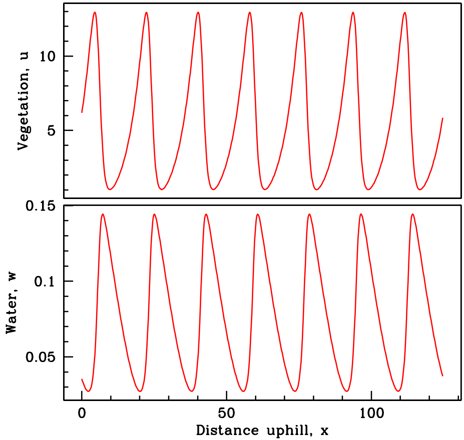
Typical Solution of the Model



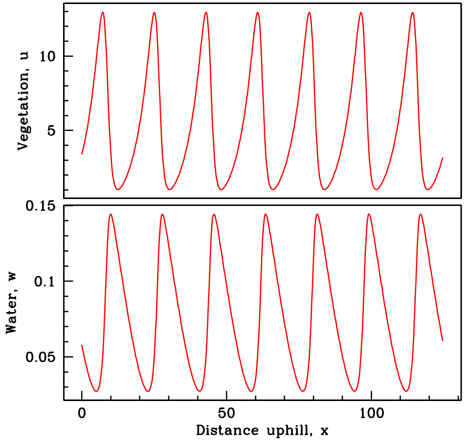
Typical Solution of the Model



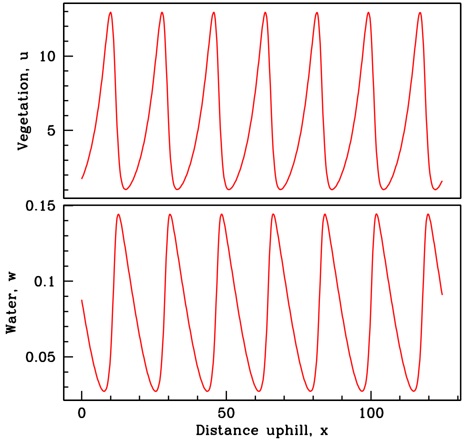
Typical Solution of the Model



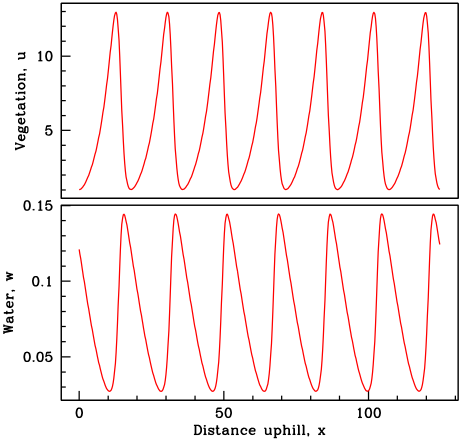
Typical Solution of the Model



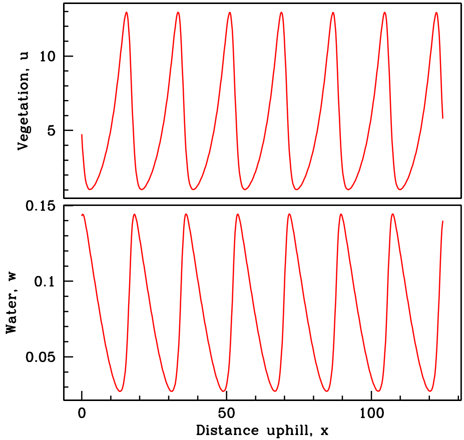
Typical Solution of the Model



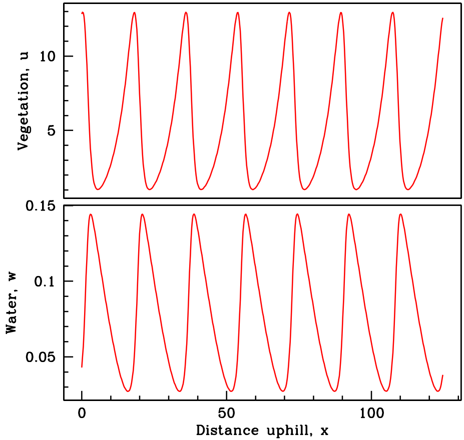
Typical Solution of the Model



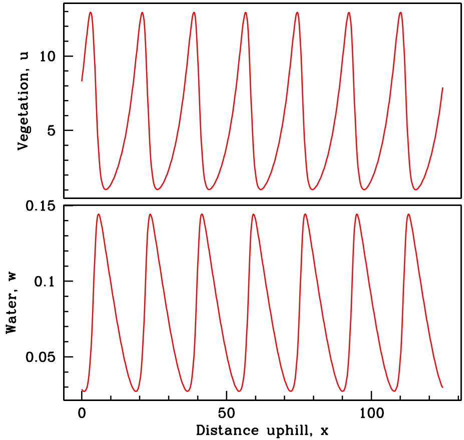
Typical Solution of the Model



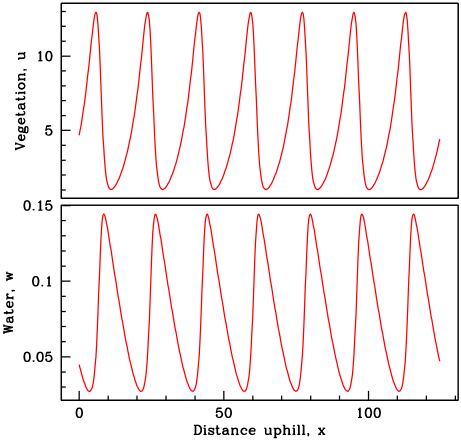
Typical Solution of the Model



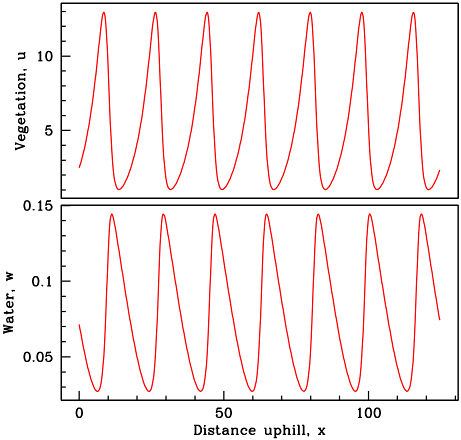
Typical Solution of the Model



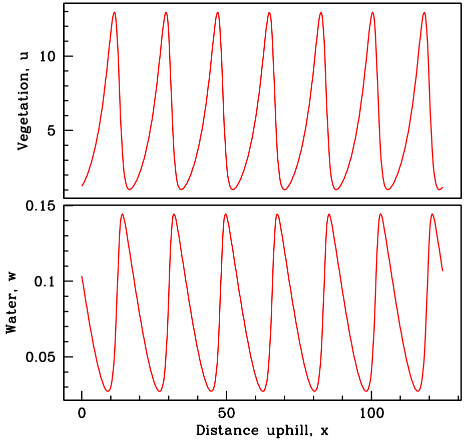
Typical Solution of the Model



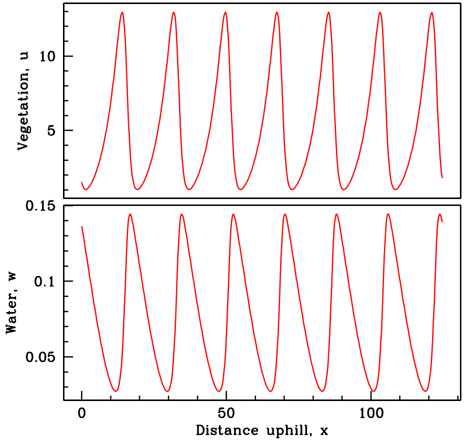
Typical Solution of the Model



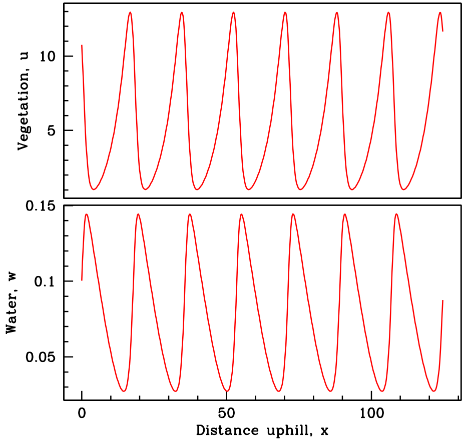
Typical Solution of the Model



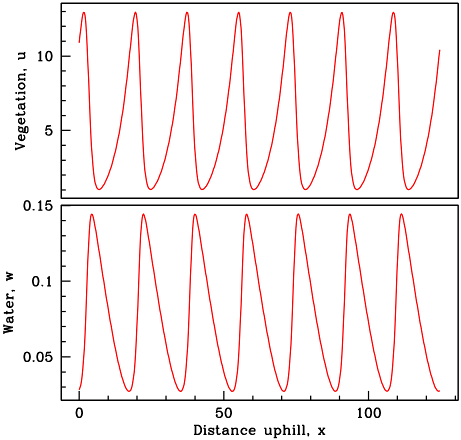
Typical Solution of the Model



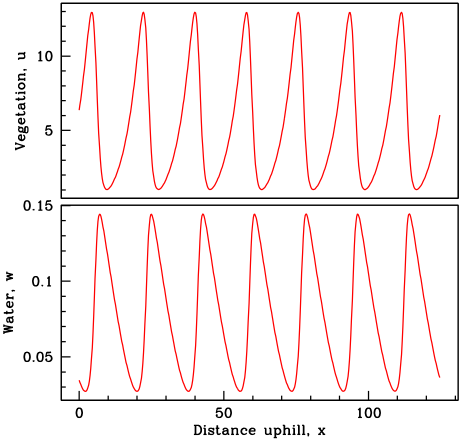
Typical Solution of the Model



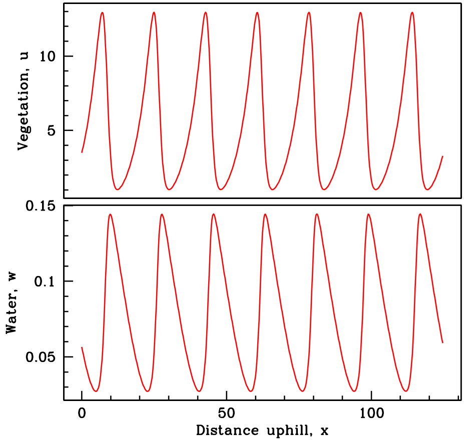
Typical Solution of the Model



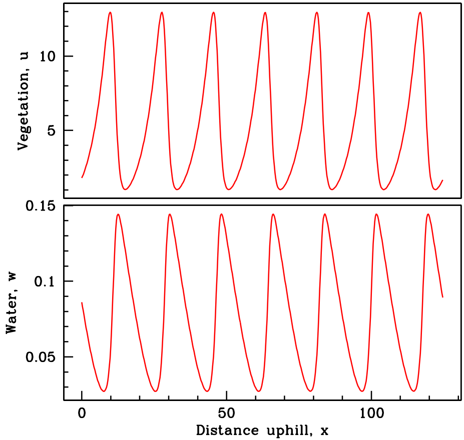
Typical Solution of the Model



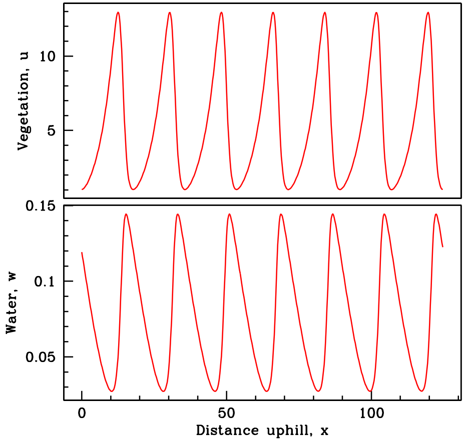
Typical Solution of the Model



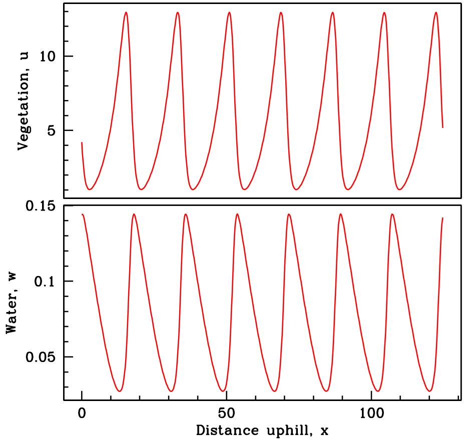
Typical Solution of the Model



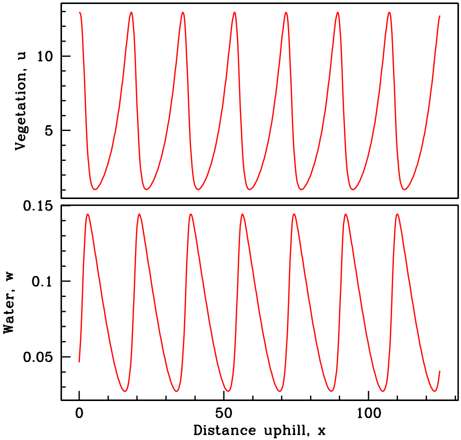
Typical Solution of the Model



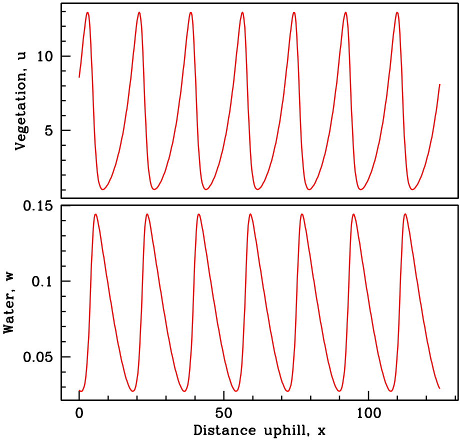
Typical Solution of the Model



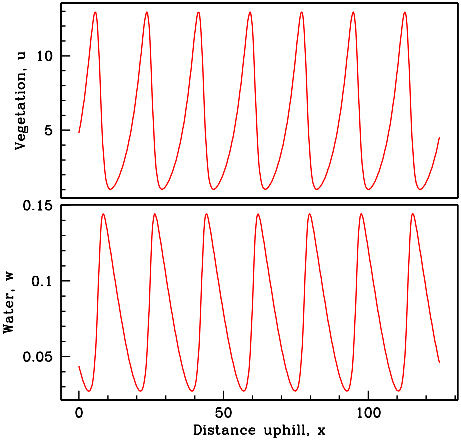
Typical Solution of the Model



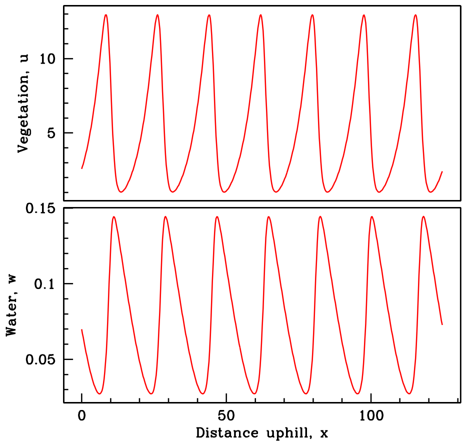
Typical Solution of the Model



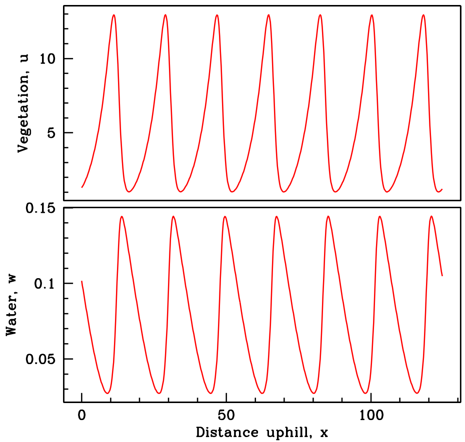
Typical Solution of the Model



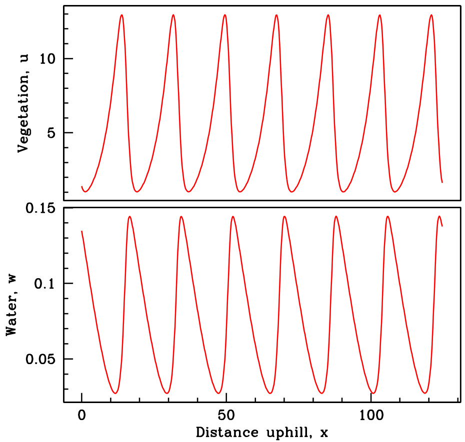
Typical Solution of the Model



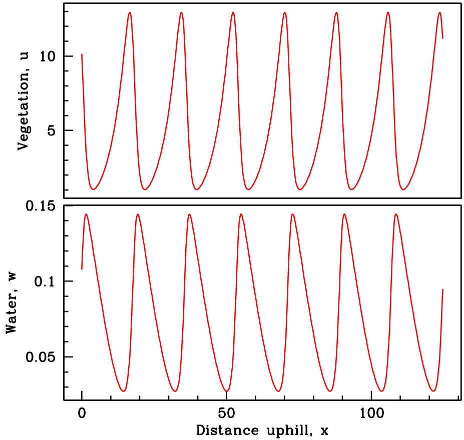
Typical Solution of the Model



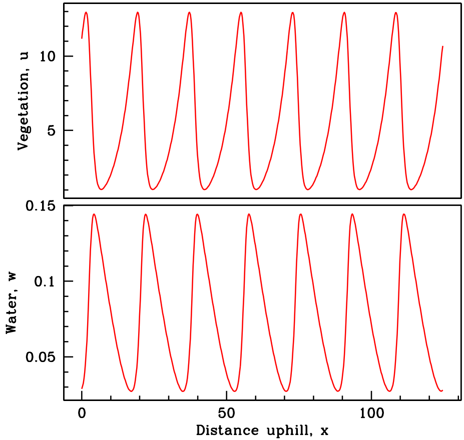
Typical Solution of the Model



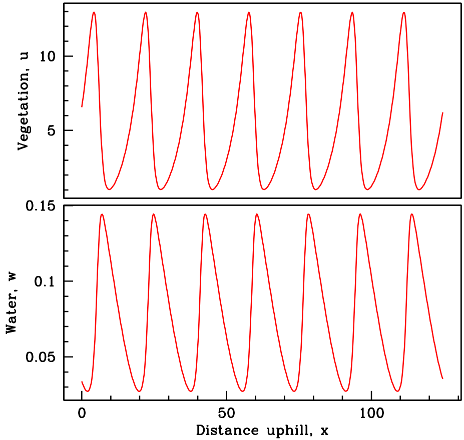
Typical Solution of the Model



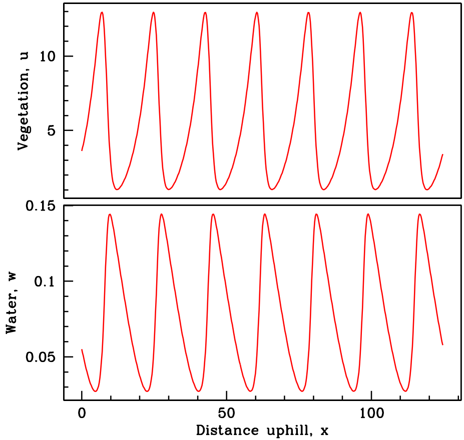
Typical Solution of the Model



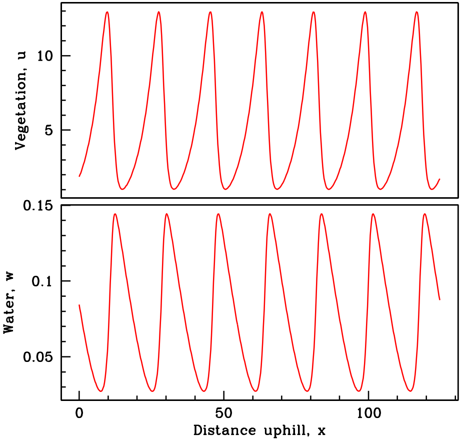
Typical Solution of the Model



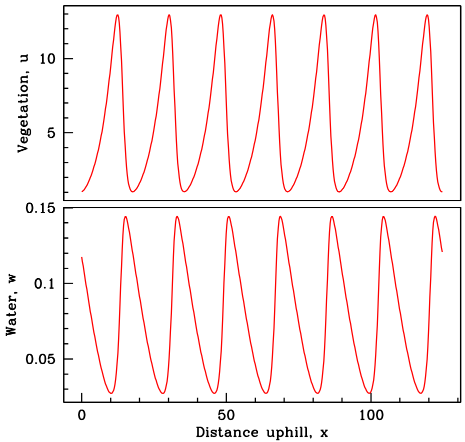
Typical Solution of the Model



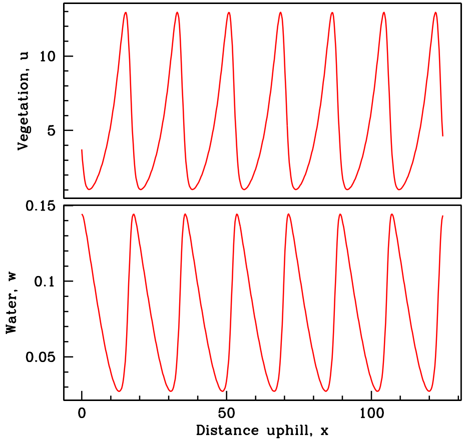
Typical Solution of the Model



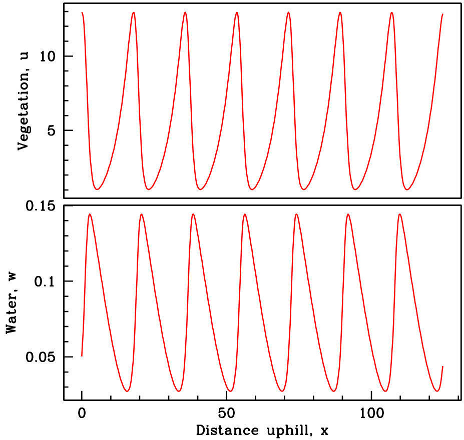
Typical Solution of the Model



Typical Solution of the Model



Typical Solution of the Model



Outline

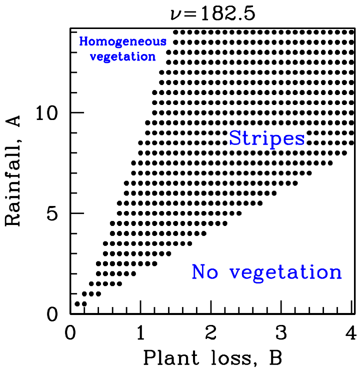
Homogeneous Steady States

- For all parameter values, there is a stable “desert” steady state $u = 0$, $w = A$.

Homogeneous Steady States

- For all parameter values, there is a stable “desert” steady state $u = 0$, $w = A$.
- When $A \geq 2B$, there are is also a non-trivial steady states. If A is relatively small, this steady state destabilises, giving patterns

An Illustration of Conditions for Patterning



Predicting Pattern Wavelength

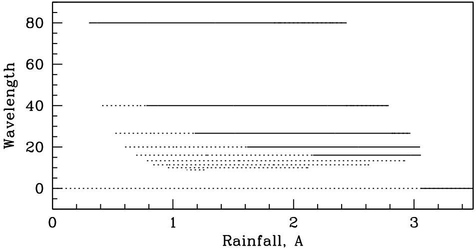
Pattern wavelength is the most accessible property of vegetation stripes in the field, via aerial photography.

Mathematical prediction of wavelength as a function of parameters (rainfall, plant loss, slope) is difficult because there are multiple pattern solutions.

Outline

Multiple Pattern Solutions

We determine pattern existence via numerical bifurcation analysis of the pattern ordinary differential equations, and pattern stability via numerical bifurcation analysis of the discretized model partial differential equations.

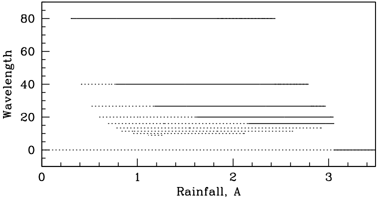


Pattern Selection

- For a range of rainfall levels, there is more than one stable pattern. Which will be selected?
- We consider initial conditions that are small perturbations of a spatially uniform state.
- All such initial conditions give a pattern, but the wavelength depends on the initial perturbation

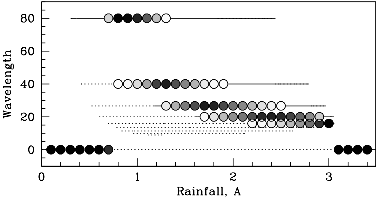
Pattern Selection

- For a range of rainfall levels, there is more than one stable pattern. Which will be selected?
- We consider initial conditions that are small perturbations of a spatially uniform state.
- All such initial conditions give a pattern, but the wavelength depends on the initial perturbation



Pattern Selection

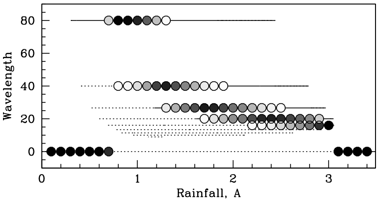
- For a range of rainfall levels, there is more than one stable pattern. Which will be selected?
- We consider initial conditions that are small perturbations of a spatially uniform state.
- All such initial conditions give a pattern, but the wavelength depends on the initial perturbation



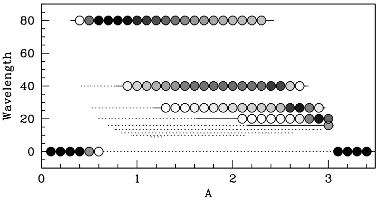
Perturbations to (u_s, v_s)

Pattern Selection

- For a range of rainfall levels, there is more than one stable pattern. Which will be selected?
- We consider initial conditions that are small perturbations of a spatially uniform state.
- All such initial conditions give a pattern, but the wavelength depends on the initial perturbation



Perturbations to (u_s, v_s)

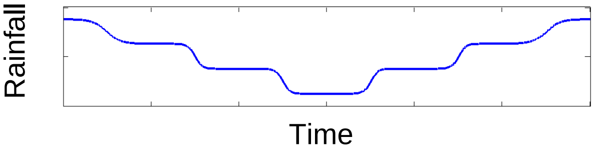


Perturbations to (u_u, v_u)

Key Result

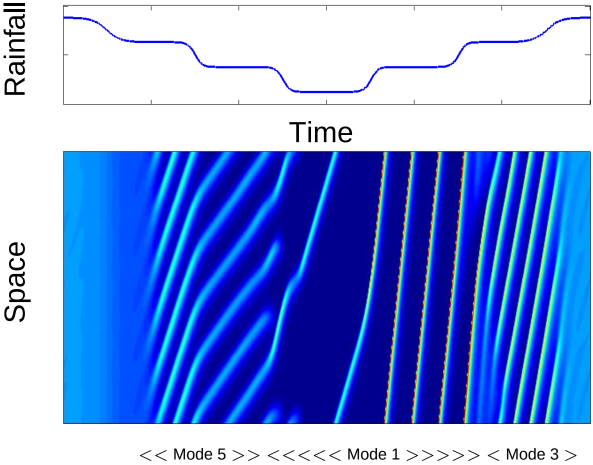
For a wide range of rainfall levels, there are multiple stable patterns.

Hysteresis

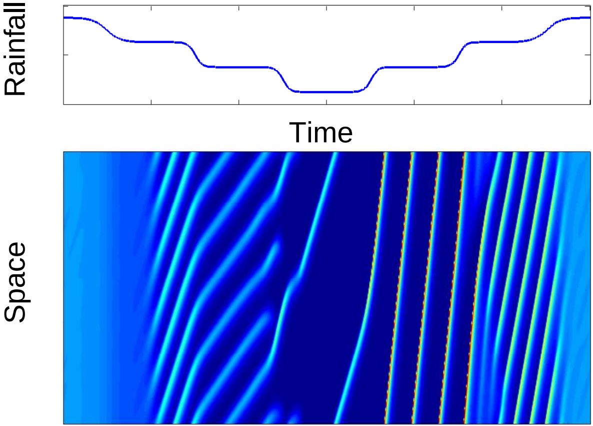


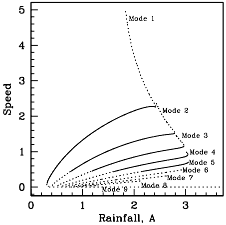
- The existence of multiple stable patterns raises the possibility of hysteresis
- We consider slow variations in the rainfall parameter A
- Parameters correspond to grass, and the rainfall range corresponds to 130–930 mm/year

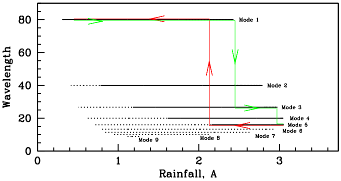
Hysteresis



Hysteresis







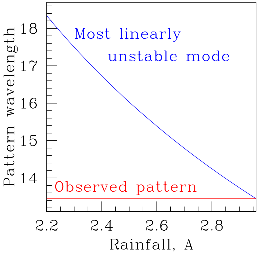
<< Mode 5 >> <<<<< Mode 1 >>>>> < Mode 3 >

Outline

Predictions of Pattern Wavelength

- In general, pattern wavelength depends on initial conditions
- When vegetation stripes arise from homogeneous vegetation via a decrease in rainfall, pattern wavelength will remain at a constant value.

$$\text{Wavelength} = \sqrt{\frac{8\pi^2}{B\nu}}$$



Other Potential Mechanisms for Vegetation Patterns

Rietkirk Klausmeier model with diffusion of water in the soil

van de Koppel Klausmeier model with grazing

Maron two variable model (plant density and water in the soil) with water transport based on porous media theory

Lejeune short range activation (shading) and long range inhibition (competition for water)

All of these models predict patterns. To discriminate between them requires a detailed understanding of each model.

List of Frames