Determining the Past and the Future of Vegetation in Semi-Deserts Using Mathematical Modelling

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Heriot-Watt University

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This talk can be downloaded from my web site
`www.macs.hw.ac.uk/~jas`

Outline

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Vegetation Patterns

Desert ecosystems provide a classic example of self-organised pattern formation.

W National Park, Niger

Average patch width is 50 m

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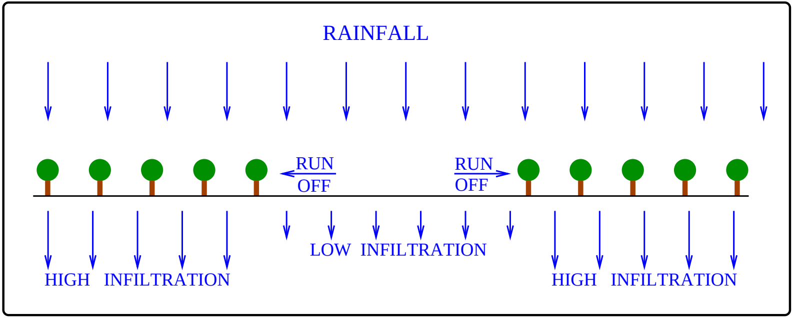

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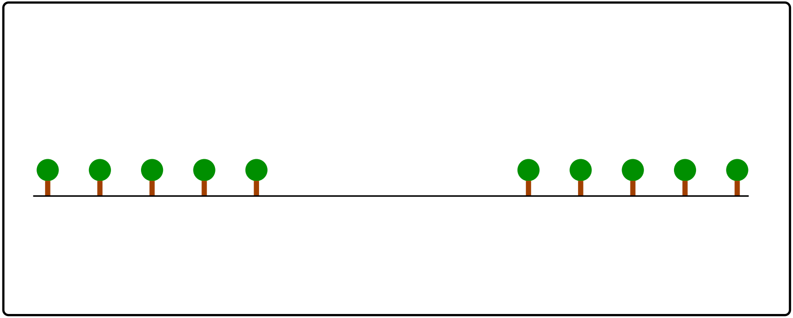

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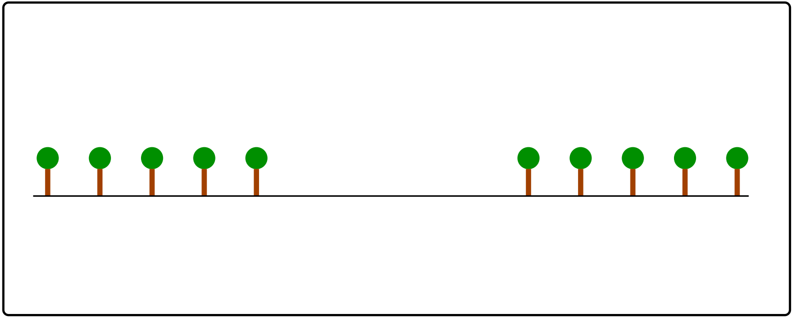

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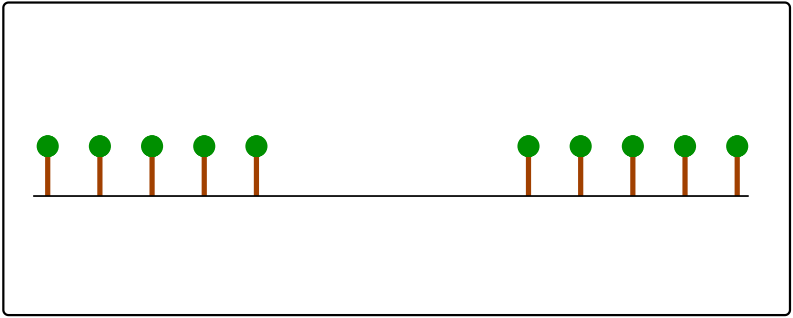

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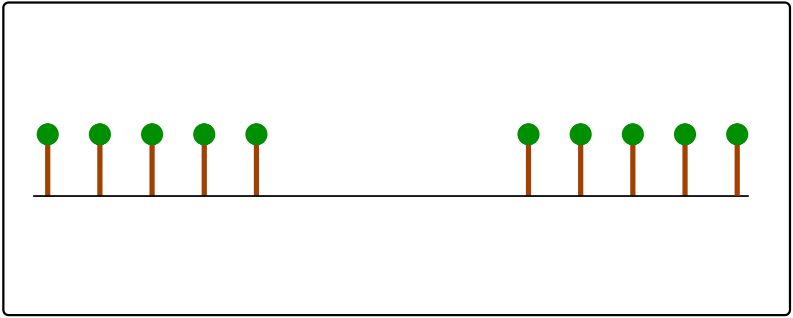

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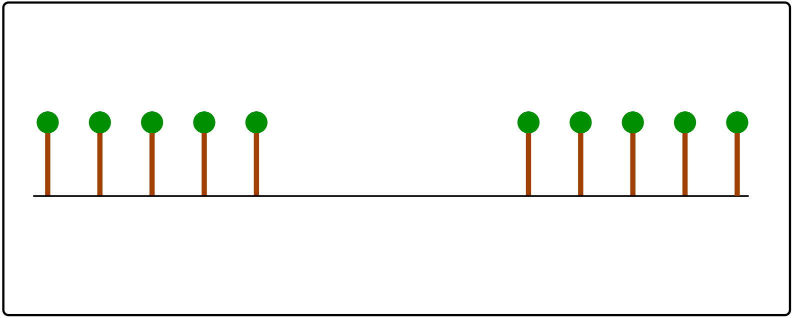

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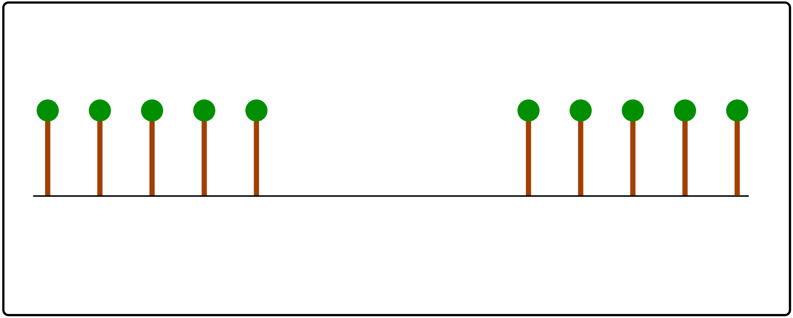


Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.

Bushy vegetation in Niger

Mitchell grass in Australia

(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, N America and Asia.

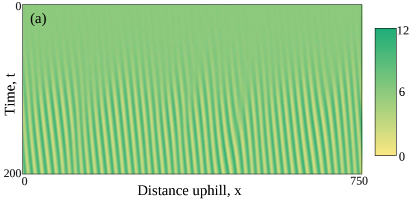
The Central Question

Bushy vegetation in Niger

For a given pattern, how can we determine its historical origin?

Possible Origins of Vegetation Patterns

Patterns can arise either via degradation of uniform vegetation



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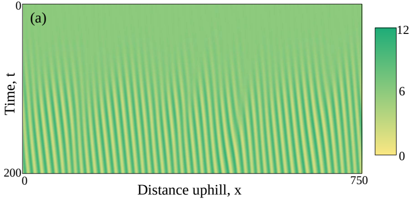
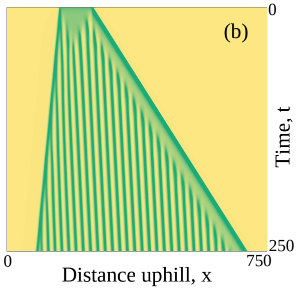


. . . or via colonisation of bare ground.



The Central Question

Bushy vegetation in Niger

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The Central Question

Bushy vegetation in Niger

For a given pattern, how can we determine its historical origin

. . . without any historical data?

Outline

Mathematical Model of Klausmeier

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\end{aligned}$$

(Klausmeier, Science 284: 1826-8, 1999)

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The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

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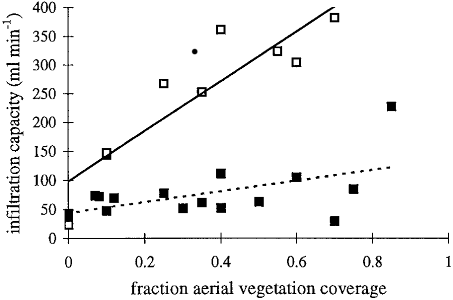


Water uptake=
Water density
$\times$ Plant density
$\times \begin{pmatrix} \text{infiltration} \\ \text{rate} \end{pmatrix}$

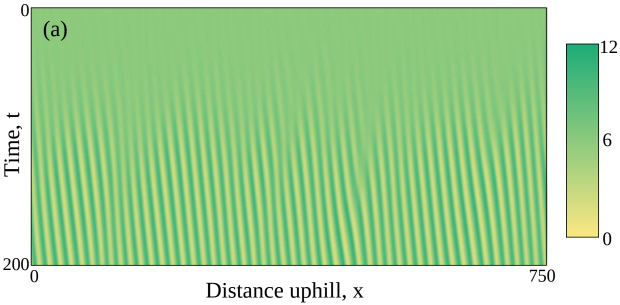
The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

Spatial Pattern Formation

The steady state (u_s, w_s) becomes unstable to spatially inhomogeneous perturbations at $A = A_{crit}$, giving patterns.

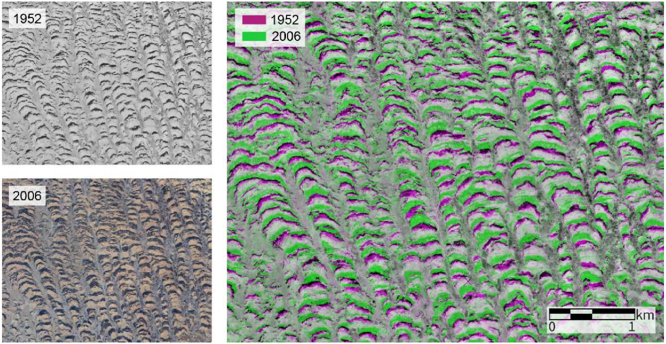
This is a Turing-Hopf bifurcation; the patterns move uphill (slowly).

A Simulation of Pattern Formation



Note that the pattern moves uphill (slowly).

Data on Pattern Migration



Vegetation near Dhahar, Somalia

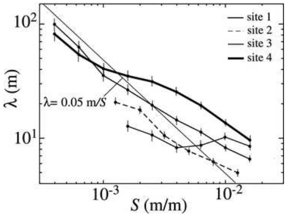
(Gandhi et al, in “Dryland Ecology” ed. D’Odorico et al, Springer, 2019)

Outline

Wavelength vs Slope

To be most useful, a method for estimating historical origin should be based on remotely sensed data

- wavelength of pattern
- slope of hillside



Data from Nevada, USA (Pelletier et al, J. Geophys. Res. 117: F04026, 2012)

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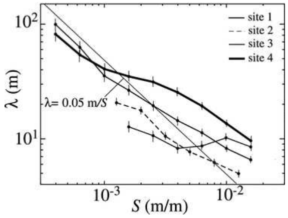

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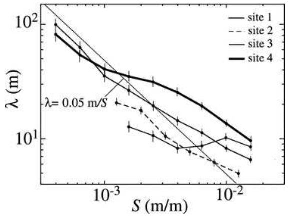

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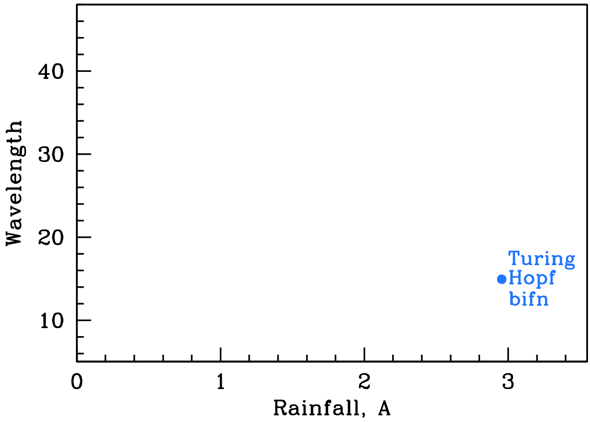
What does the model predict for wavelength vs slope?

Problem: in the model there is a range of possible pattern wavelengths

Existence and Stability of Patterns



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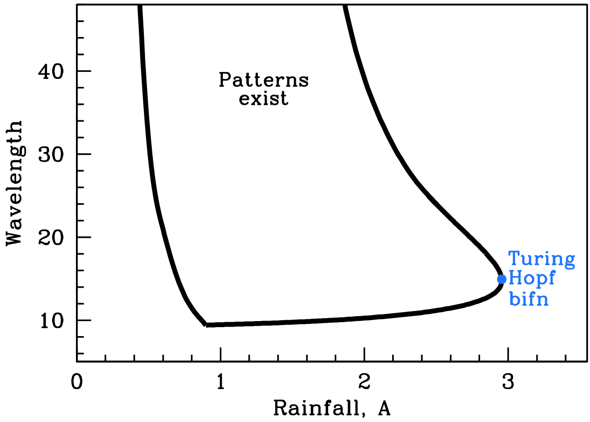

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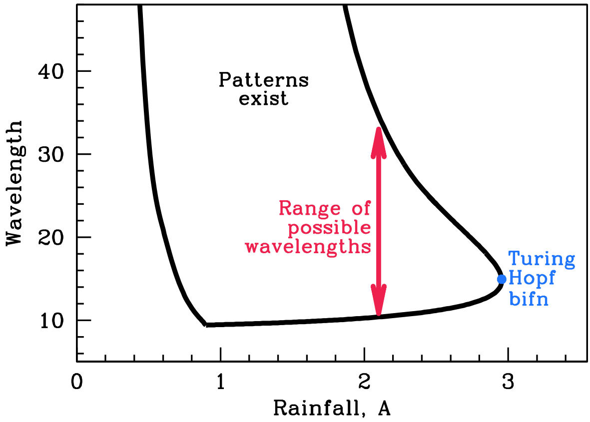

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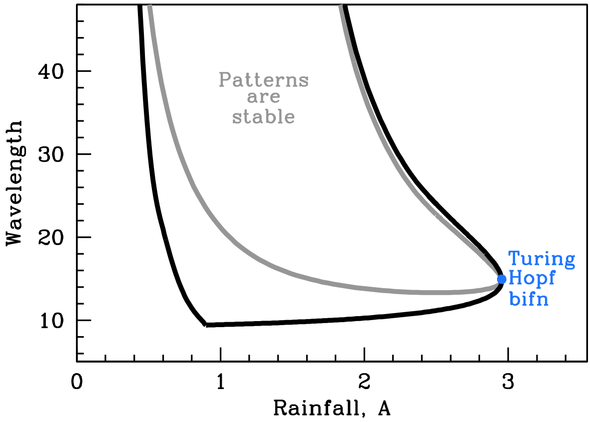

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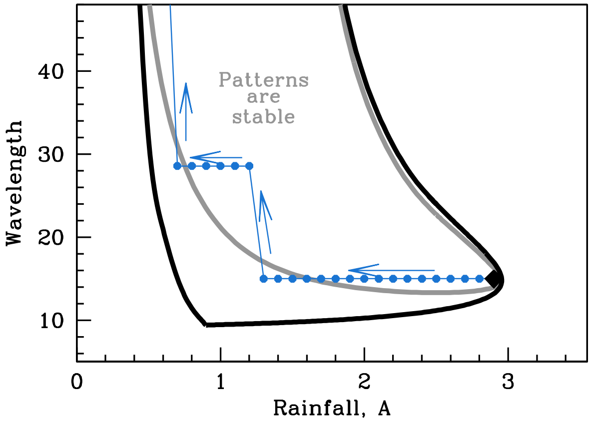
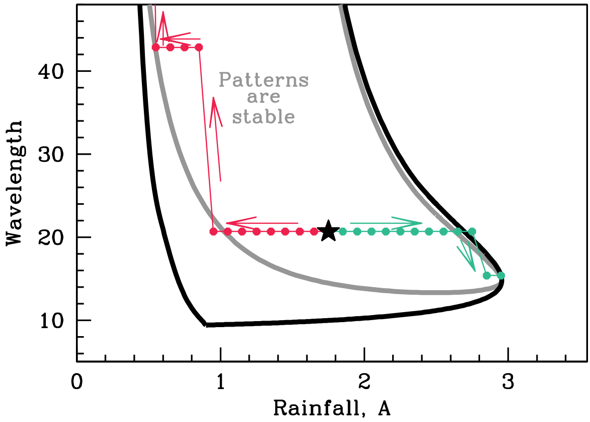


♦ Starting point: degradation of uniform vegetation

Existence and Stability of Patterns



★ Starting point: colonisation of bare ground

Existence and Stability of Patterns

Objective: what does the model predict for wavelength vs slope?

Problem: in the model there is a range of possible pattern wavelengths.

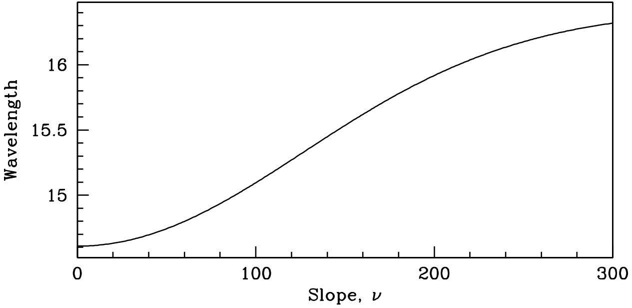
Resolution: for wavelength vs slope we must study the onset of patterning.

Case 1: degradation of uniform vegetation.

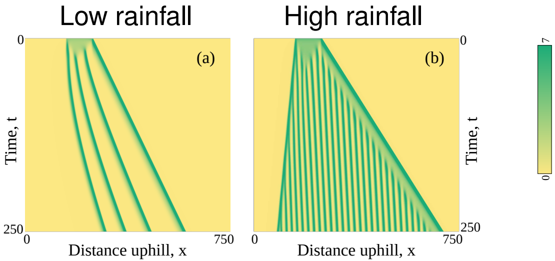
Case 2: colonisation of bare ground.

Wavelength vs Slope for Degradation of Uniform Vegetation

Linear stability analysis $\Rightarrow$ wavelength increases with slope at pattern onset from degradation of uniform vegetation.



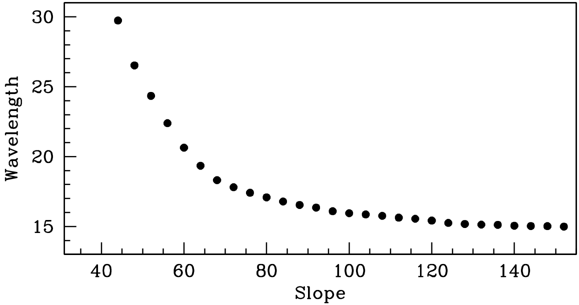
When Does Vegetation Colonise Bare Ground?



The critical rainfall for colonisation can be calculated numerically $\Rightarrow$ wavelength at pattern onset.

Wavelength vs Slope for Colonisation of Bare Ground

Wavelength decreases with slope at pattern onset from colonisation of bare ground.



Conclusions

Wavelength is positively correlated with slope $\Rightarrow$ vegetation pattern originated by degradation of uniform vegetation

Wavelength is negatively correlated with slope $\Rightarrow$ vegetation pattern originated by colonisation of bare ground

Example: The African Sahel

- Patterned vegetation is widespread in the Sahel
- Several studies of banded vegetation show wavelength ↓ as slope ↑

Rainfall History in the Sahel

- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.

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How do we know this?

- There is no direct data on rainfall before c. 1850.

- Proxy data: (i) lake levels, esp. Lake Chad; (ii) historical chronologies, e.g. Bornu Empire; (iii) memories of local peoples.

Rainfall History in the Sahel

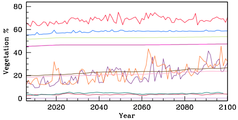
- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.
- Reasonable assumption: areas with vegetation patterns today had uniform vegetation at the end of the 17th century.
- Since wavelength decreases with slope, my results imply that vegetation must have died out and then recolonised since the end of the 17th century.
- The most severe drought since 1700 was c. 1738-1756. So today's vegetation patterns result from recolonisation since 1760.

Outline

Question: How will vegetation levels in the Sahel region of Africa change over the remainder of the century?

Global Climate Models

- Prediction of future climate is an active research area: 60 models in CMIP5, results of CMIP6 due in ~~2020~~ 2021
- Some of these models include "dynamic vegetation" (12/60 in CMIP5)
- But: spatial grid cells (∼100km) are too large to deal effectively with patterned vegetation
- This is demonstrated by the huge variability in predictions of future vegetation levels in the Sahel.



- So: a different approach is needed

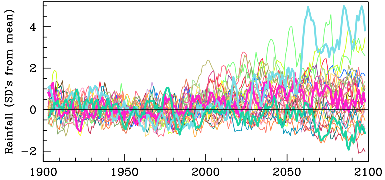
Approaches to Predicting Future Vegetation

1. Improve the spatial resolution in global climate models: in progress but a resolution suitable for patterned vegetation lies well in the future.
2. Improve models for patterned vegetation, to include some climate data or feedbacks
   (e.g. work of Mara Baudena & Max Rietkerk)

My approach: use predictions of future rainfall from global climate models (CMIP5) as a forcing term in a simple model for semi-arid vegetation.

Rainfall Predictions for the Sahel

- Predictions of future rainfall for the Sahel are highly variable.



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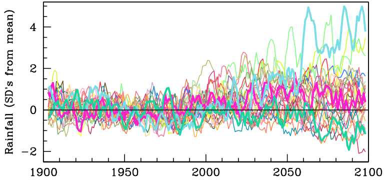


- In view of this, is it possible to make meaningful predictions of future vegetation?
- Another complication: the history-dependence of vegetation patterns means that historical data is needed to predict future behaviour.

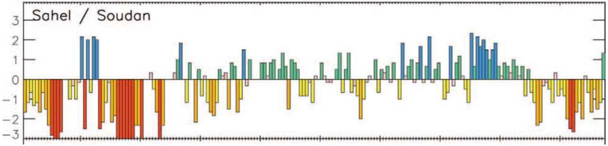
Rainfall History in the Sahel

History-dependence makes a starting point difficult.
But: a very severe drought occurred in the Sahel c. 1738-1756: a reasonable starting point for simulations is zero vegetation in 1750.

Historical Rainfall Data Set

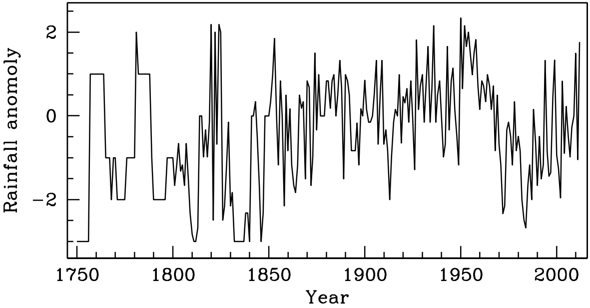
I base my historical data set on work by Sharon Nicholson (Florida State U) on rainfall history in the Sahel.

Sahel "wetness index" 1800-2000



- Extension back to 1750 is based on historical work of Stefan Norrgård (Turku)
- Extension forwards to present is based on recent rain gauge data
- I use linear correlation of data for overlapping years to combine the data sets

Historical Rainfall Data Set



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Simulation Approach

$$
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\end{aligned}
$$

- Note: no advection of water (flat ground)

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- I run simulations from 1750-2100
- *A* varies over time to reflect the historical rainfall data set, and predictions of future rainfall levels (CMIP5)

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\end{aligned}$$

- I allow for colonisation: when the vegetation level is small, there is a small probability of extra vegetation being added.

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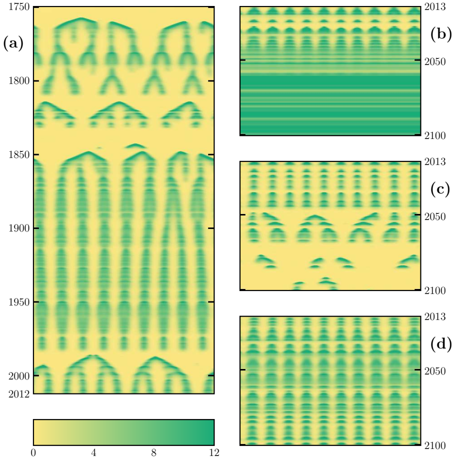
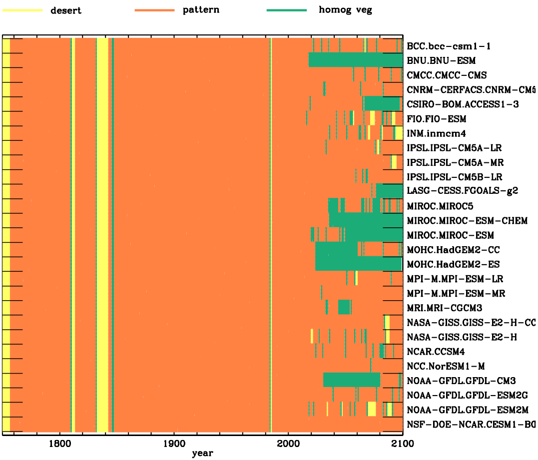
- I consider a wide array of parameter values, with all runs done for 27 CMIP5 datasets
  – a total of 46 000 simulations

Example Simulations



Classification of Vegetation



Predictions on Desertification

Percentage of years with (almost) no vegetation
Historical (1750-present): 10%
Future (present-2100): 3.5%

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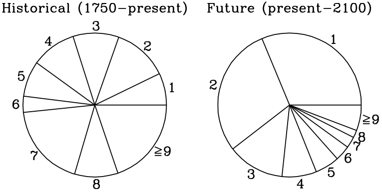

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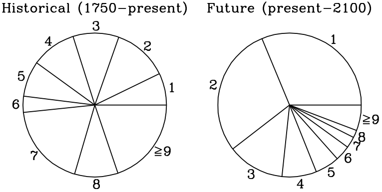

Conclusion: the vast majority of simulations imply relatively high vegetation levels throughout the 21st century, with much lower levels of desertification than for the previous 2.5 centuries.

List of Frames