Wavetrain Patterns
In Cyclic Populations

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Lorenz Centre, December 2014

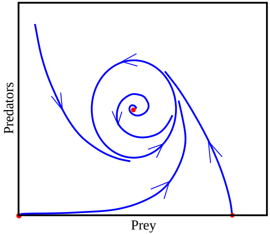
This talk can be downloaded from my web site

`www.ma.hw.ac.uk/∼jas`

Cyclic Predator-Prey Systems

The interaction between a predator population and its prey can cause population cycles. This has been modelled extensively using systems of two coupled ODEs:

$$\begin{array}{rrcl} \boxed{\text{predators}} & du/dt & = & f(u,v) \\ \boxed{\text{prey}} & dv/dt & = & g(u,v) \end{array}$$



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$$\begin{array}{rrcl} \boxed{\text{predators}} & \partial u/\partial t & = & f(u,v) + D_u\,\partial^2 u/\partial x^2 \\ \boxed{\text{prey}} & \partial v/\partial t & = & g(u,v) + D_v\,\partial^2 v/\partial x^2 \end{array}$$

With diffusion, the model predicts wavetrain patterns.

What is a Wavetrain?

A wavetrain is a soln of form $f(x \pm ct)$, with $f(.)$ periodic.

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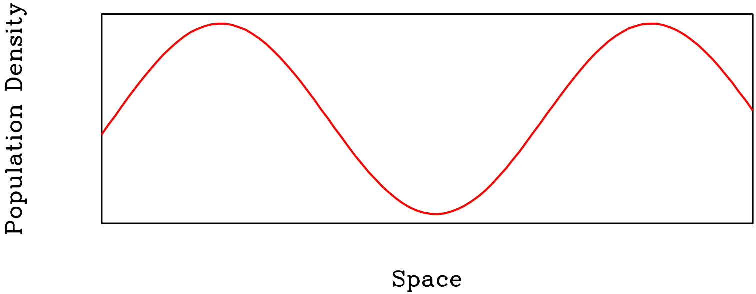

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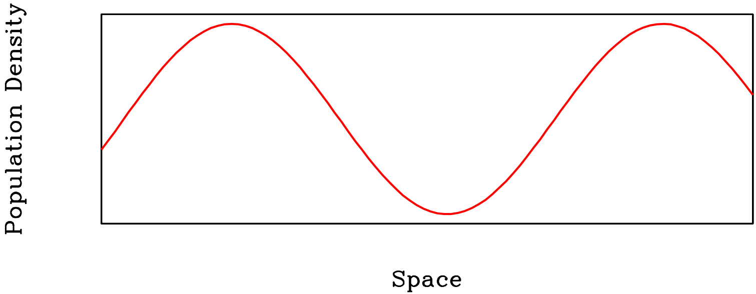

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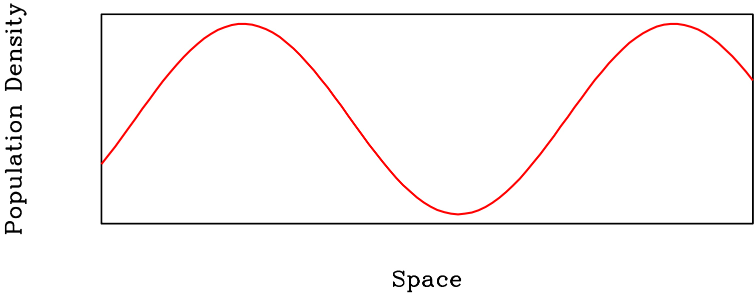

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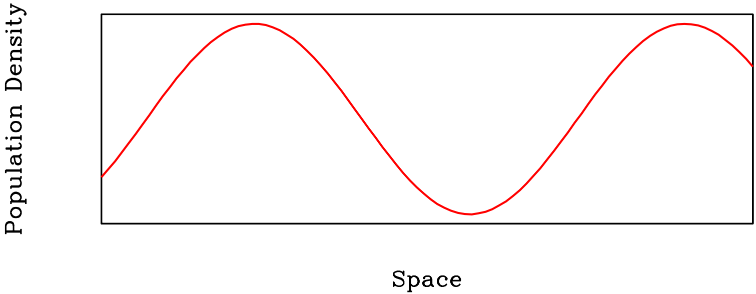

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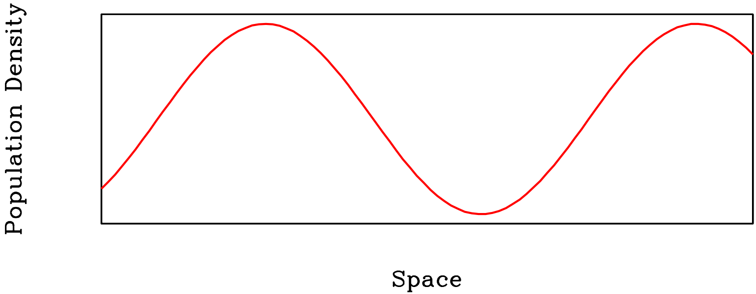

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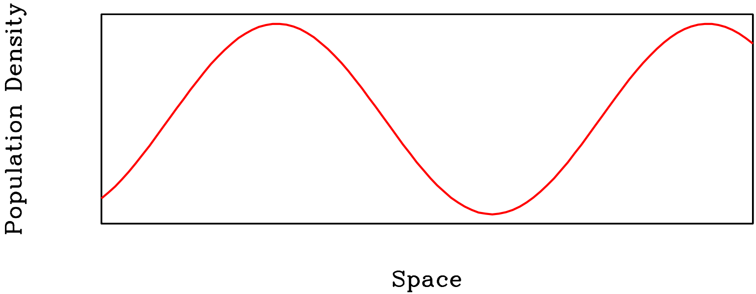

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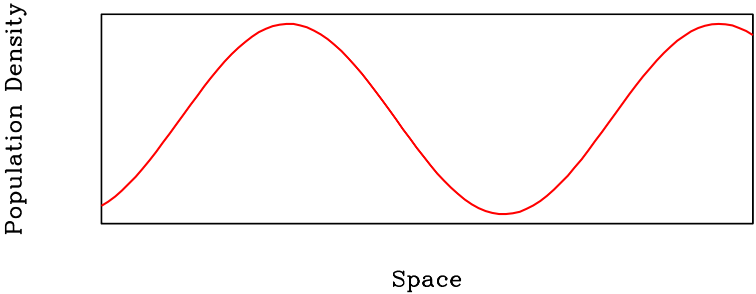

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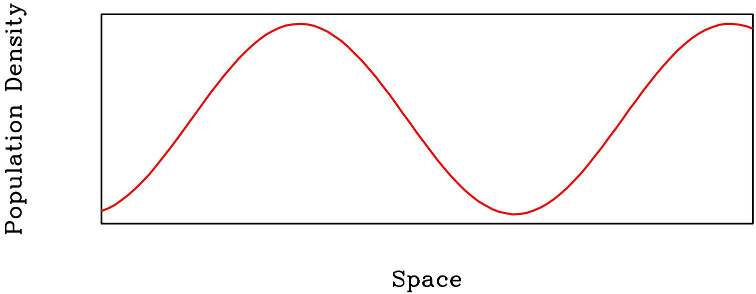

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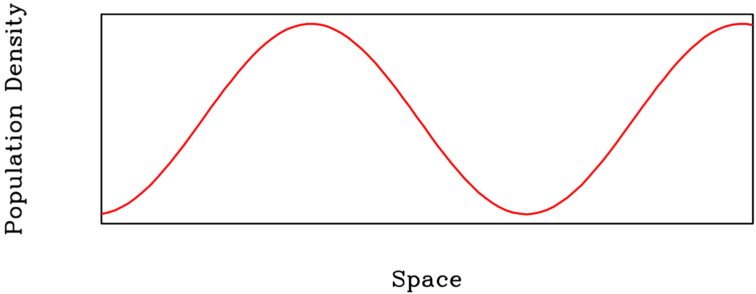

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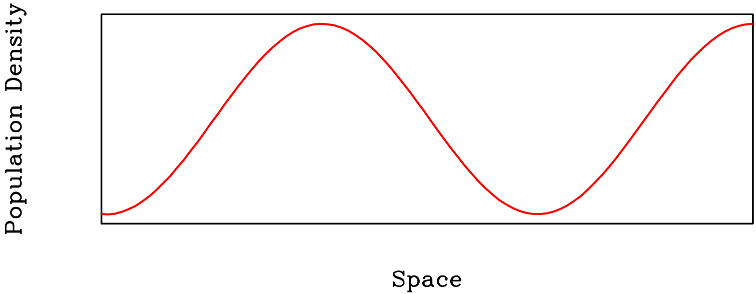

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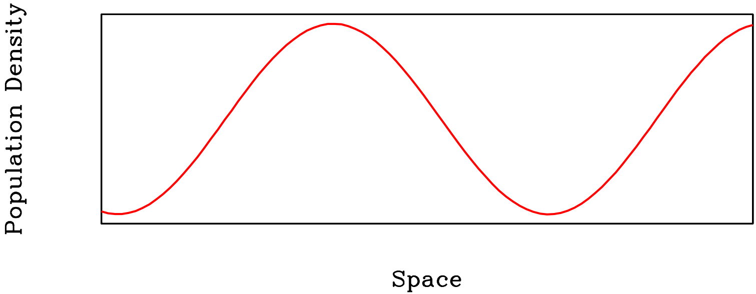

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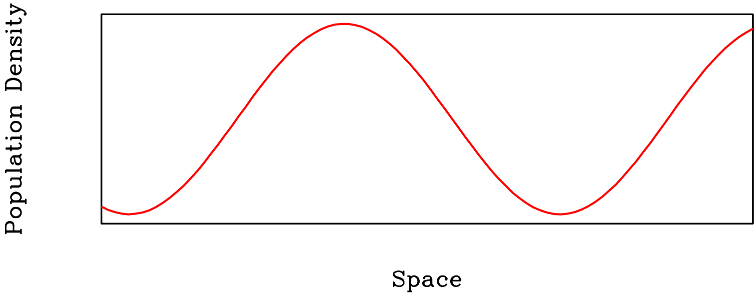

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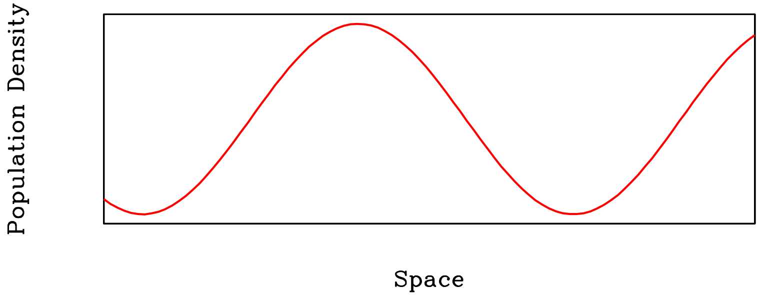

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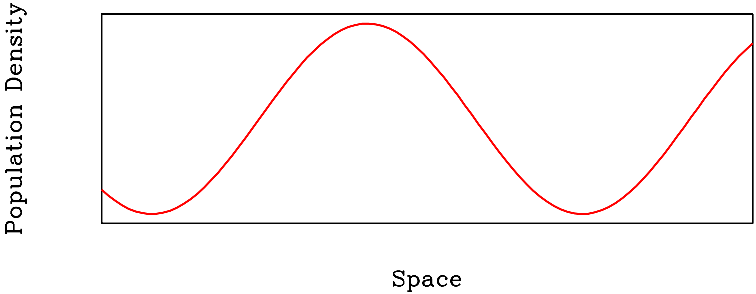

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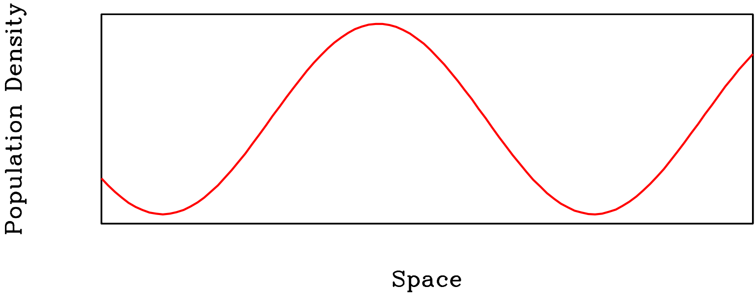

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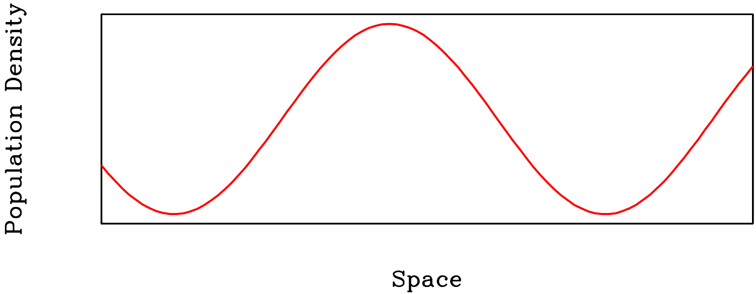

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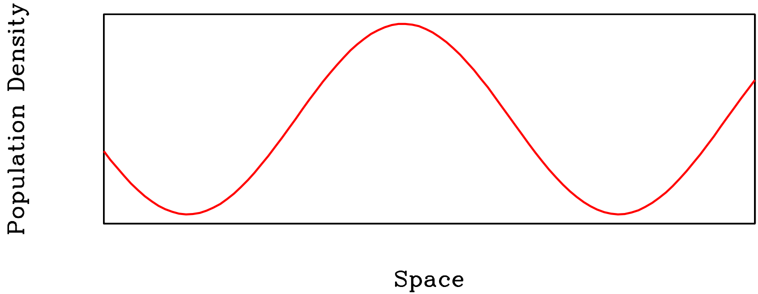

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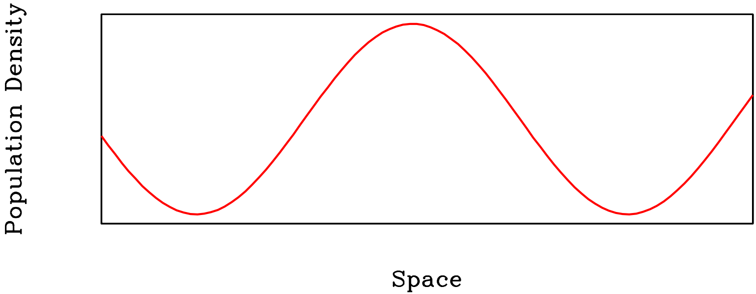

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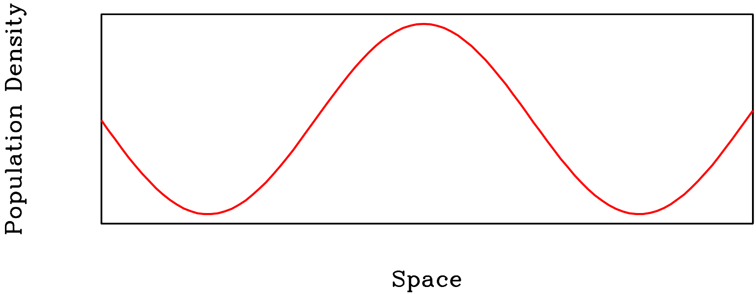

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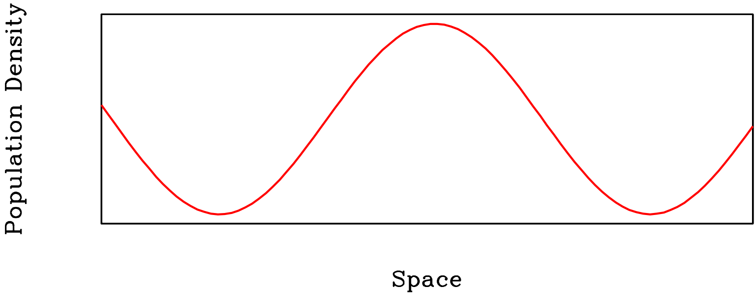

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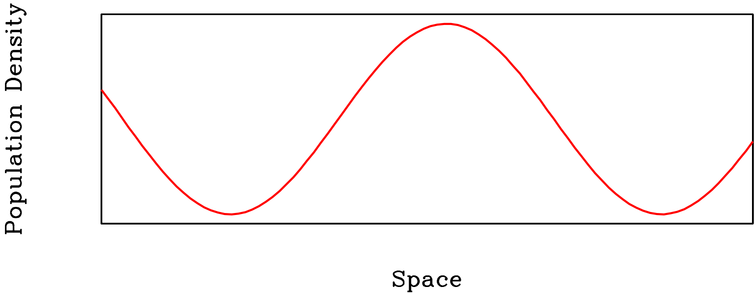

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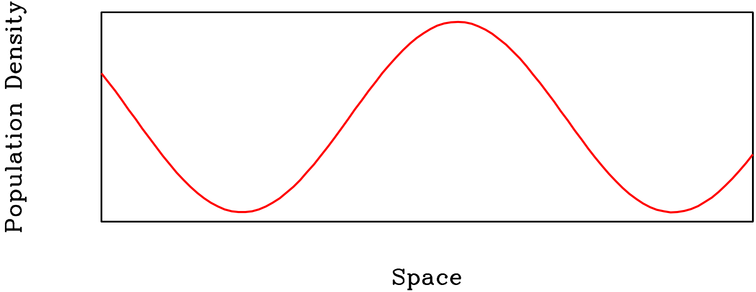

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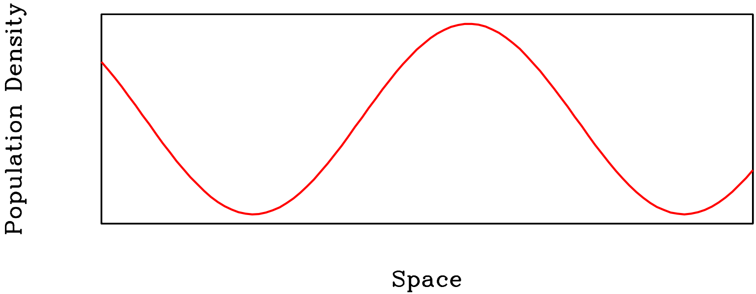

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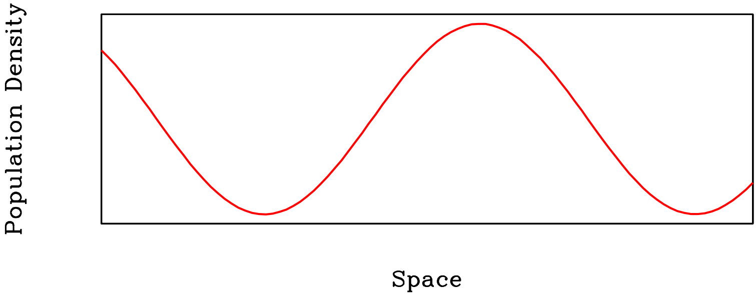

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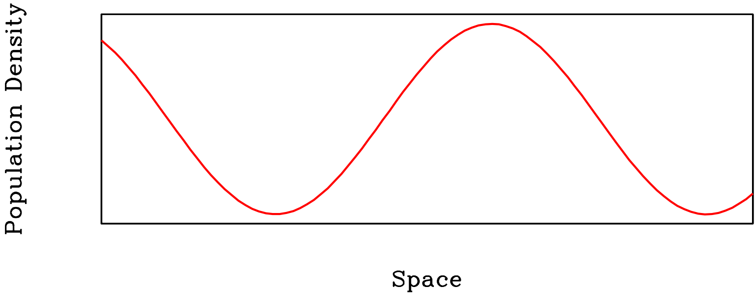

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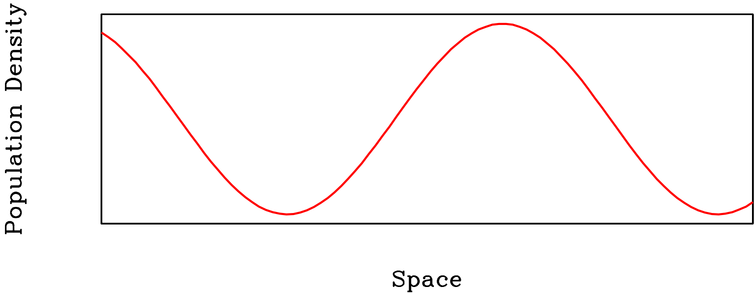

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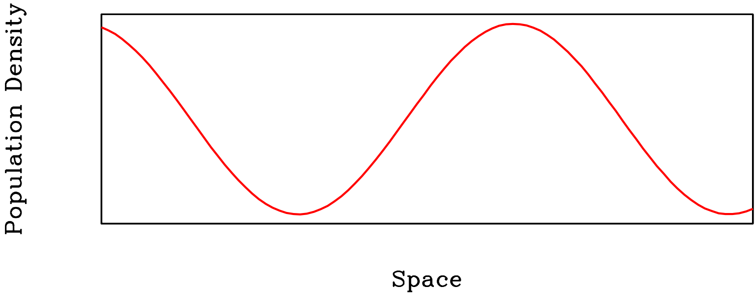

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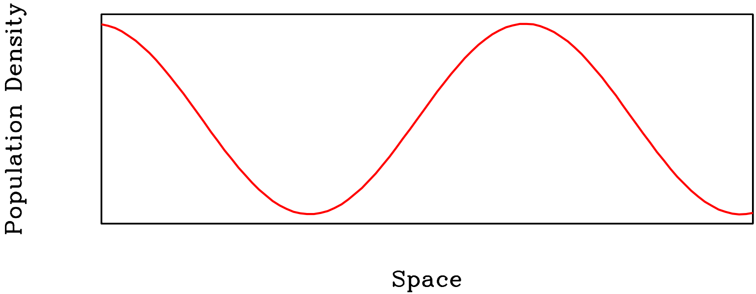

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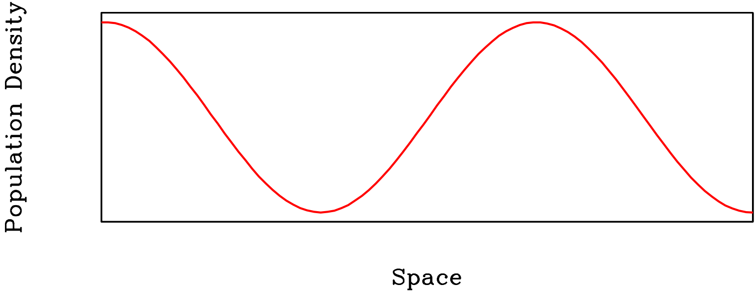

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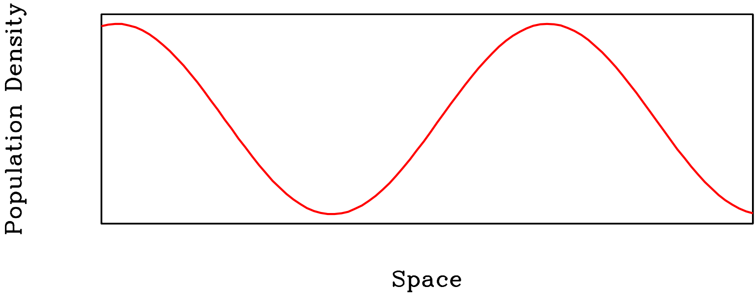

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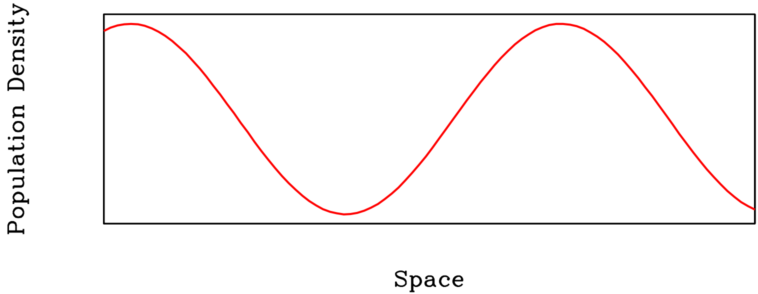

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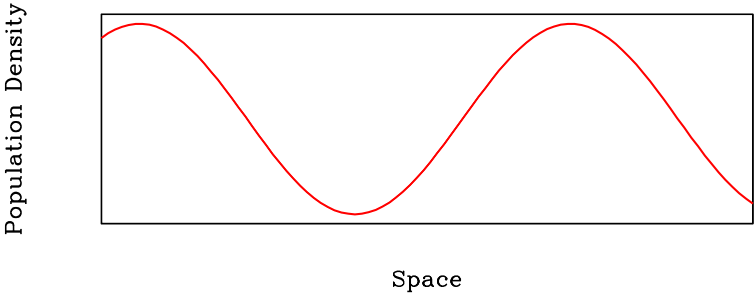

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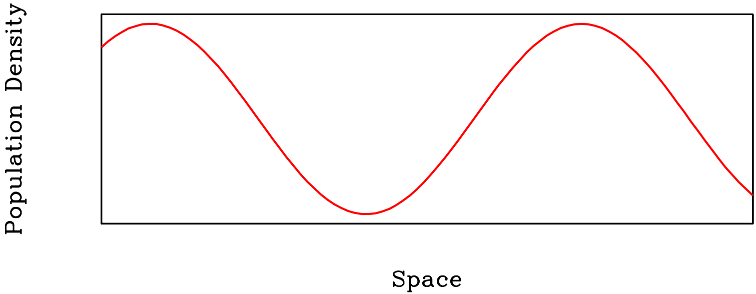

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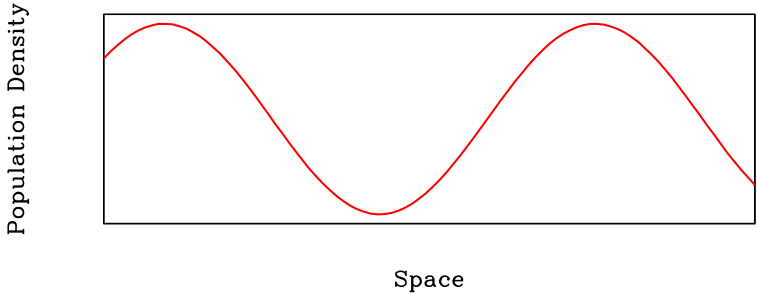

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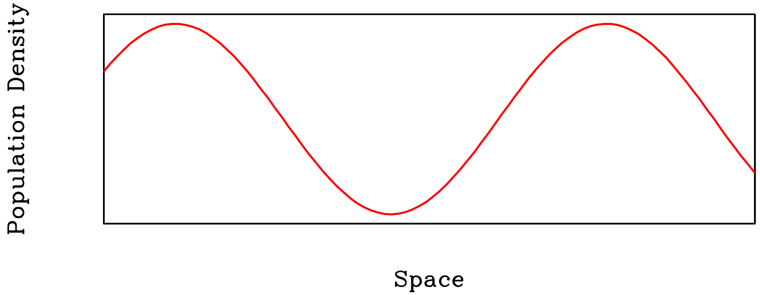

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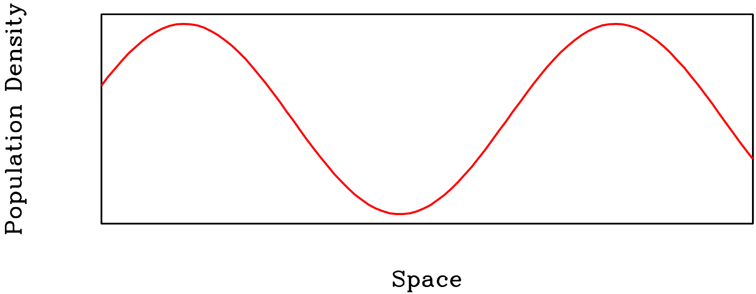

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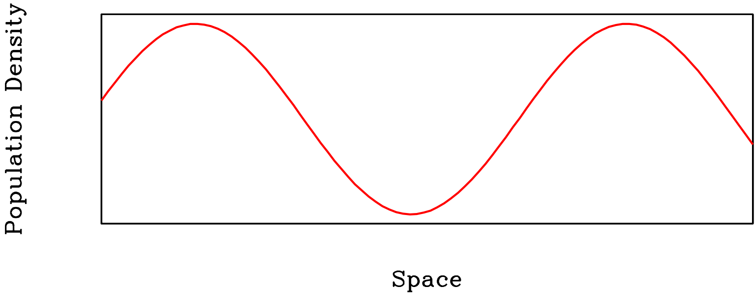

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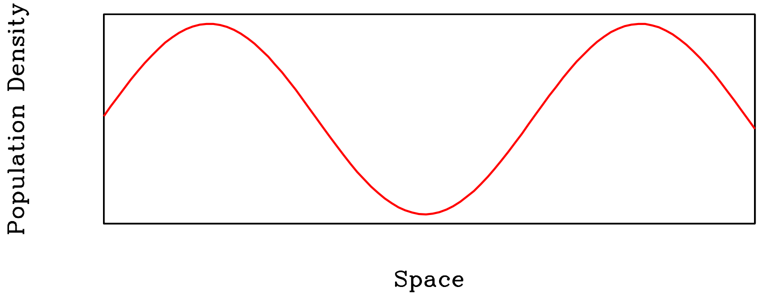

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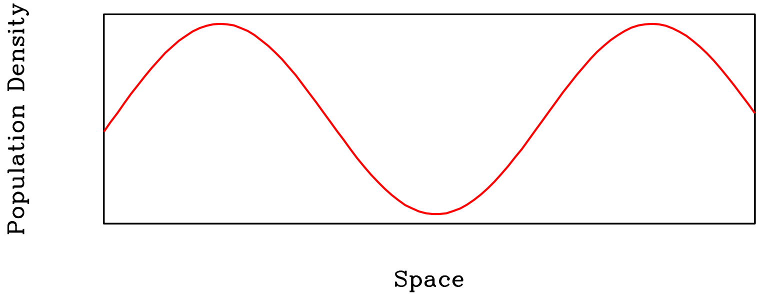


Wavetrains in Ecology

Fennoscandian voles

Clethrionomys glareolus

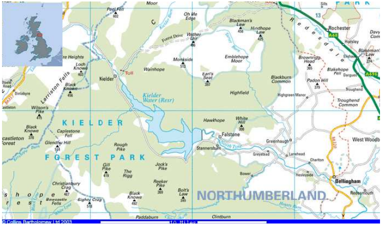
Fennoscandia

Ranta & Kaitala: Travelling waves in vole population dynamics. *Nature* 390:456 (1997).

Wavetrains in Ecology

Field vole

Microtus agrestis



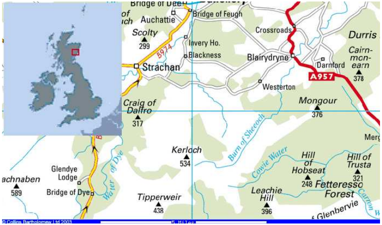
Kielder forest, UK

Lambin et al: Spatial asynchrony and periodic travelling waves in cyclic populations of field voles. *Proc. R. Soc. Lond.* B 265:1491 (1998).

Wavetrains in Ecology

Red grouse

Lagopus lagopus scoticus



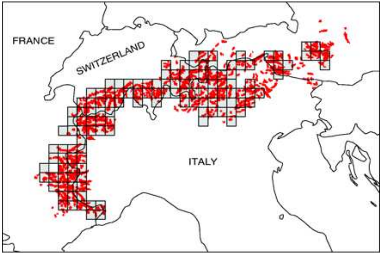
Kerloch moor, UK

Mougeot et al: Experimentally increased aggressiveness reduces population kin structure and subsequent recruitment in red grouse *Lagopus lagopus scoticus*. *J. Anim. Ecol.* 74:488 (2005).

Wavetrains in Ecology

Larch budmoth

Zeiraphera diniana



Central European Alps

Bjørnstad et al: Waves of larch budmoth outbreaks in the European Alps. *Science* 298:1020 (2002).

Wavetrains in Ecology

Autumnal moth

Epirrita autumnata

Northern Norway

Tenow et al: Waves and synchrony in *Epirrita autumnata/Operophtera brumata* outbreaks I. *J. Anim. Ecol.* 76:258 (2007).

Wavetrains in Mathematical Models

Wavetrain patterns are well studied in reaction-diffusion models:

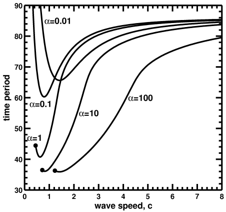
- in “realistic” ecological models
- in toy models, especially CGLE with real diffusion (“λ–ω systems”)

Key Mathematical Questions

- What are the wavetrain solutions?

  General: there is a wavetrain family (1970s)

  λ–ω: exact forms for the family (1970s)

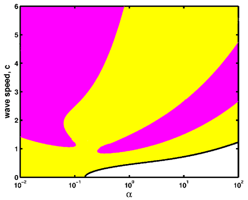


Key Mathematical Questions

- Which members of the wavetrain family are stable?

  General: some analytical results (1980s), and detailed numerical results (Rademacher, Sandstede & Scheel 2007; Smith & Sherratt 2007)

  λ–ω: exact condition (1970s)

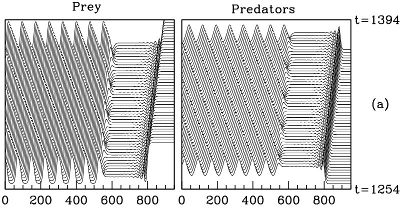


Key Mathematical Questions

- Wavetrain selection by initial / boundary conditions

  General: wavetrains are generated by invasions (1993), Dirichlet bc's (2002) and Robin bc's (2008)

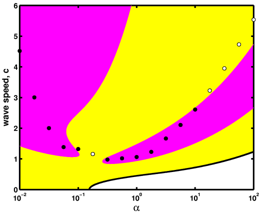
  λ–ω: exact solution of the wave selection problem for invasions (1994), Dirichlet bc's (2003) and Robin bc's (2008)



Open Problem: Wavetrain selection

Open problem: what is the mechanism of wavetrain selection by invasions / bc's?

Recent work: Merchant & Nagata (2010, 2014)



Open Problem: Absolute stability of wavetrains

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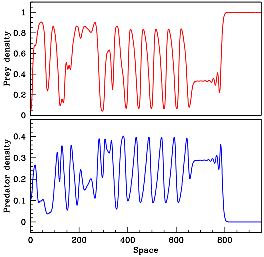
Recent work: numerical calculation of absolute stability for λ–ω systems (Smith, Rademacher & Sherratt 2009)



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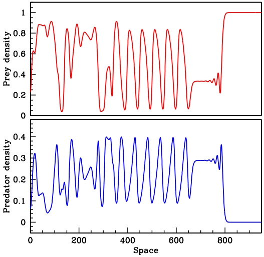

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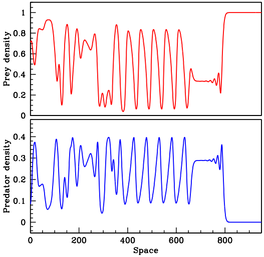

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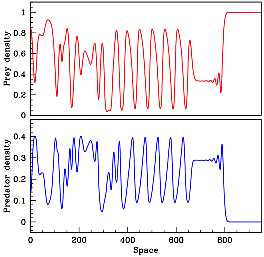

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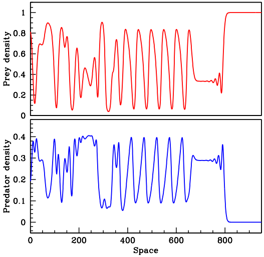

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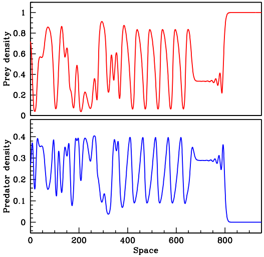

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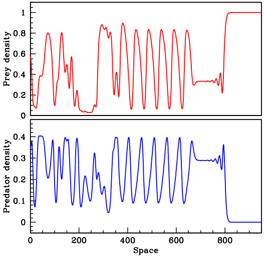

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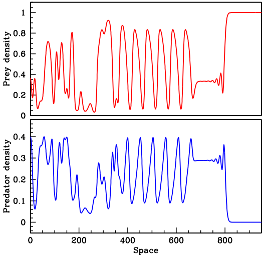

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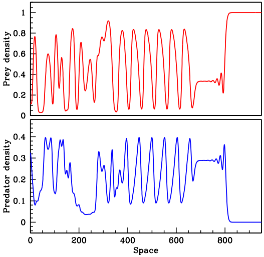

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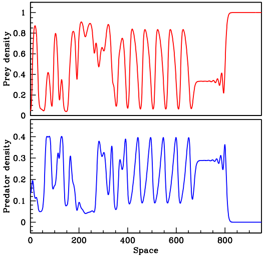

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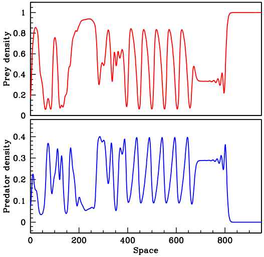

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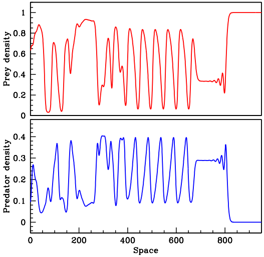

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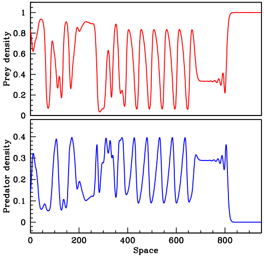

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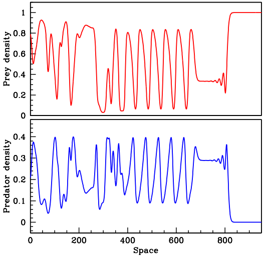

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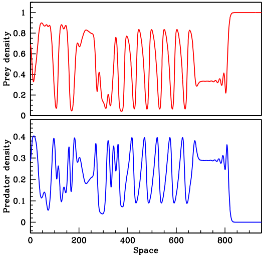

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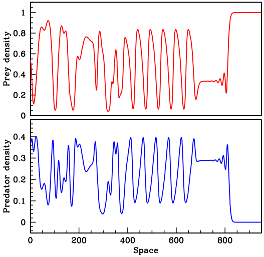

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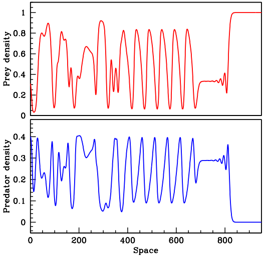

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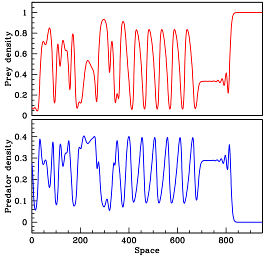

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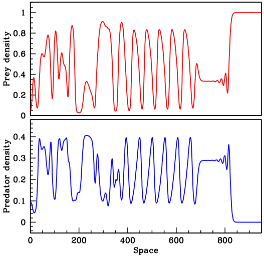

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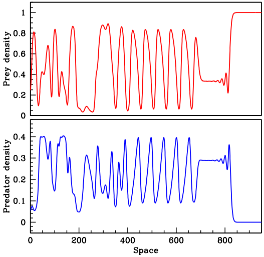

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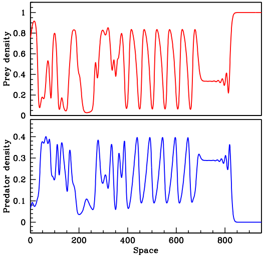

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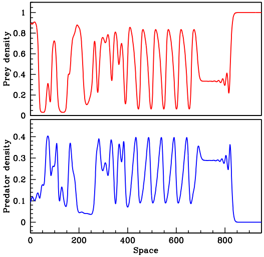

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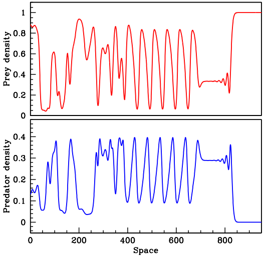

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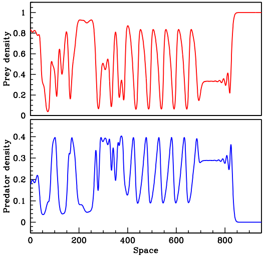

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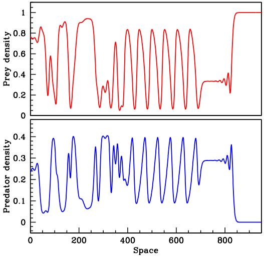

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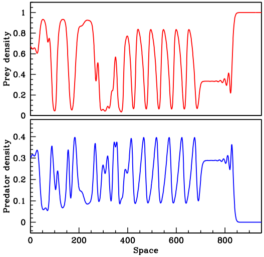

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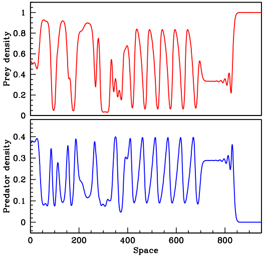

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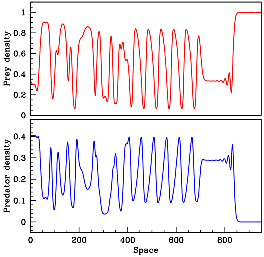

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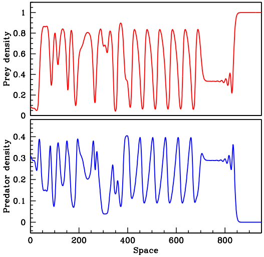

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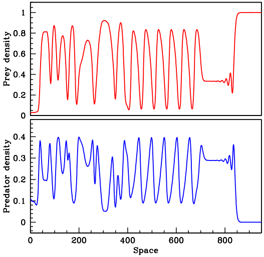

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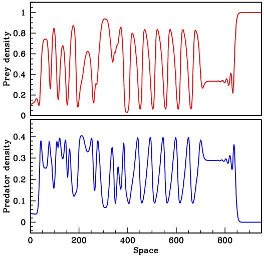

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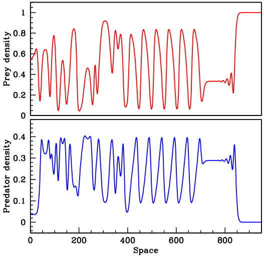

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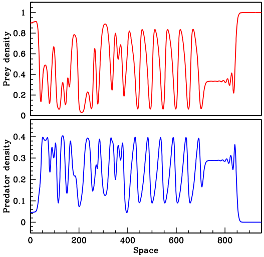

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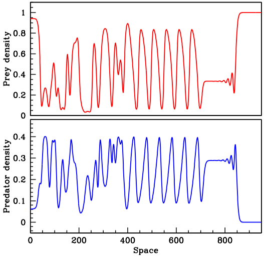

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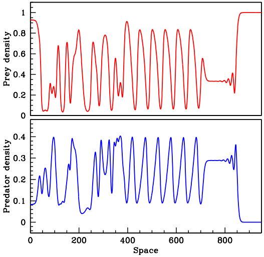

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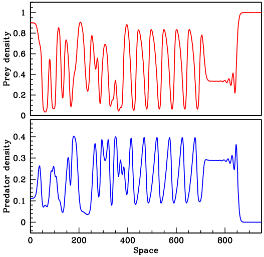

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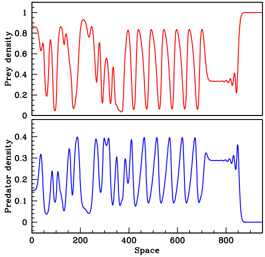

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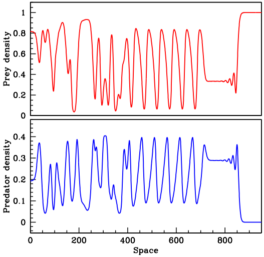

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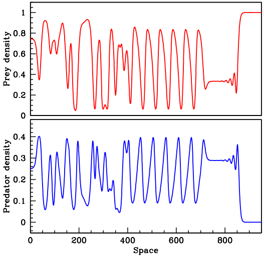

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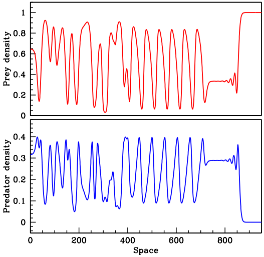

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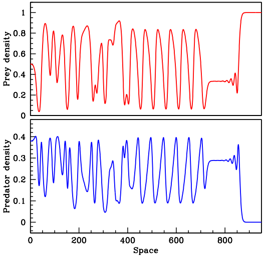

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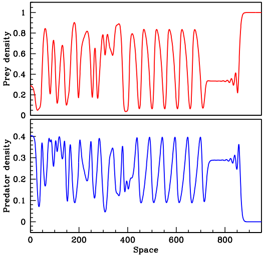

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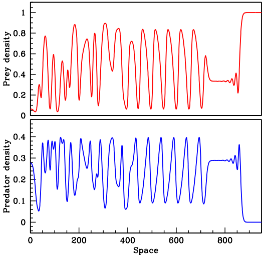

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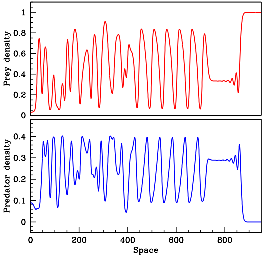

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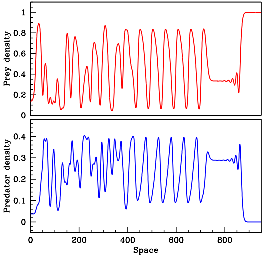

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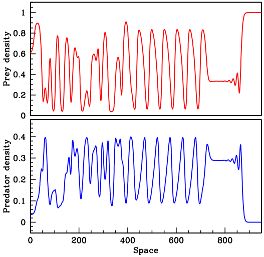

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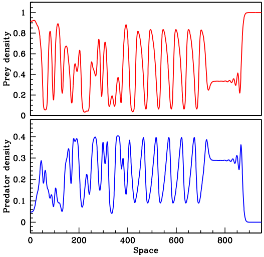

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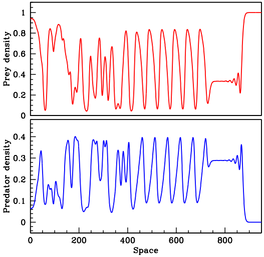

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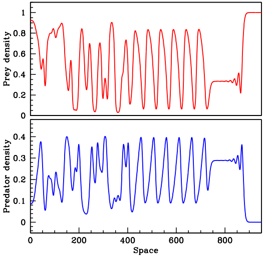

Open Problem: Wavetrains in Other Types of Models

Open problem: understanding wavetrain patterns in ecological models other than reaction-diffusion equations

Recent work: equations with non-local dispersal (Sherratt, SIADS 13: 1517-1541, 2014)

$$\begin{aligned}\partial u/\partial t &= f(u,v) + D\int_{-\infty}^{+\infty} K(x-y)u(y)\,dy \\ \partial v/\partial t &= g(u,v) + D\int_{-\infty}^{+\infty} K(x-y)v(y)\,dy\end{aligned}$$

Wavetrains in Equations with Non-local Dispersal

$$
\begin{aligned}
\partial u/\partial t &= \overbrace{f(u,v)}^{\substack{\lambda\text{–}\omega \\ \text{kinetics}}} + D\int_{-\infty}^{+\infty} K(x-y)u(y)\,dy \\
\partial v/\partial t &= g(u,v) + D\int_{-\infty}^{+\infty} K(x-y)v(y)\,dy
\end{aligned}
$$

For Laplace and Gaussian kernels, wavetrain existence and stability mirrors that for λ–ω reaction-diffusion eqns

Laplace	$K(s) = (1/2a)\exp(-\lvert s\rvert/a)$
Gaussian	$K(s) = (1/a\sqrt{\pi})\exp(-s^2/a^2)$

Wavetrains in Equations with Non-local Dispersal

$$\begin{aligned}\partial u/\partial t &= \overbrace{f(u,v)}^{\substack{\lambda\text{–}\omega\\ \text{kinetics}}} + D\int_{-\infty}^{+\infty} K(x-y)u(y)\,dy \\ \partial v/\partial t &= g(u,v) + D\int_{-\infty}^{+\infty} K(x-y)v(y)\,dy\end{aligned}$$

But in general there are significant differences between wavetrains in the models with non-local dispersal and diffusion. E.g. for the top hat kernel, Eckhaus stability $\not\Rightarrow$ stability.

Top hat kernel: $K(s) = (1/2a)$ if $|s| < a$, 0 otherwise

Wavetrains in Equations with Non-local Dispersal

$$\begin{aligned}
\partial u/\partial t &= \overbrace{f(u,v)}^{\substack{\lambda\text{–}\omega\\ \text{kinetics}}} + D\int_{-\infty}^{+\infty} K(x-y)u(y)\,dy \\
\partial v/\partial t &= g(u,v) + D\int_{-\infty}^{+\infty} K(x-y)v(y)\,dy
\end{aligned}$$

And for the Cauchy kernel, all non-constant wavetrains are unstable.

Cauchy kernel: $K(s) = \left[\pi a\left(1 + s^2/a^2\right)\right]^{-1}$

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