# Using Mathematics and History to Predict the Future of Semi-Arid Vegetation

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March 2017

*This talk can be downloaded from my web site*

www.ma.hw.ac.uk/∼jas

In this talk I will discuss the use of mathematical models to:

- predict future vegetation levels in semi-arid regions
- infer the historical origin of vegetation in semi-arid regions

# Outline

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# Vegetation Patterns

Desert ecosystems provide a classic example of self-organised pattern formation.

W National Park, Niger

Average patch width is 50 m

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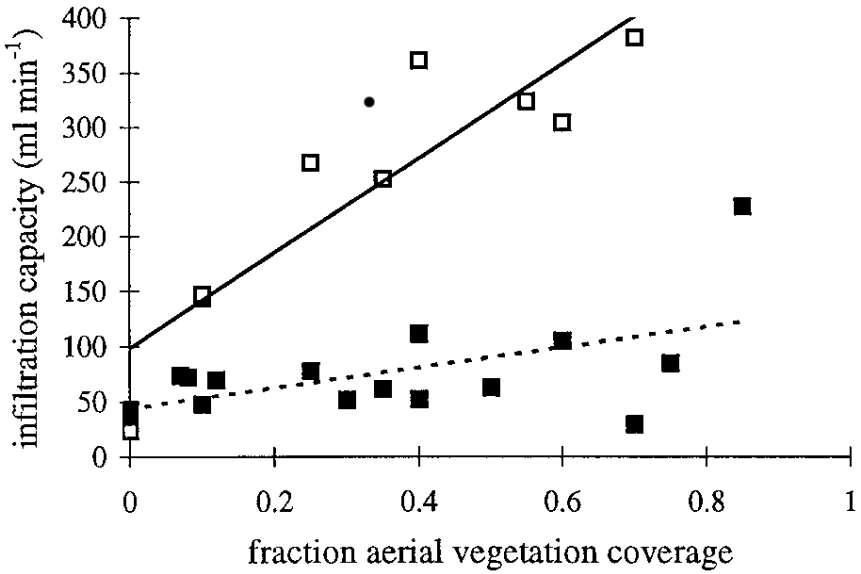


Data from Burkina Faso

Rietkerk et al
Plant Ecology 148: 207-224, 2000

More plants $\Rightarrow$ more roots and organic matter in soil
$\Rightarrow$ more infiltration of rainwater

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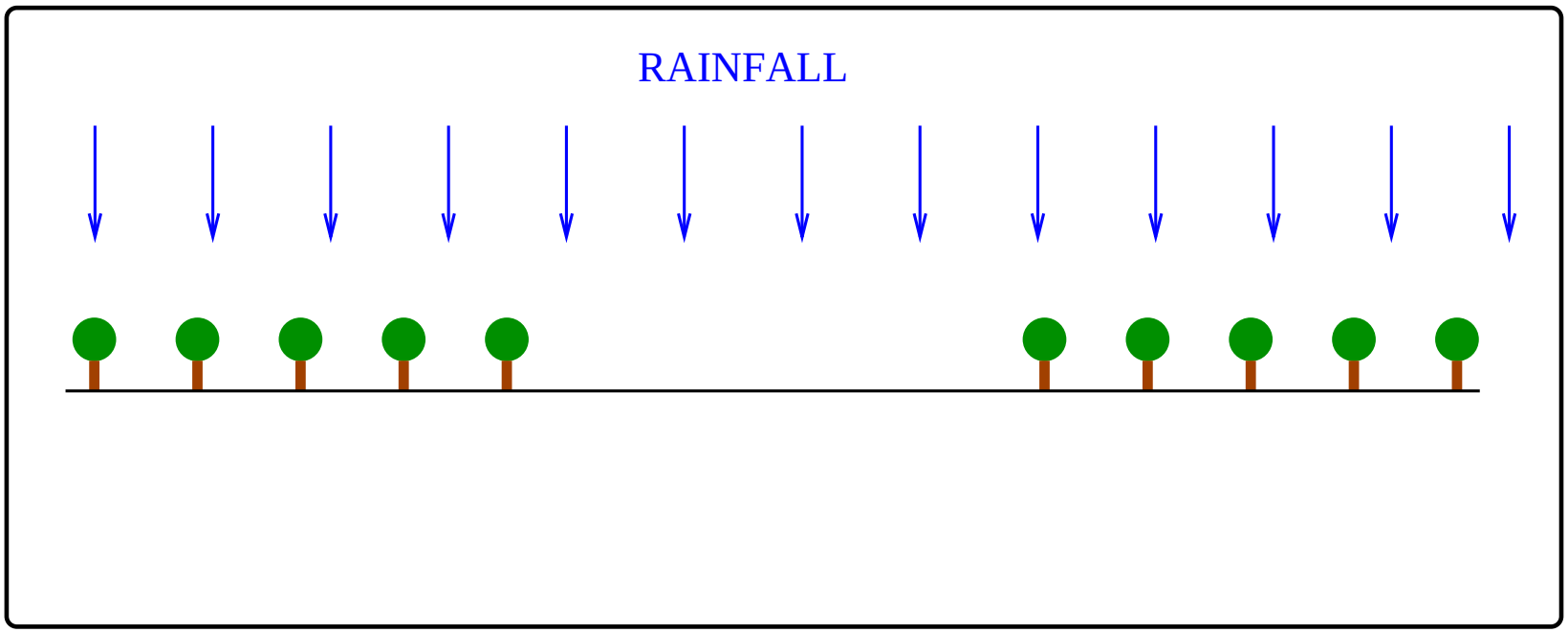

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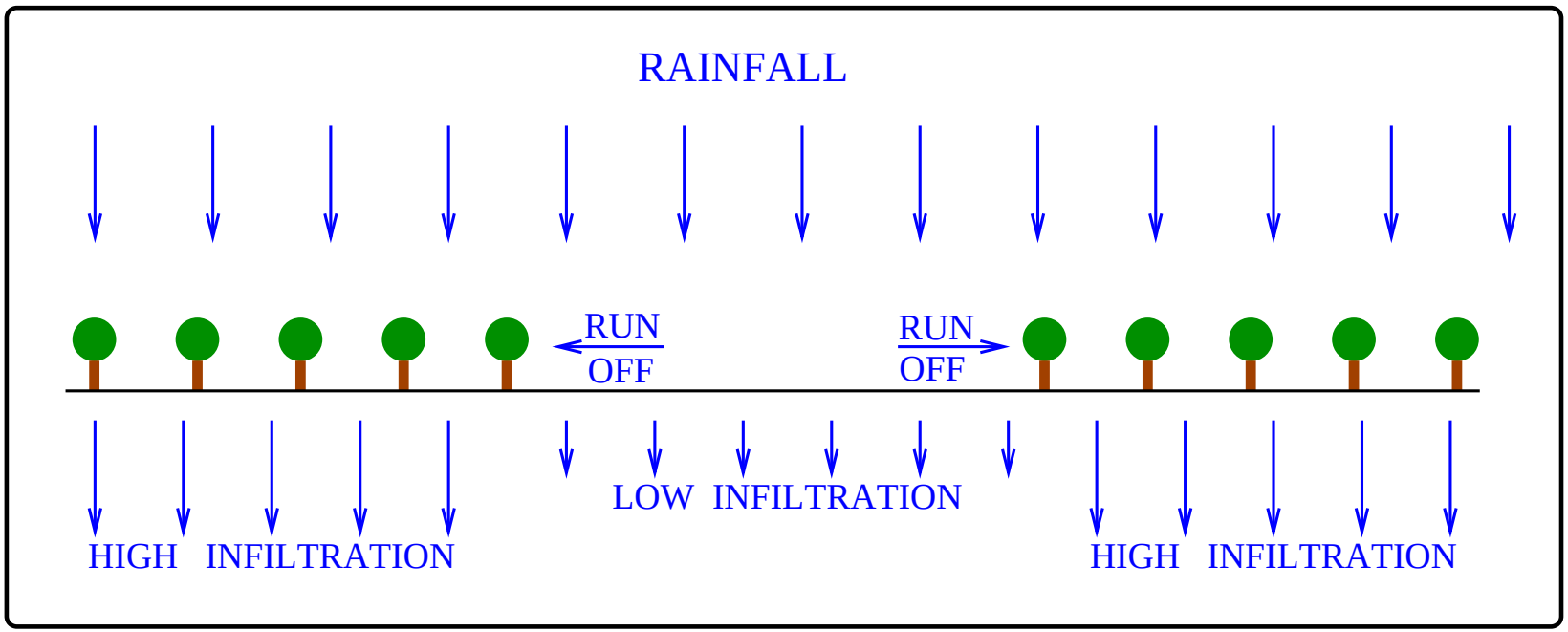

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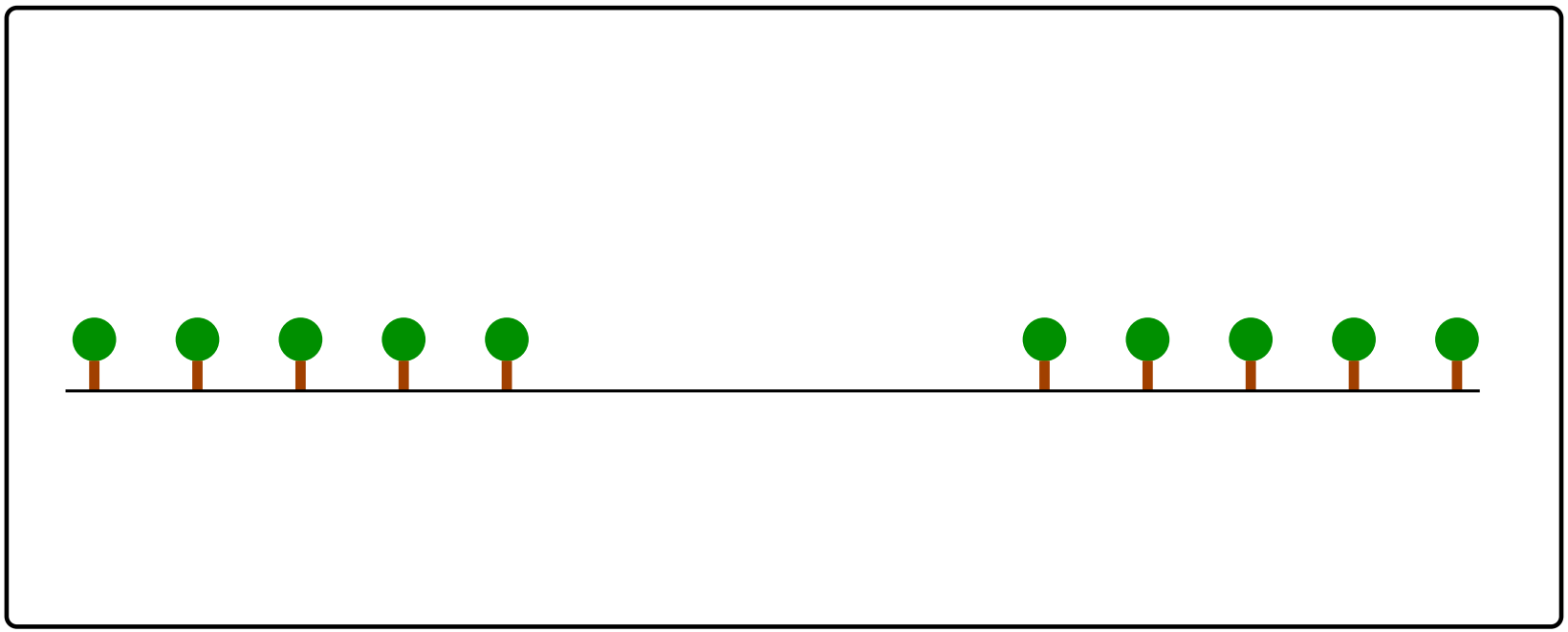

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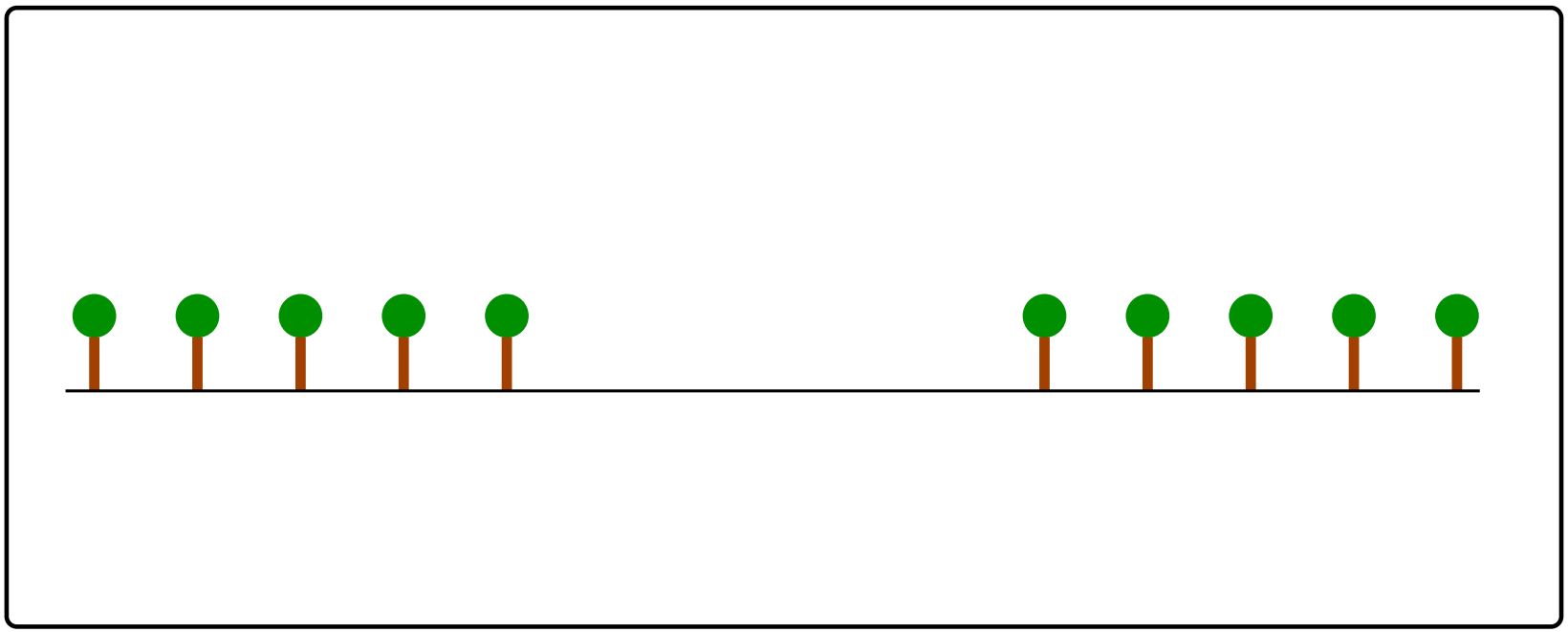

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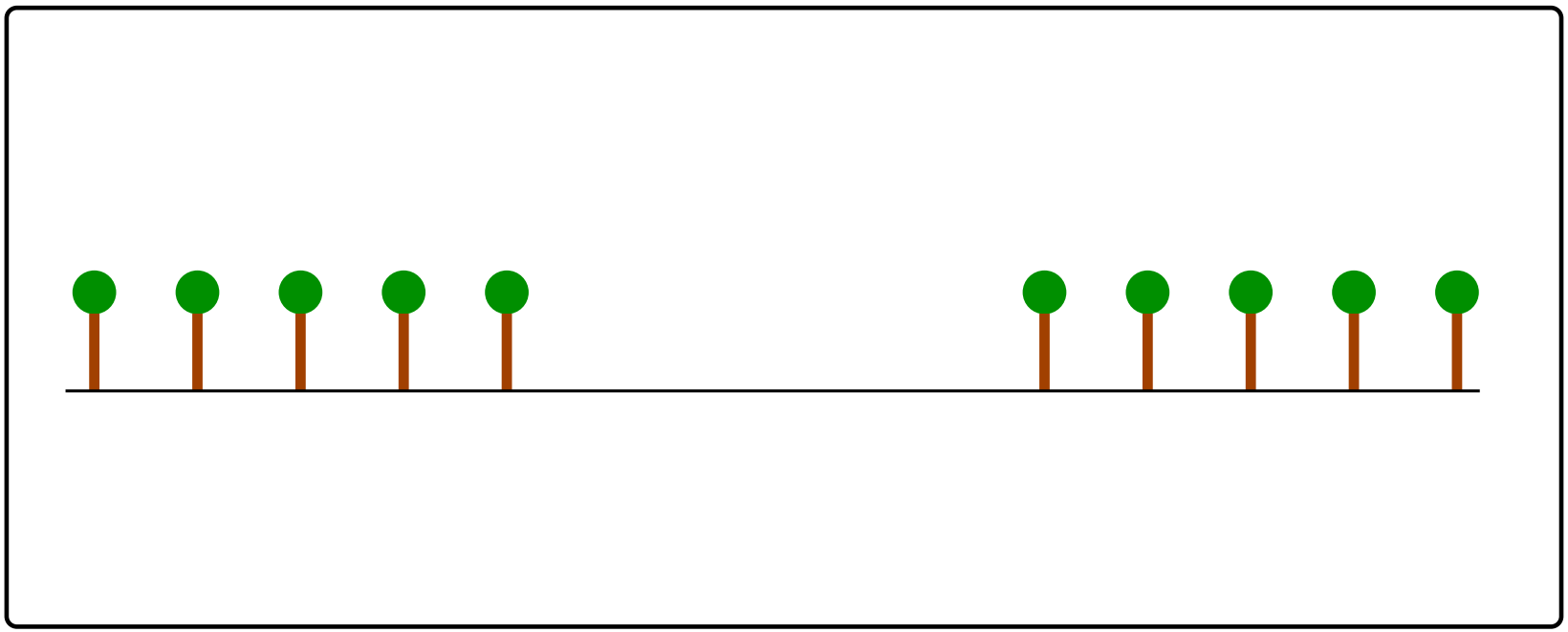

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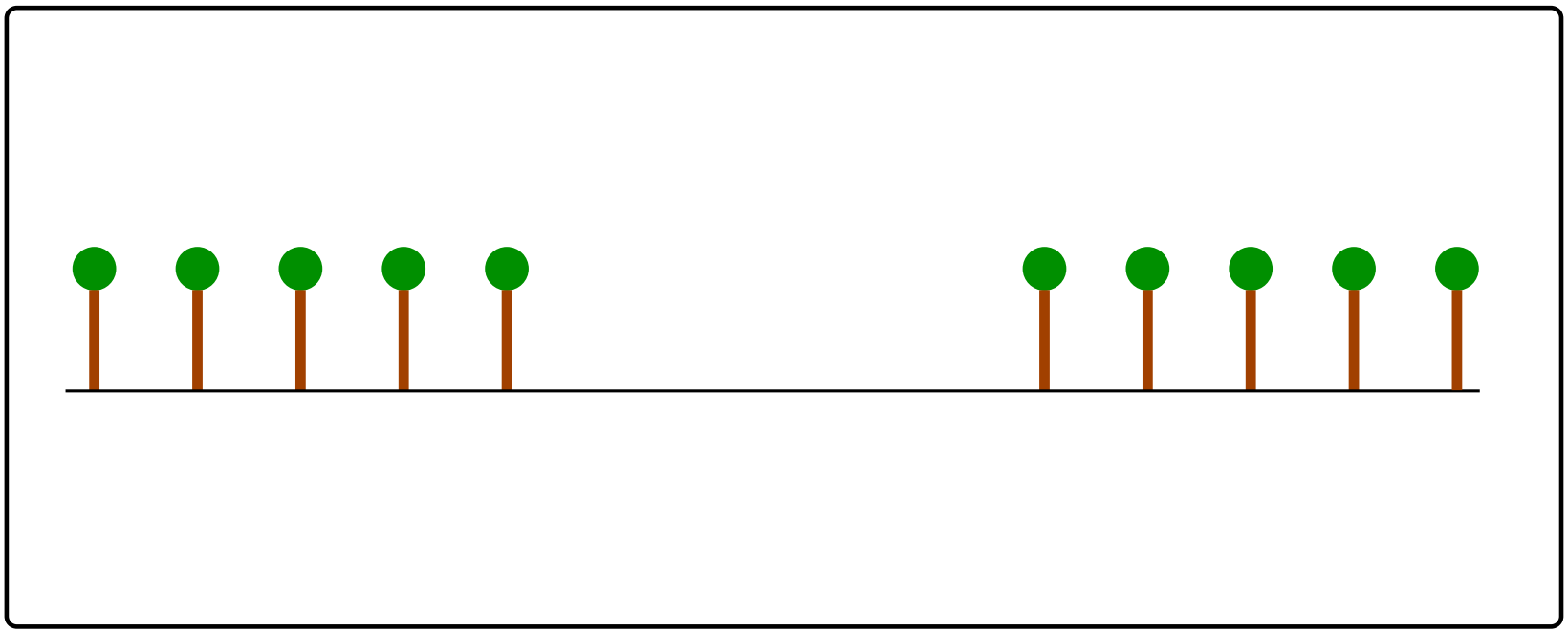

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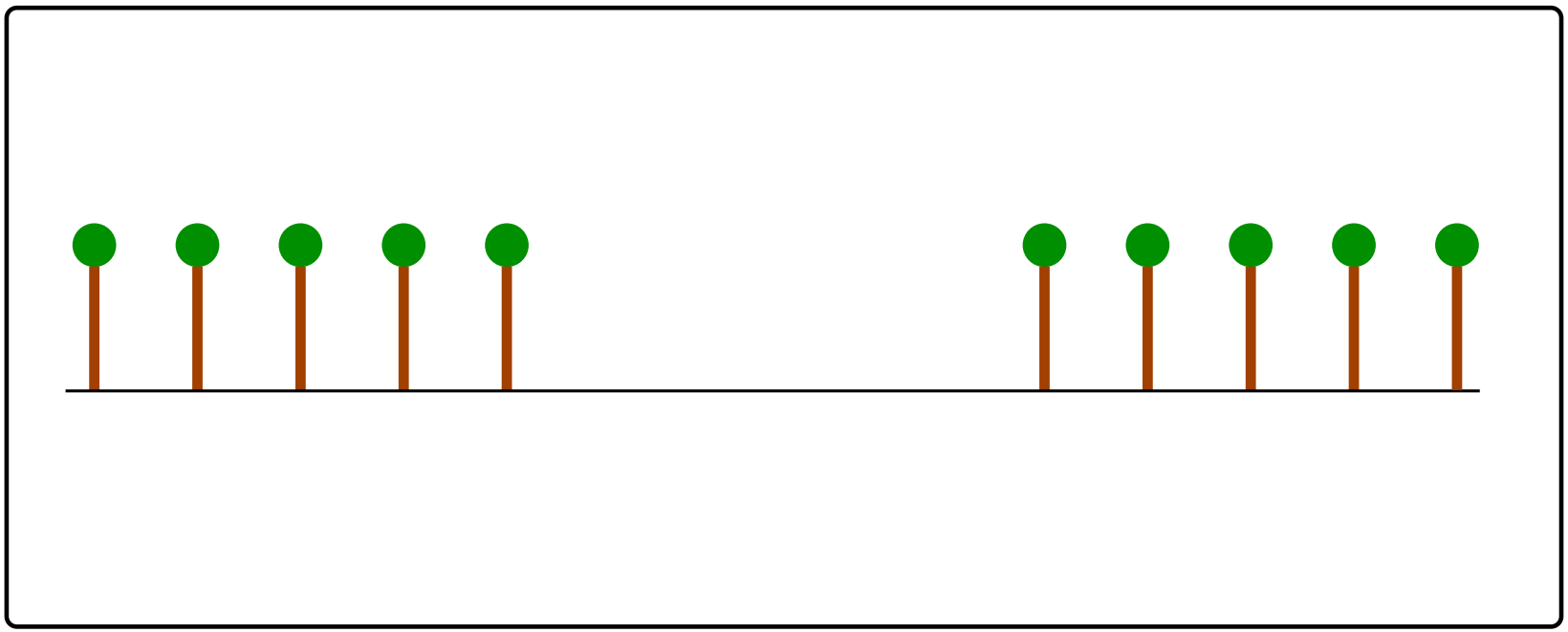

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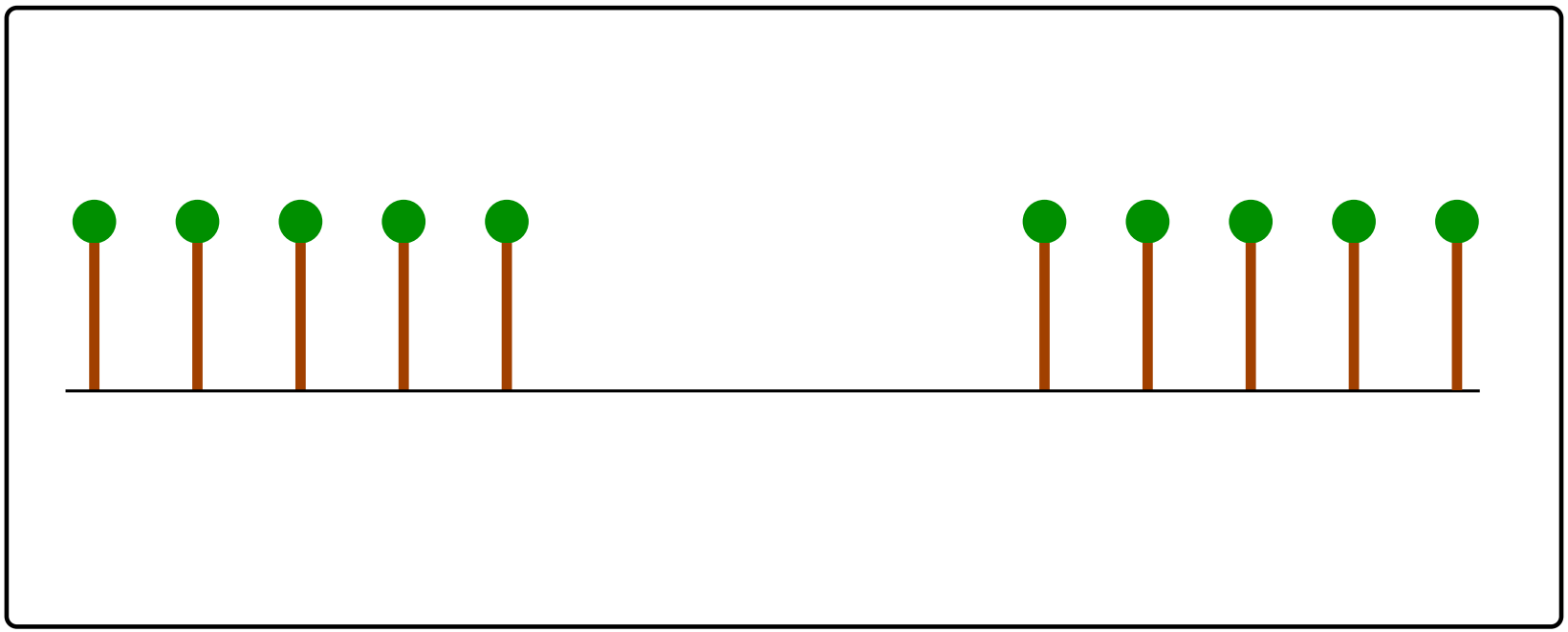

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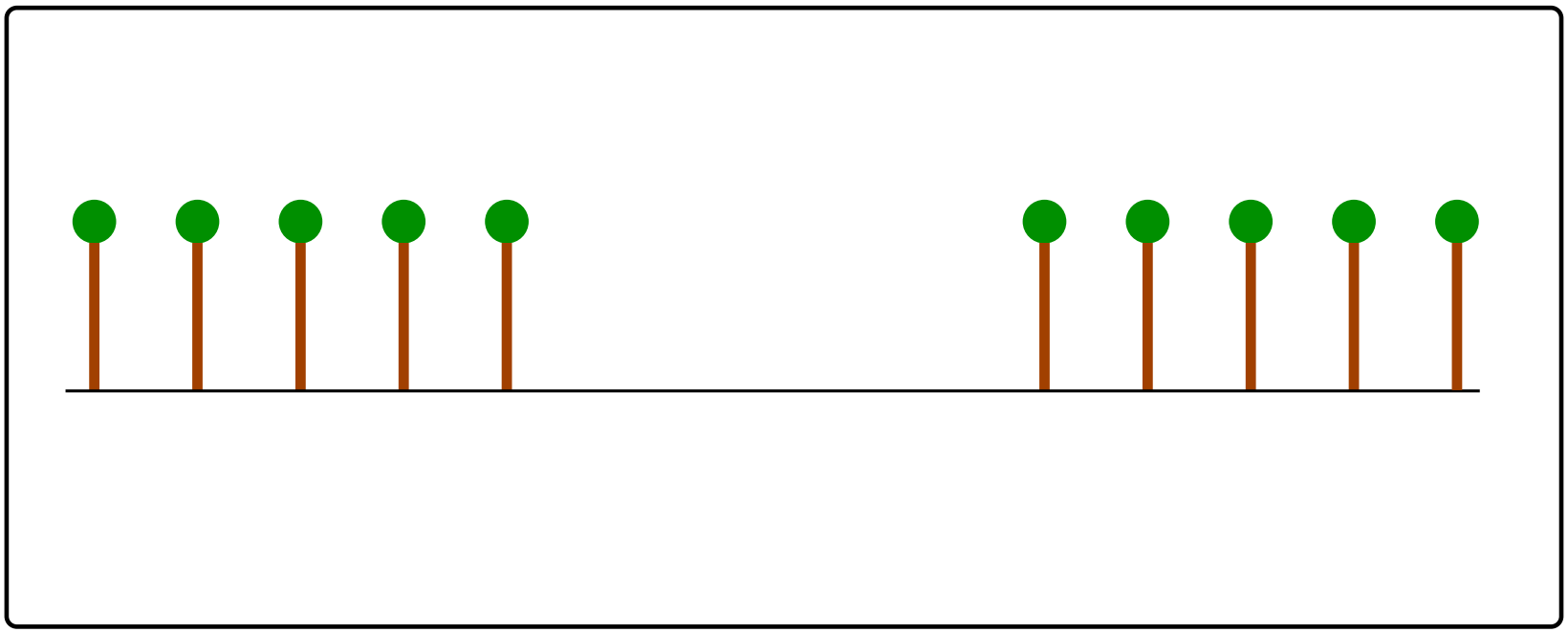

# Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.

Bushy vegetation in Niger

Mitchell grass in Australia

(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, Mexico and S-W USA.

# Outline

# Mathematical Model of Klausmeier

$$\begin{aligned} \partial u/\partial t &= \overbrace{wu^2}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\partial^2 u/\partial x^2}^{\text{plant dispersal}} \\ \partial w/\partial t &= \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{D\,\partial^2 w/\partial x^2}_{\text{diffusion of water}} \end{aligned}$$

(Klausmeier, Science 284: 1826-8, 1999)

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The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

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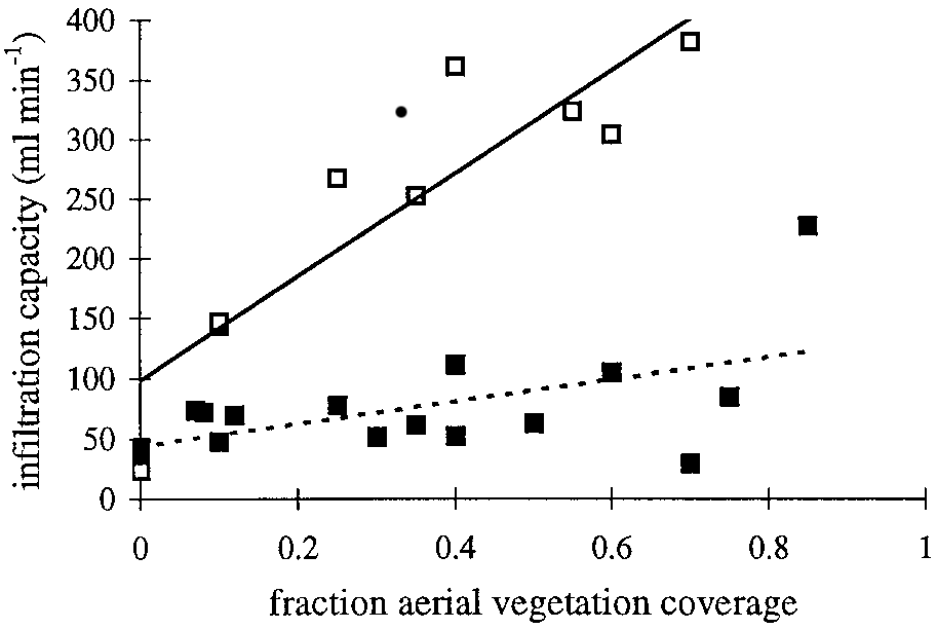


Water uptake=
Water density
$\times$ Plant density
$\times$ $\begin{pmatrix}\text{infiltration}\\ \text{rate}\end{pmatrix}$

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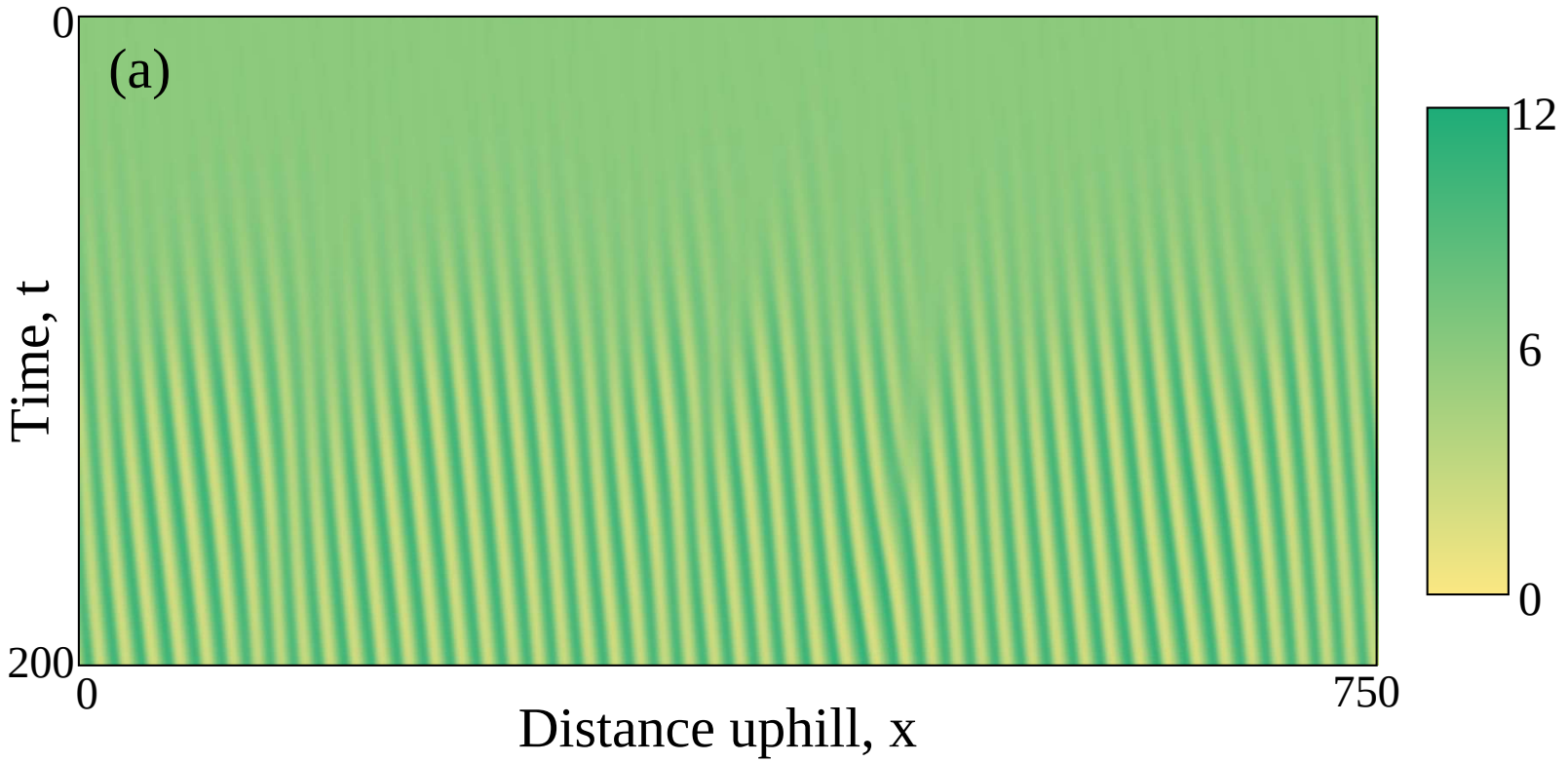
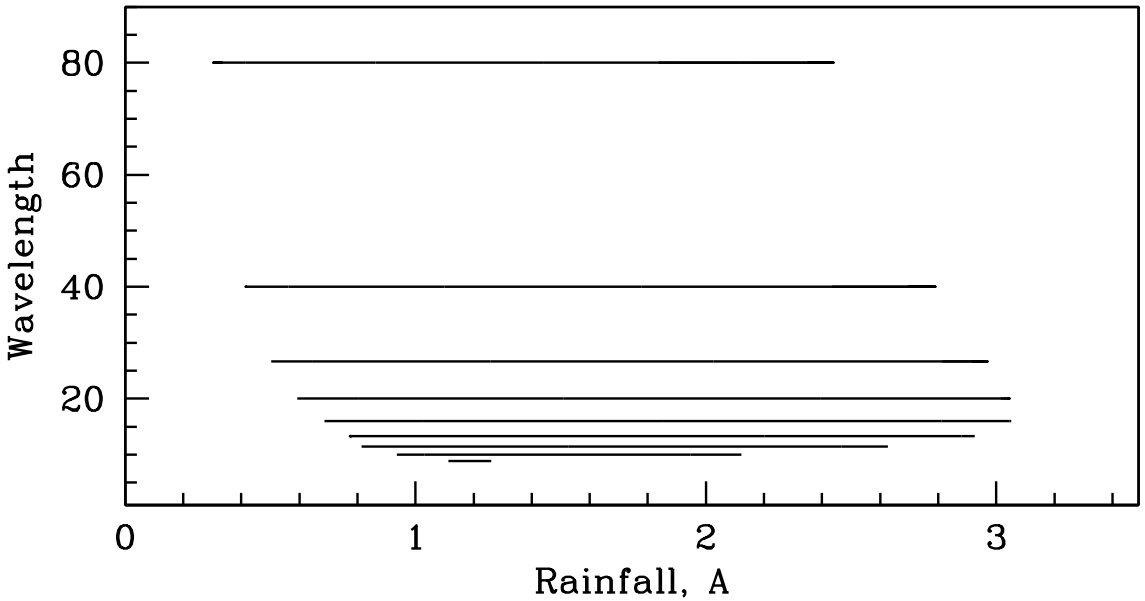
# Typical Solution of the Model



# Pattern Existence and Stability

High rainfall $\Rightarrow$ uniform vegetation
Low rainfall $\Rightarrow$ no vegetation
Medium rainfall $\Rightarrow$ patterns



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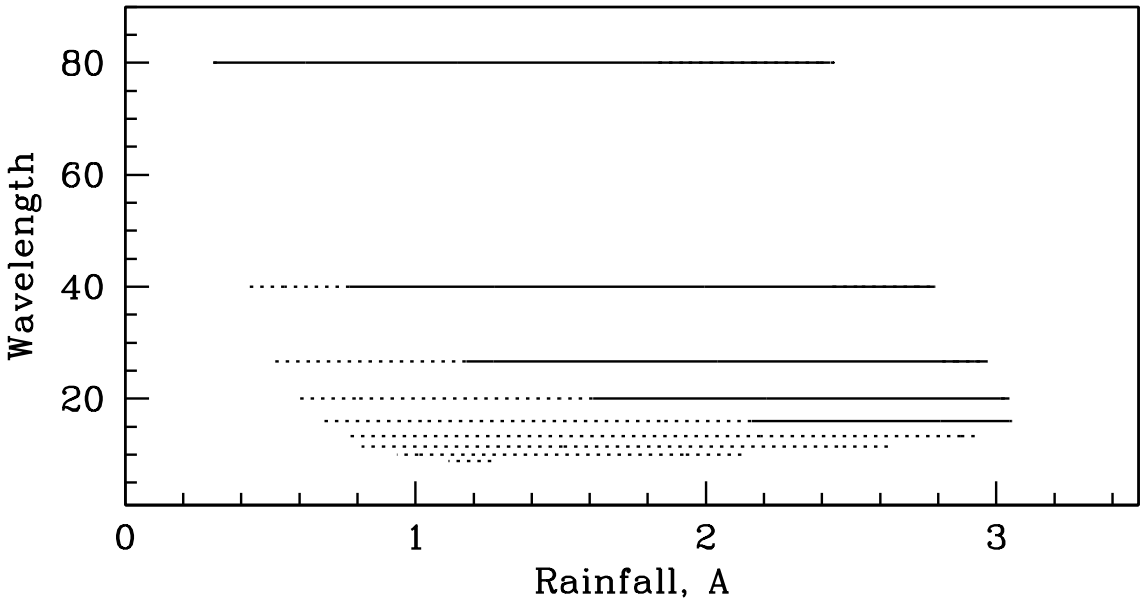


# Pattern Stability: The Key Result

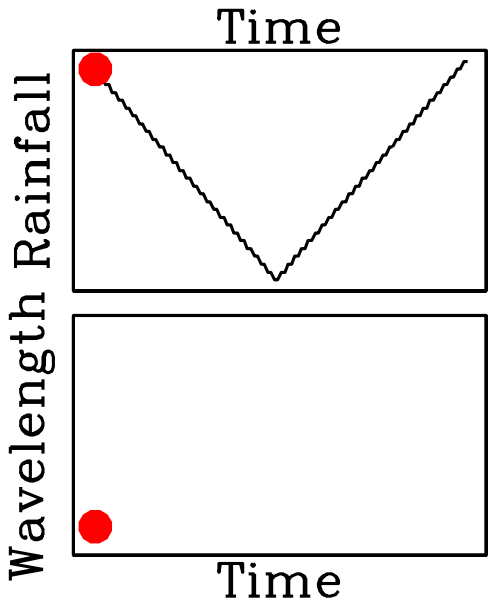
## Key Result

Many of the possible patterns are unstable and thus will never be seen.

However, for a wide range of rainfall levels, there are multiple stable patterns.

# Variations in Rainfall: Hysteresis

Model prediction: as rainfall is varied within the range giving patterns, abrupt changes in pattern wavelength occur.

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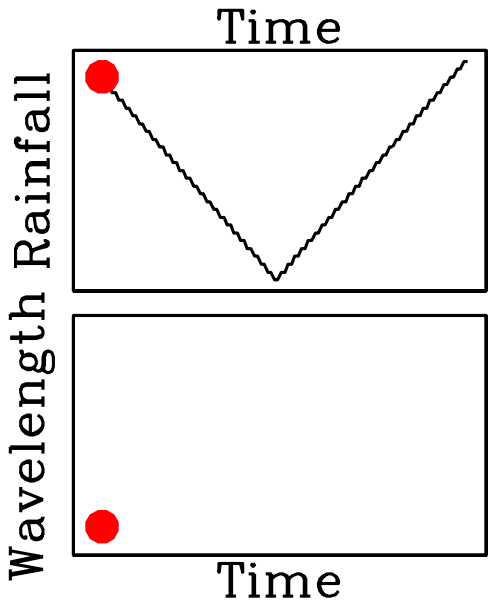


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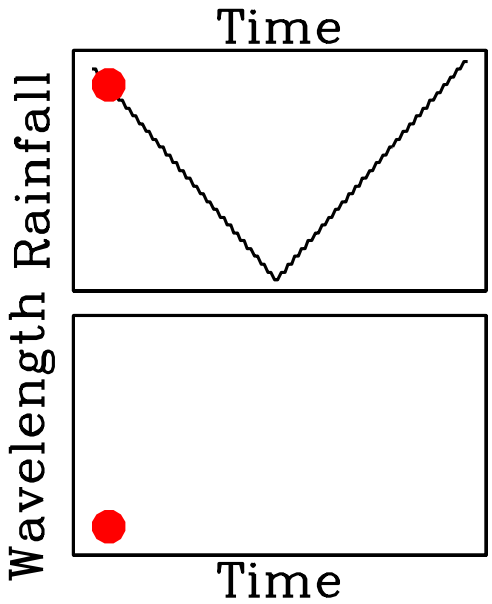


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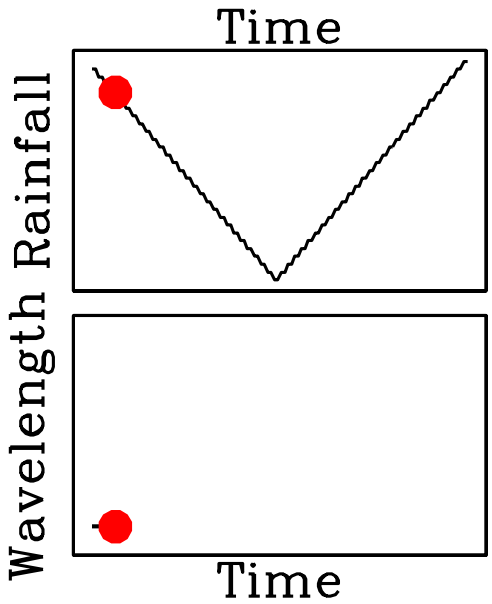


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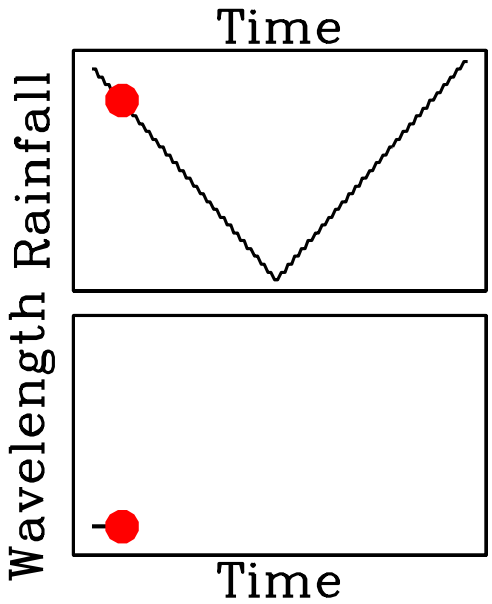


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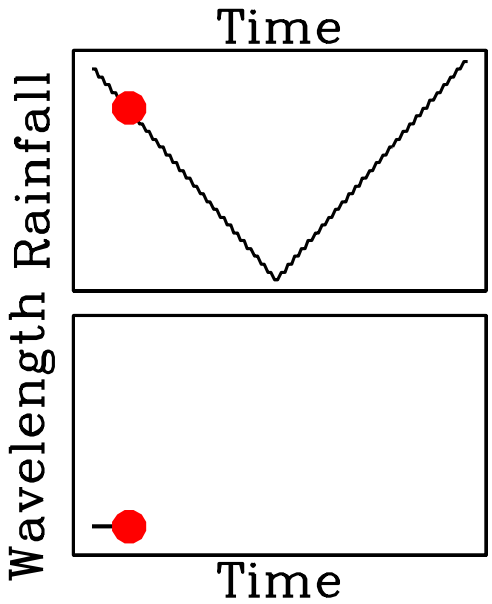


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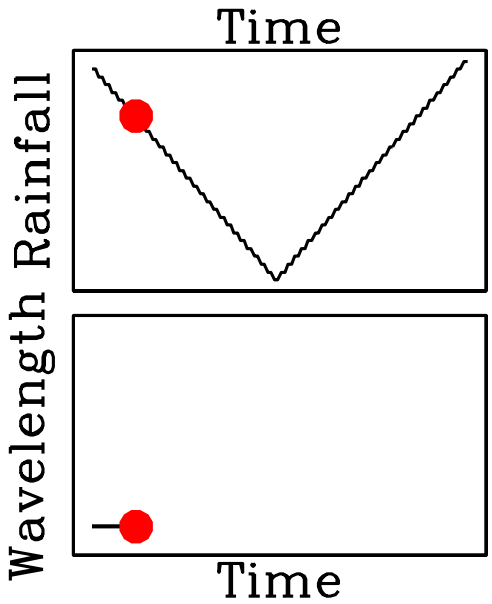


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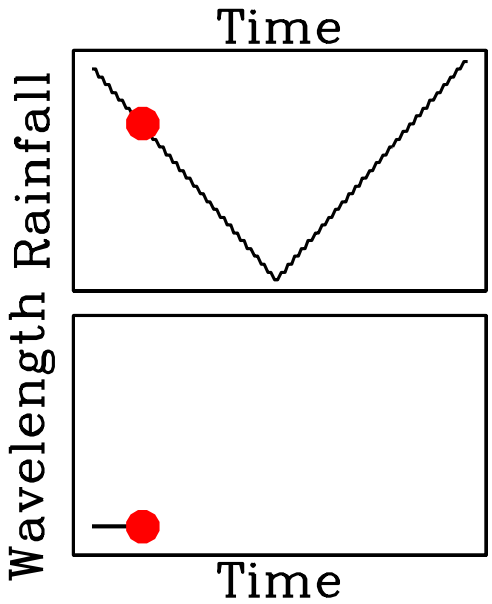


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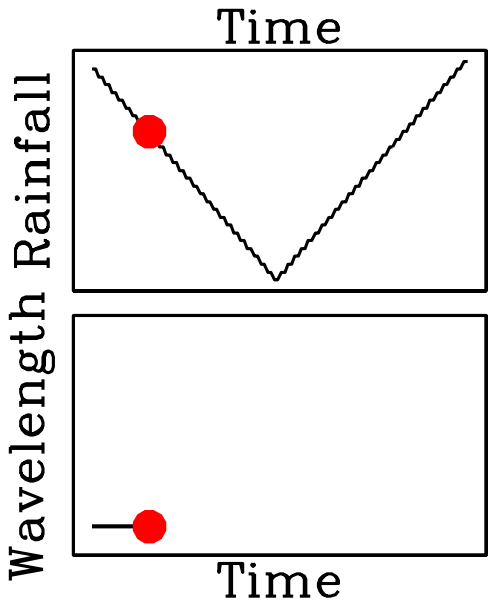


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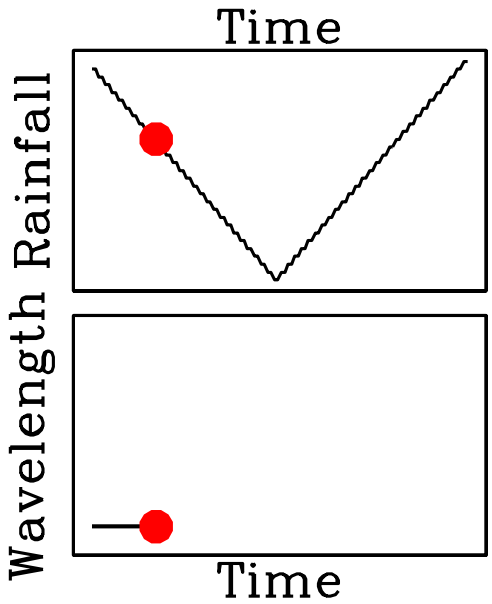


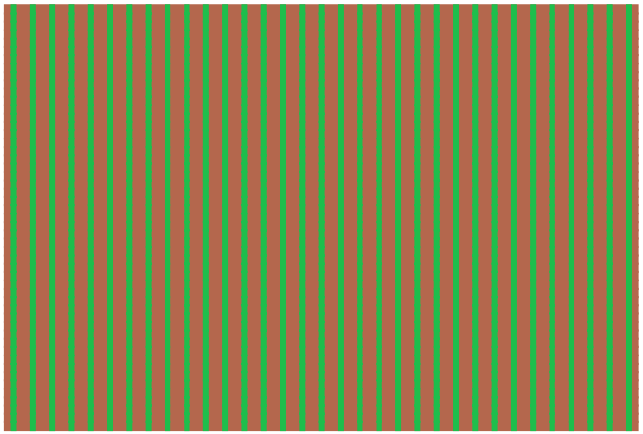

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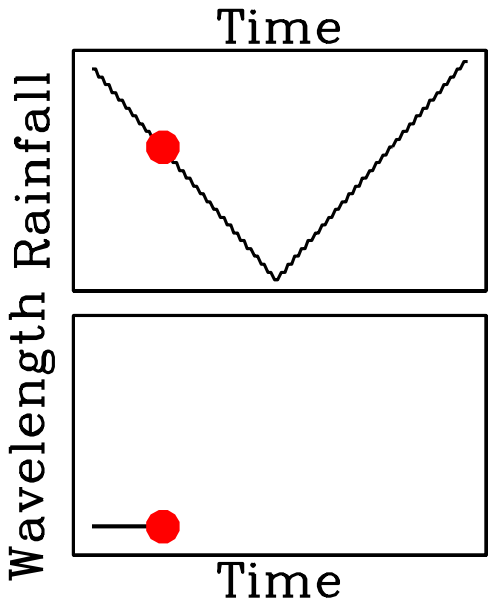


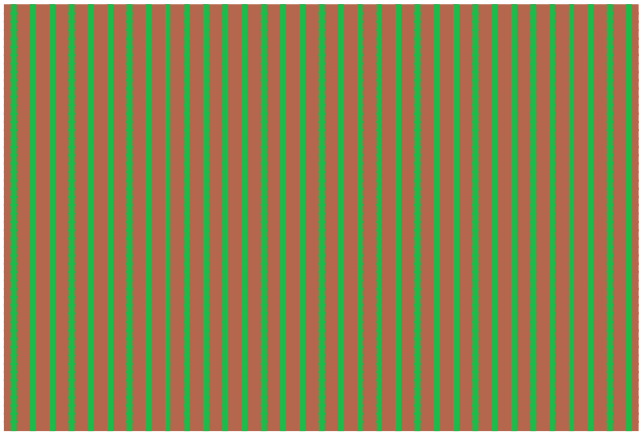

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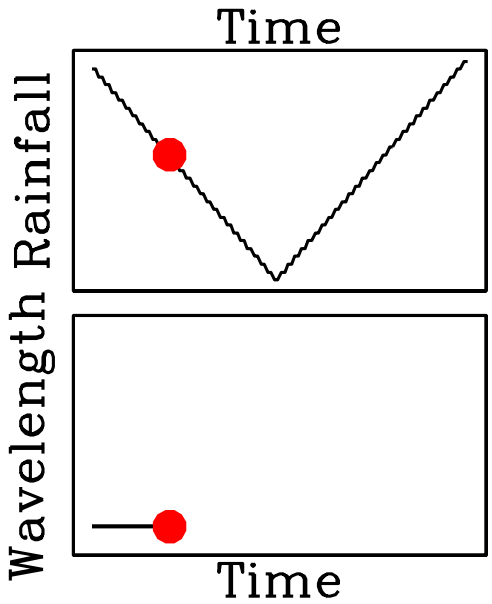


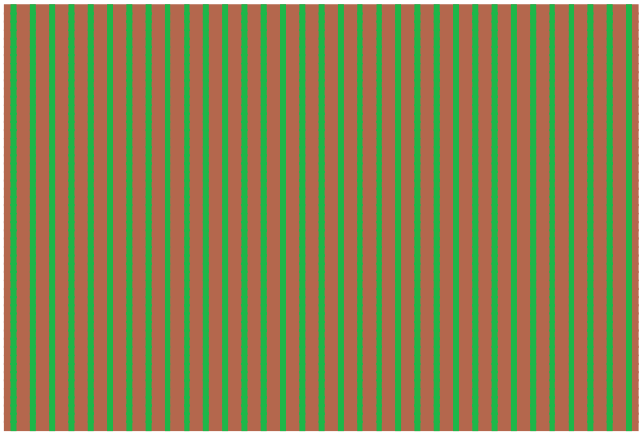

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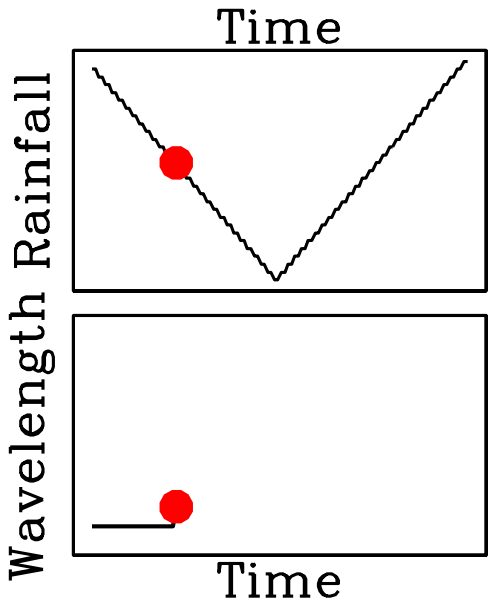


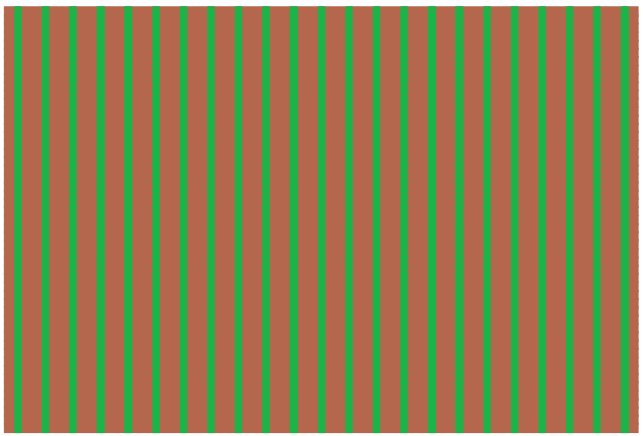

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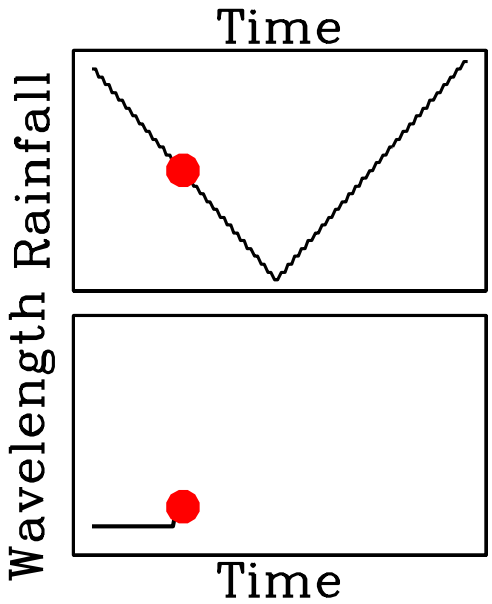


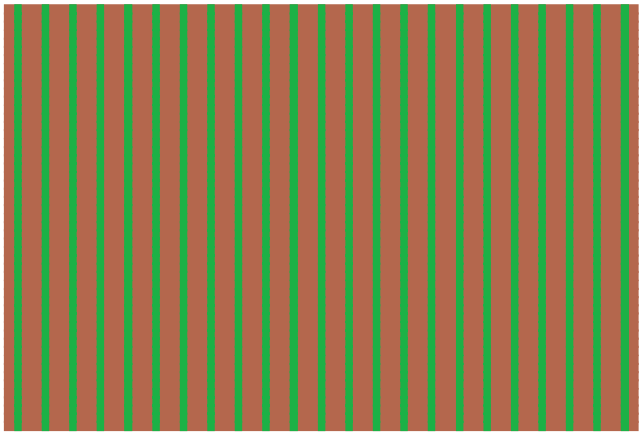

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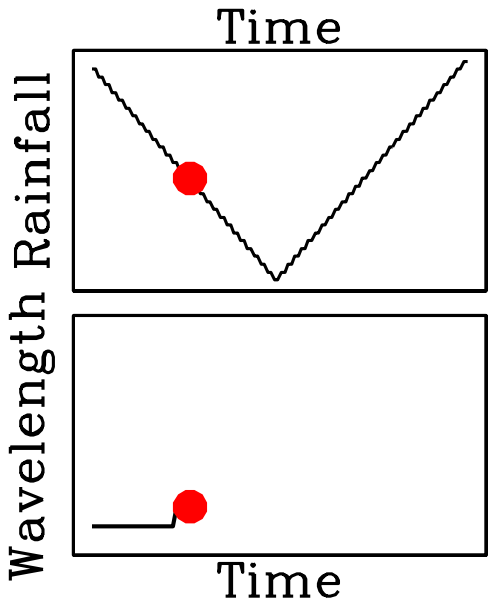


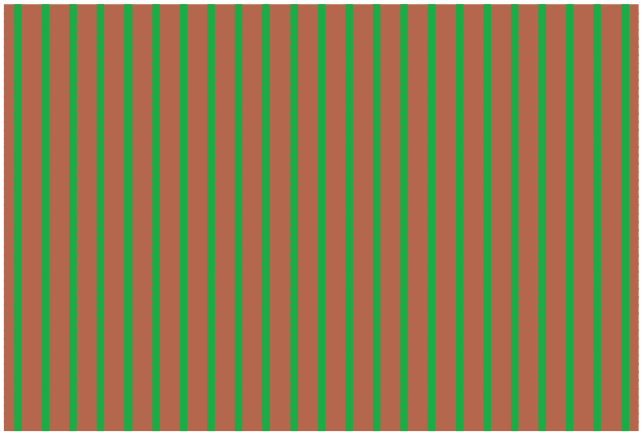

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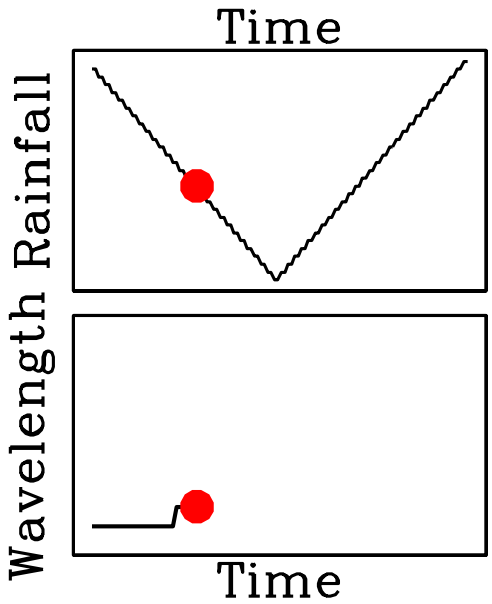


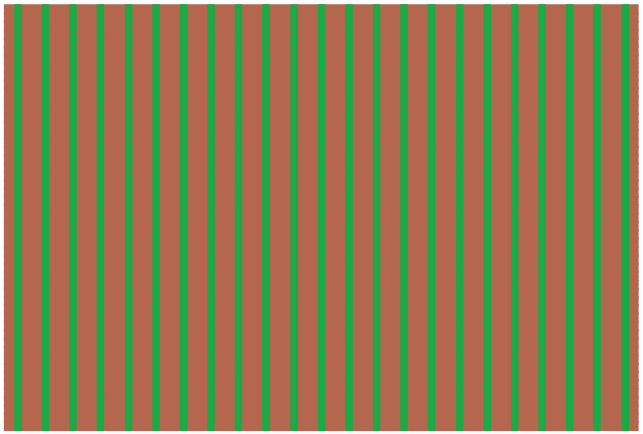

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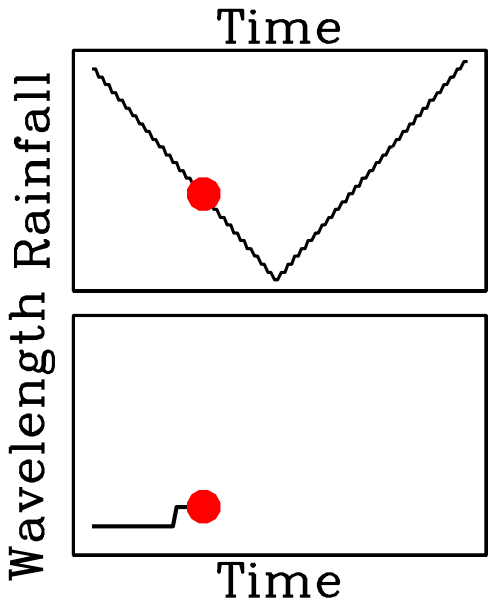


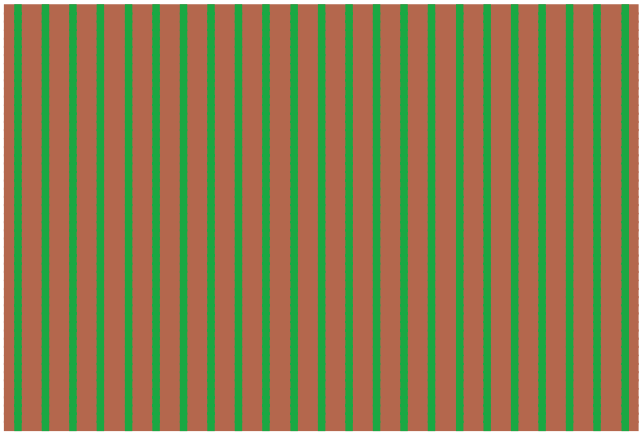

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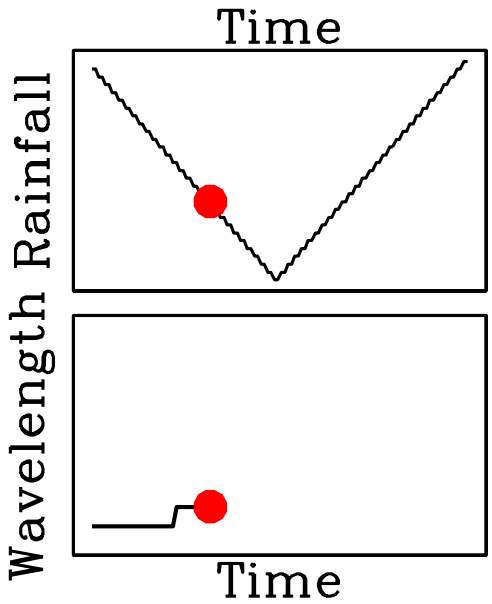


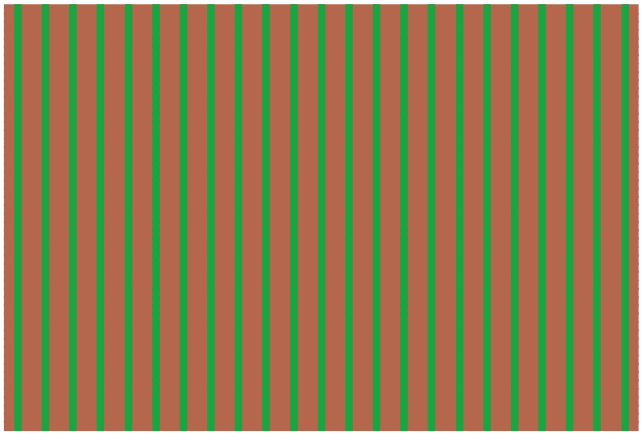

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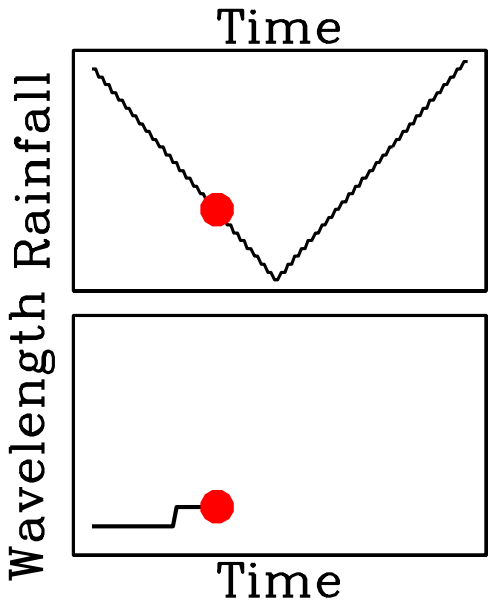


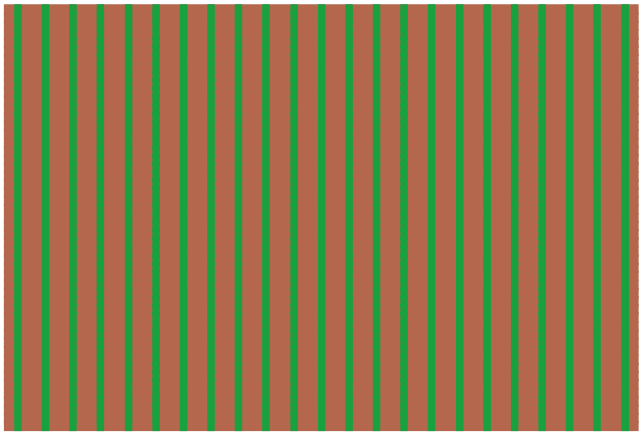

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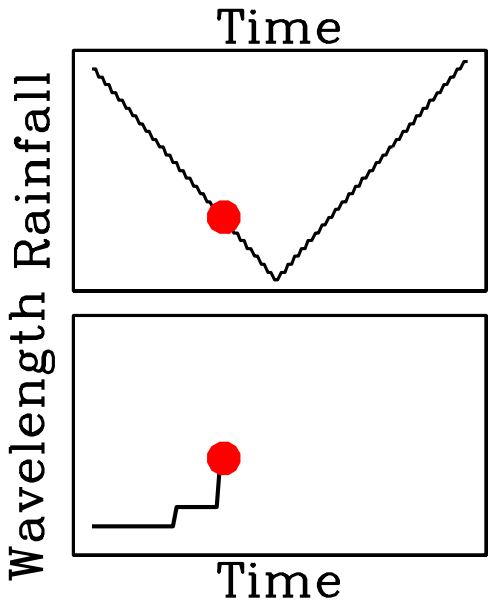


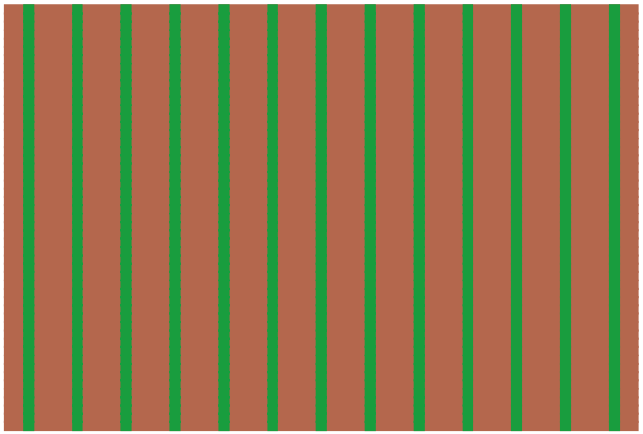

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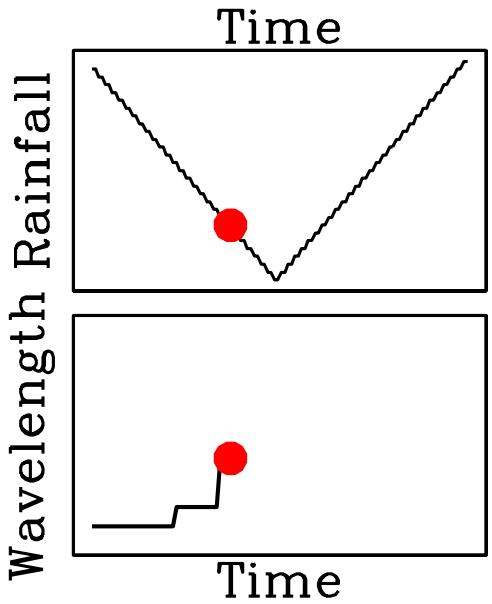


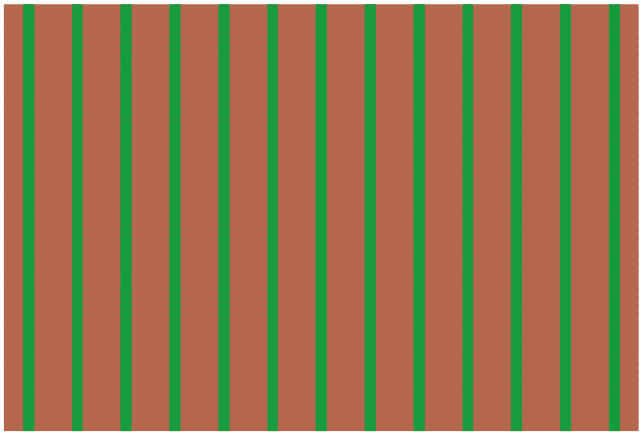

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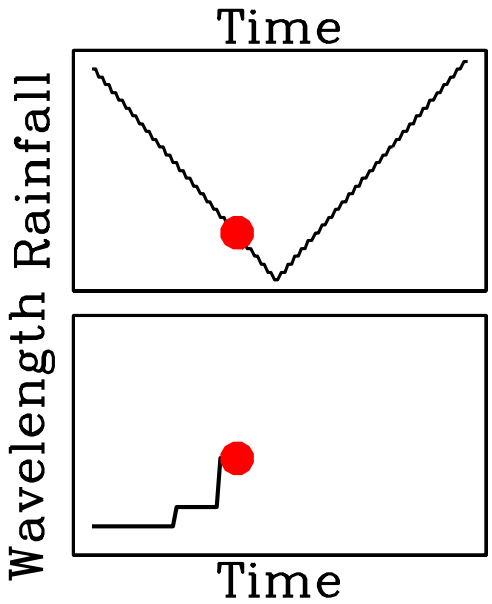


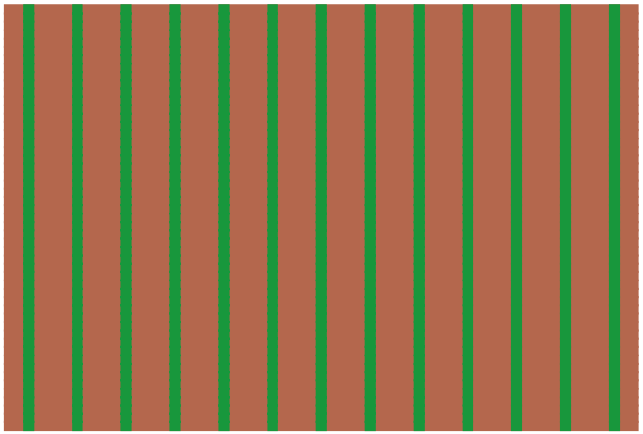

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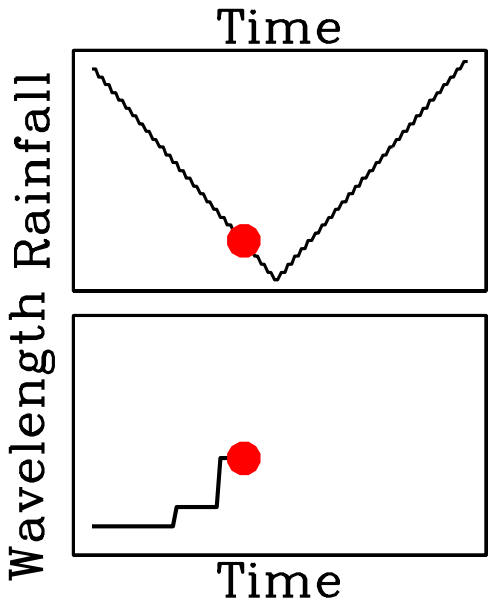


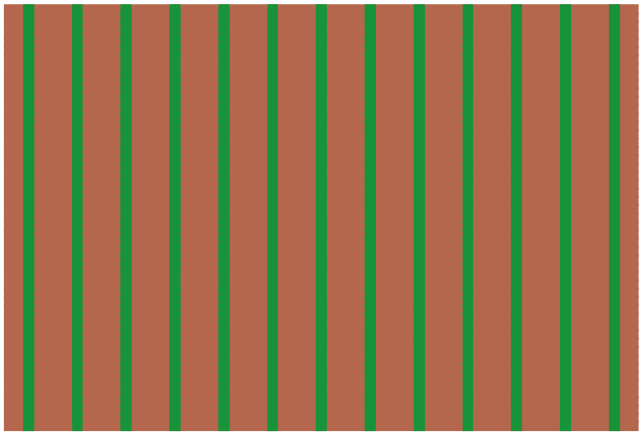

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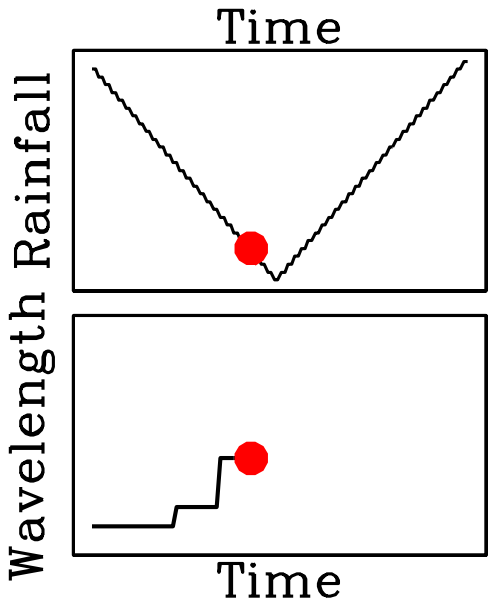


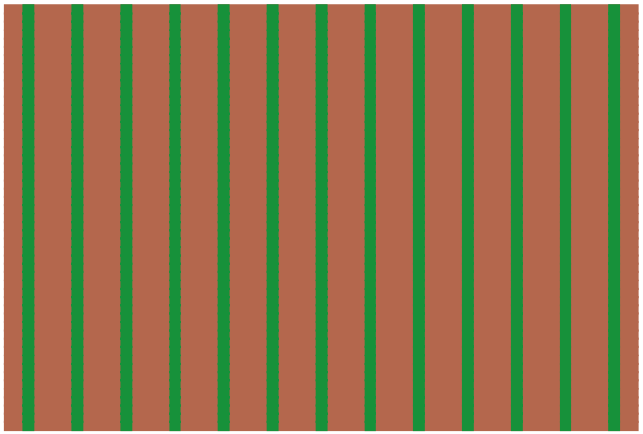

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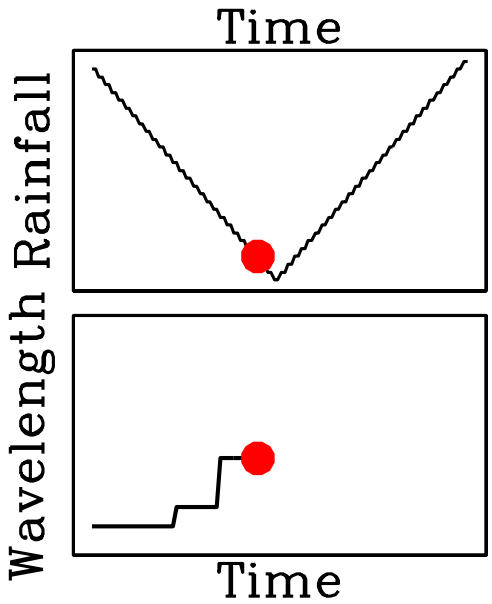


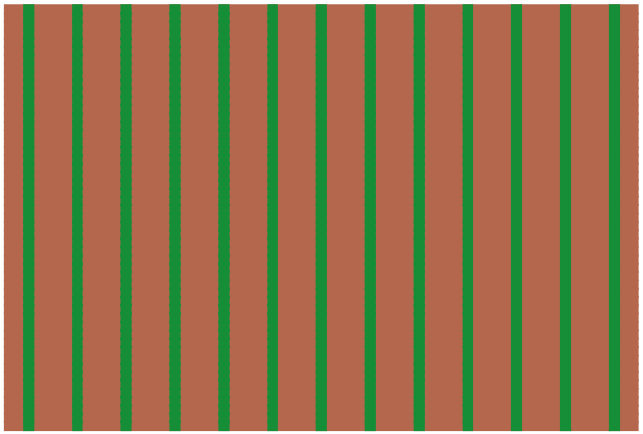

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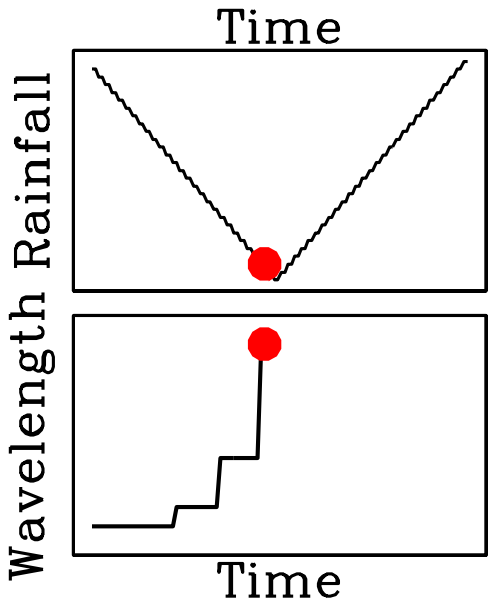


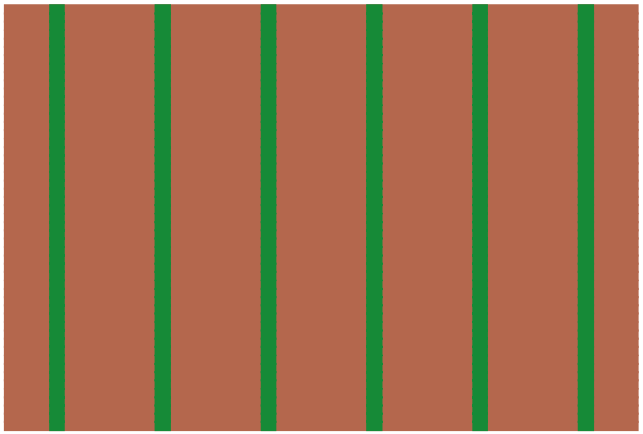

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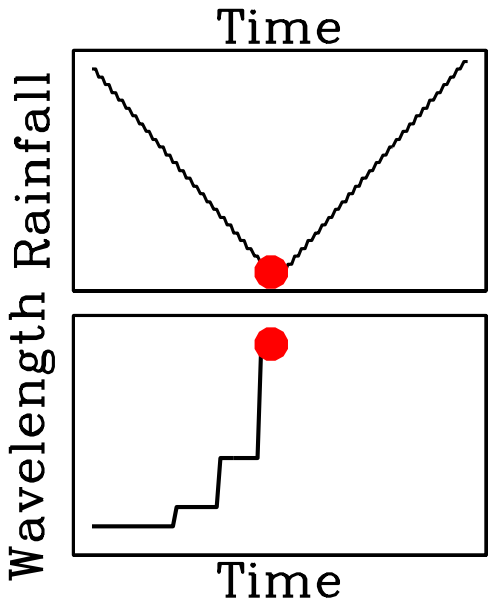


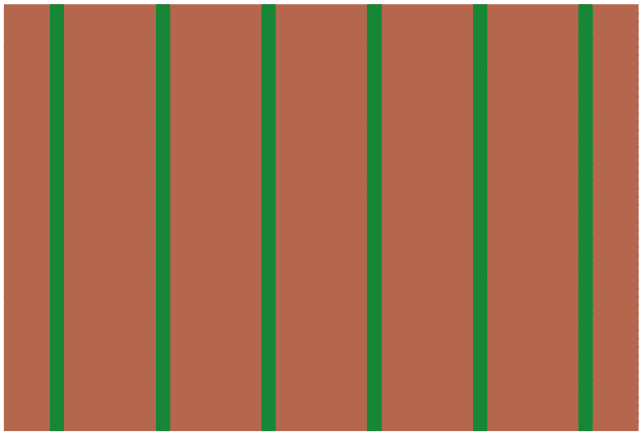

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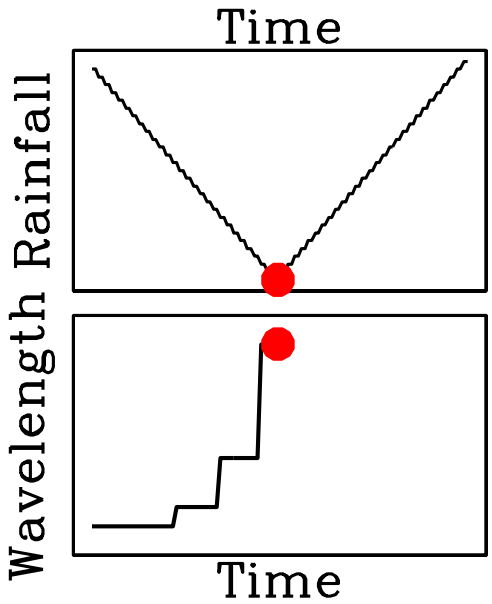


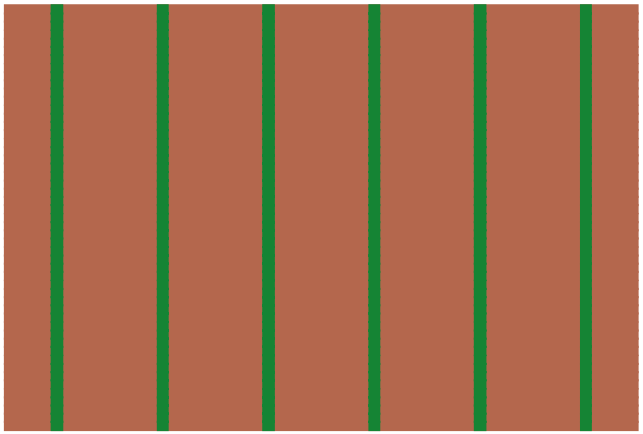

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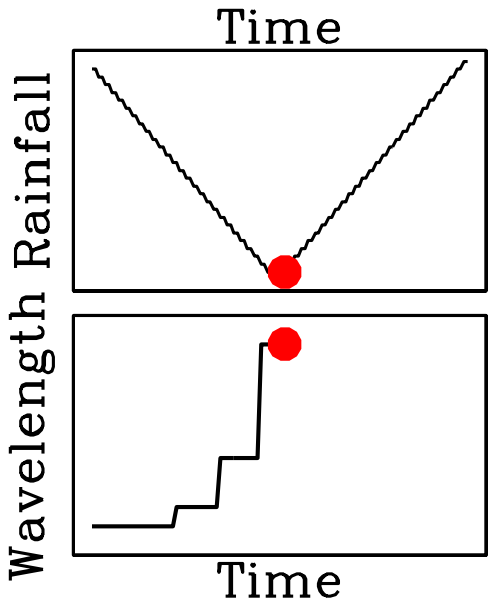


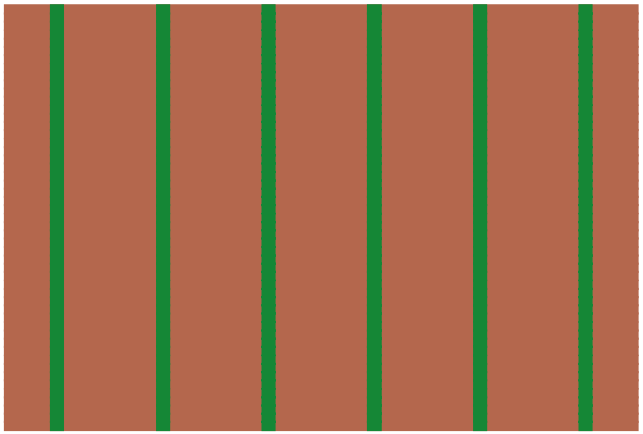

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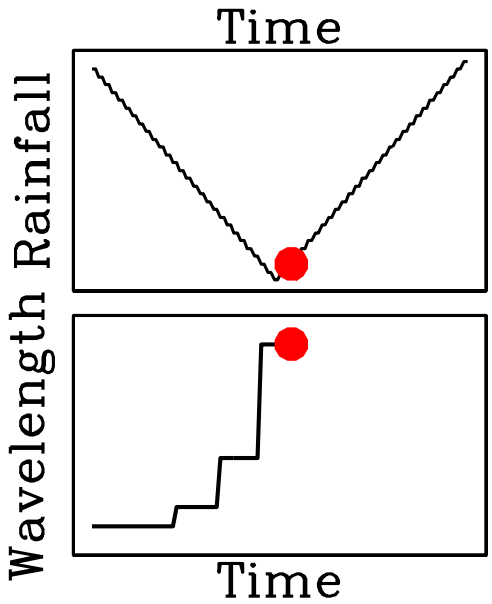


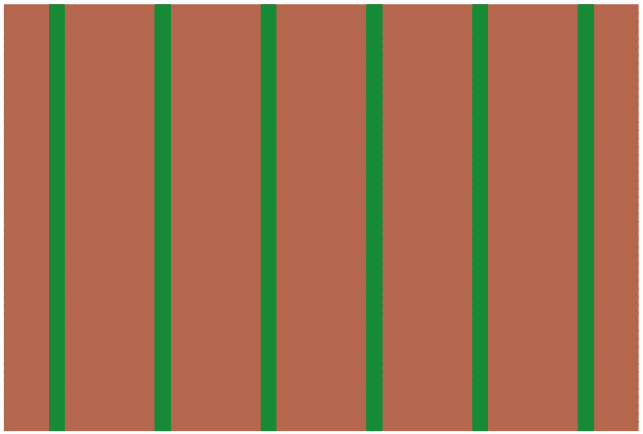

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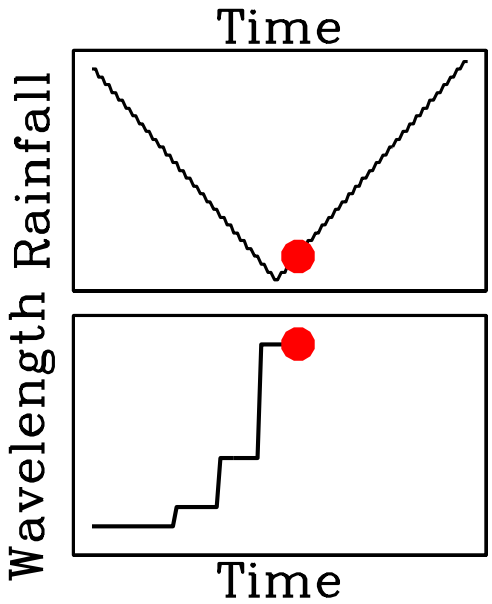


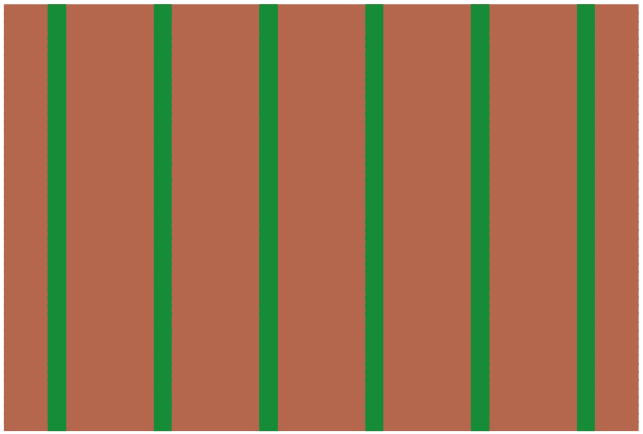

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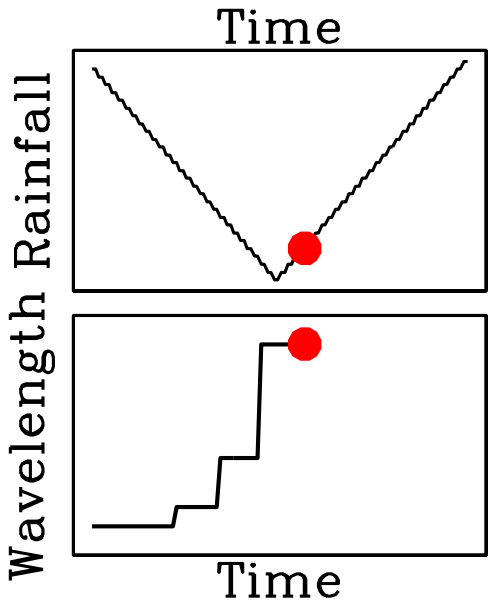


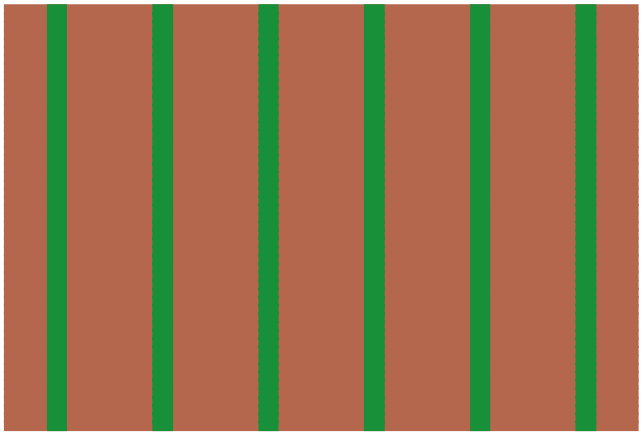

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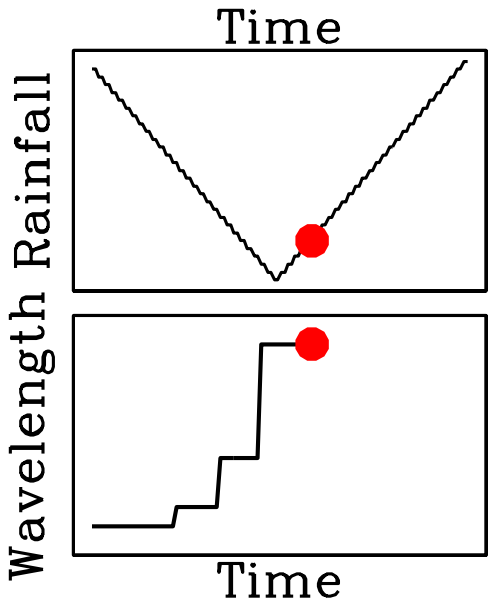


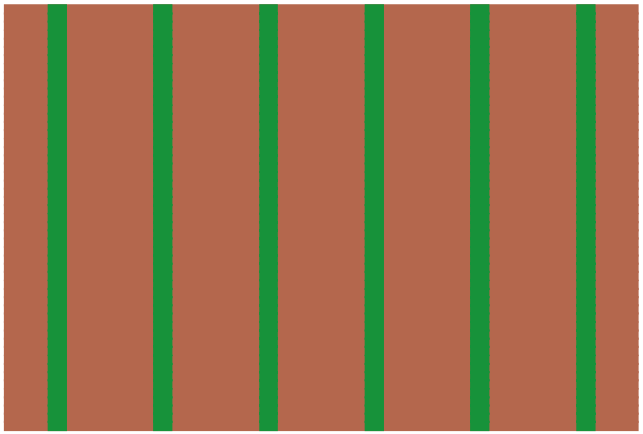

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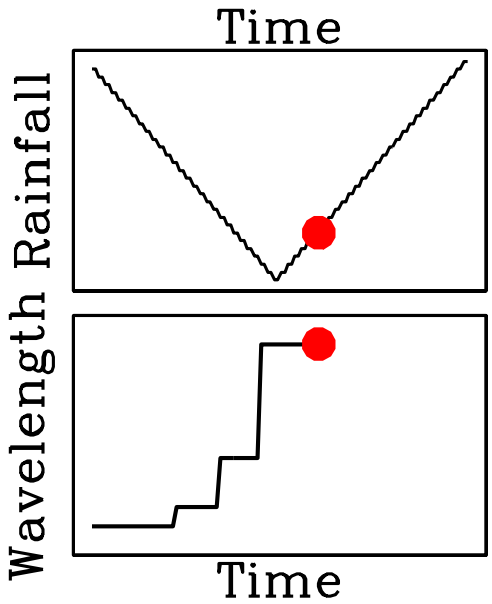


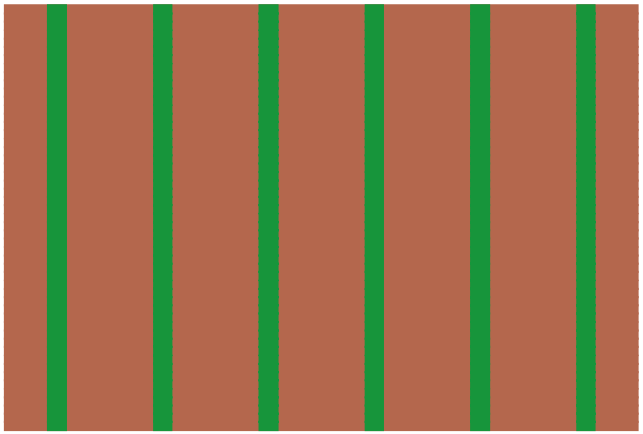

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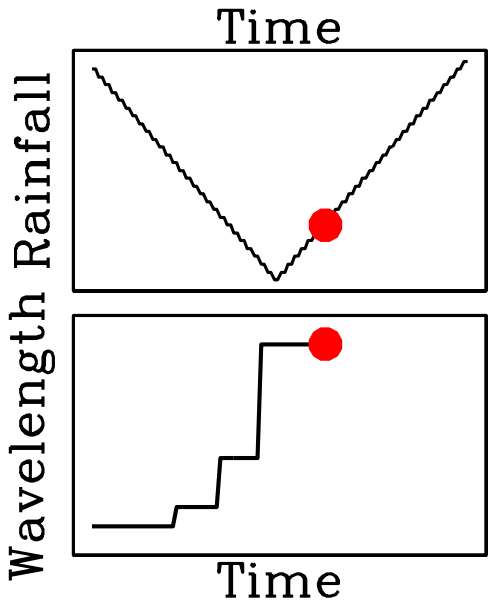


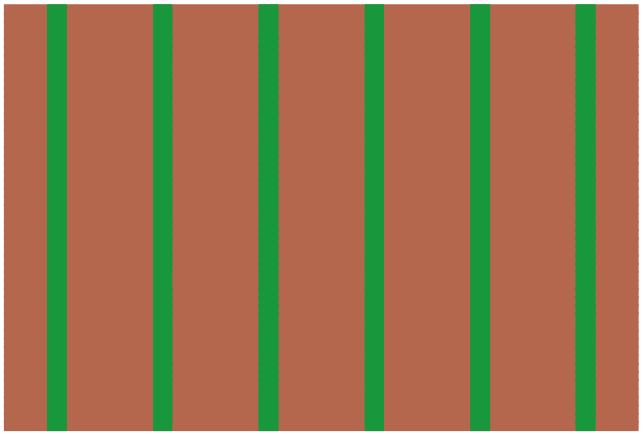

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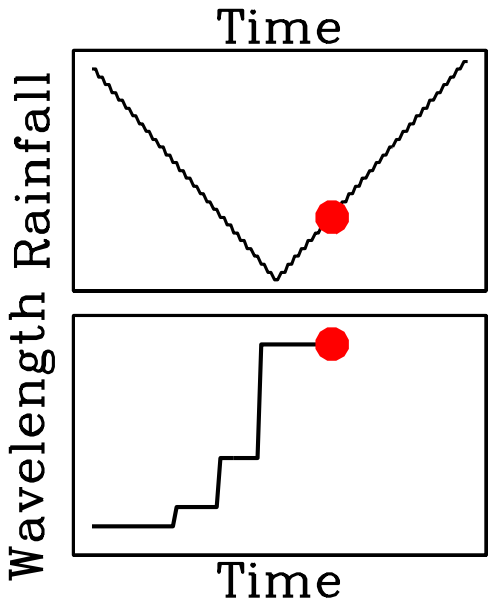


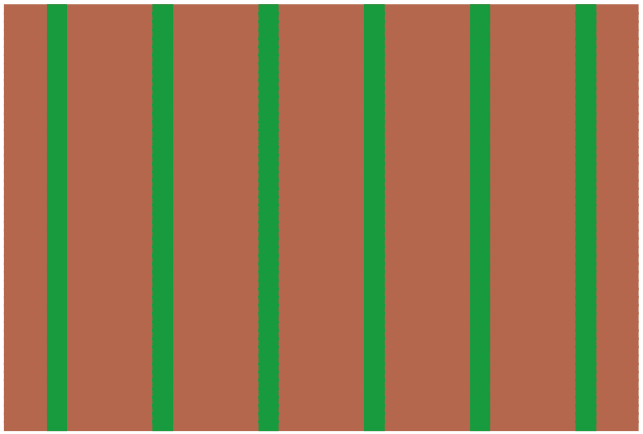

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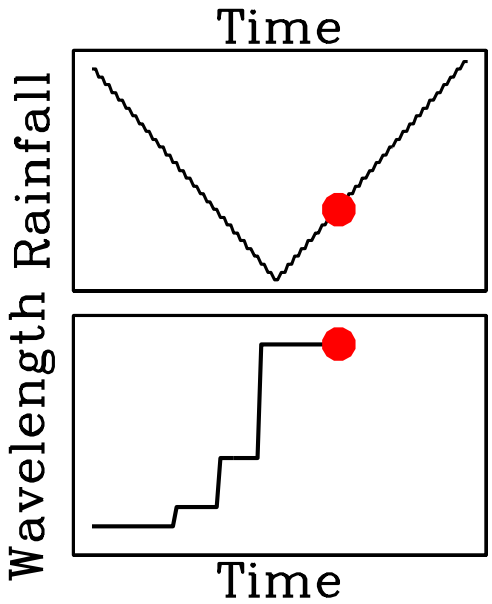


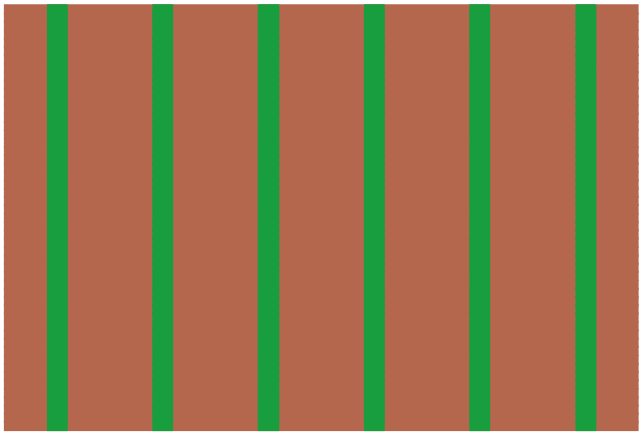

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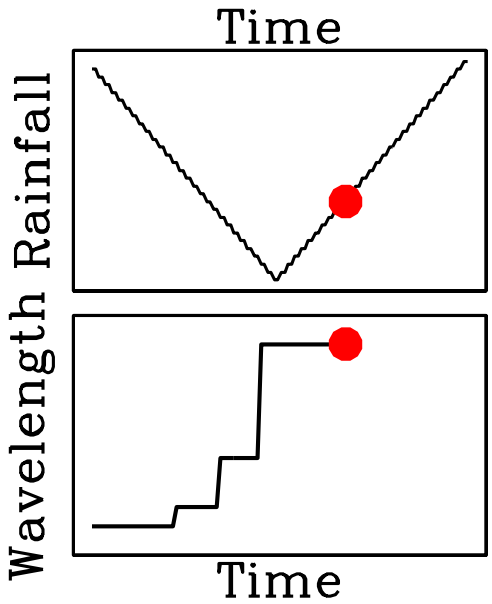


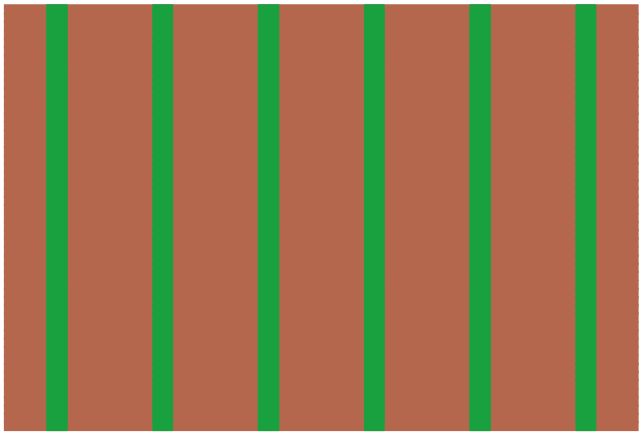

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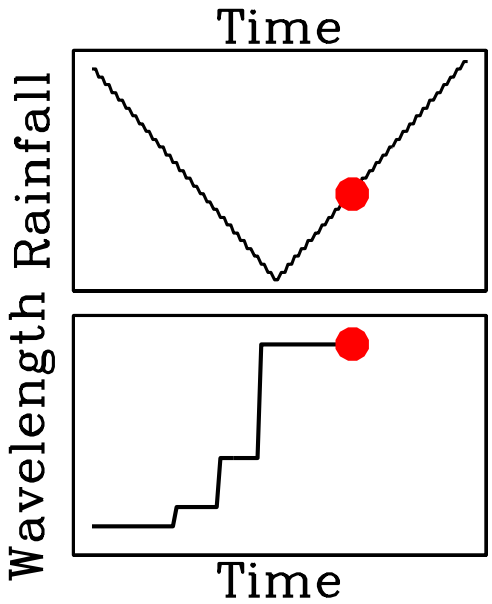


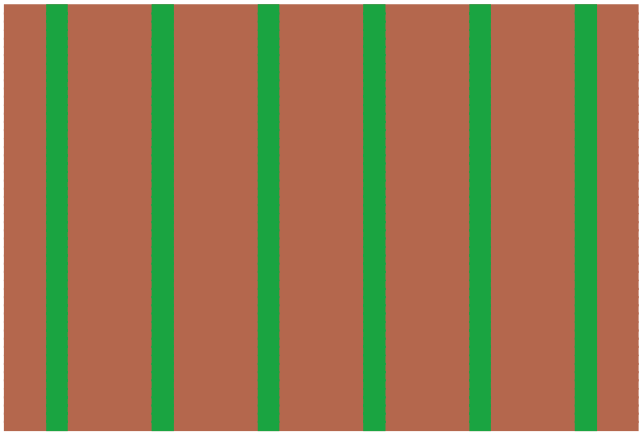

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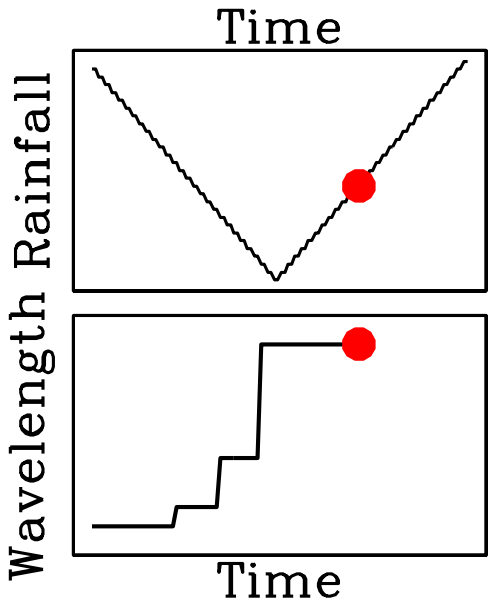


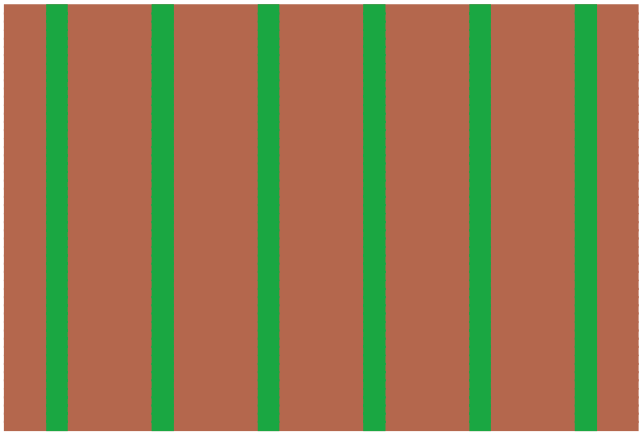

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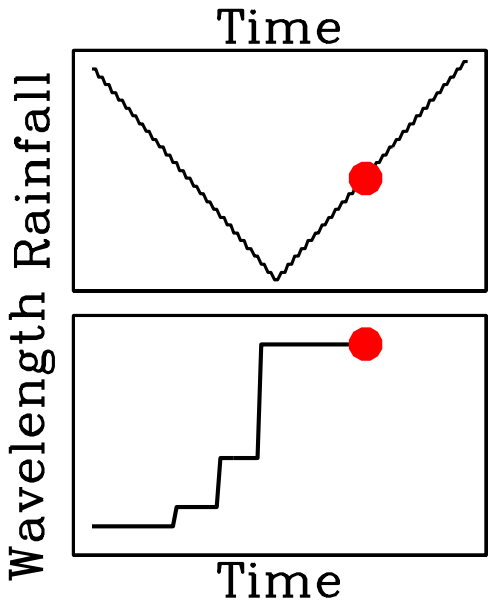


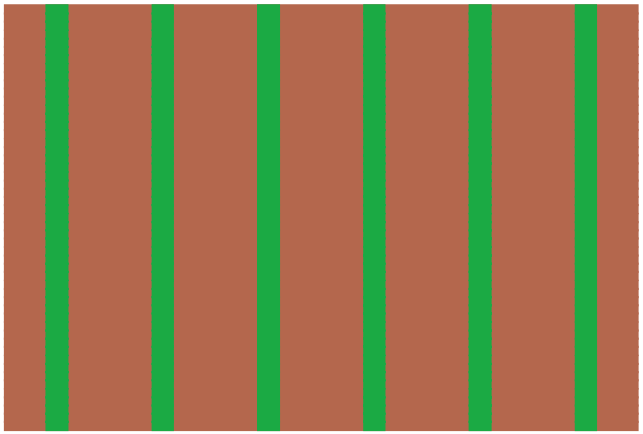

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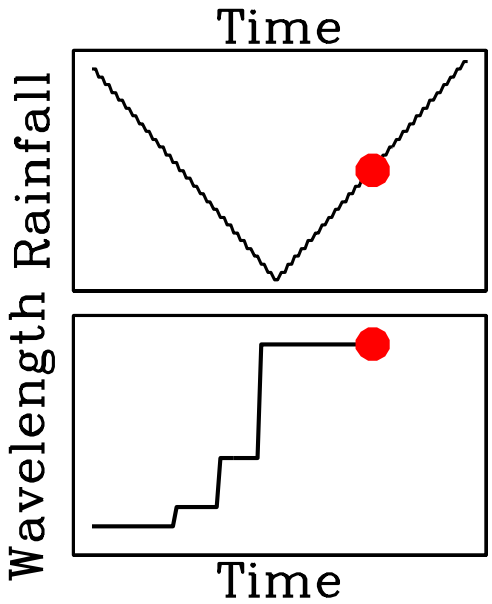


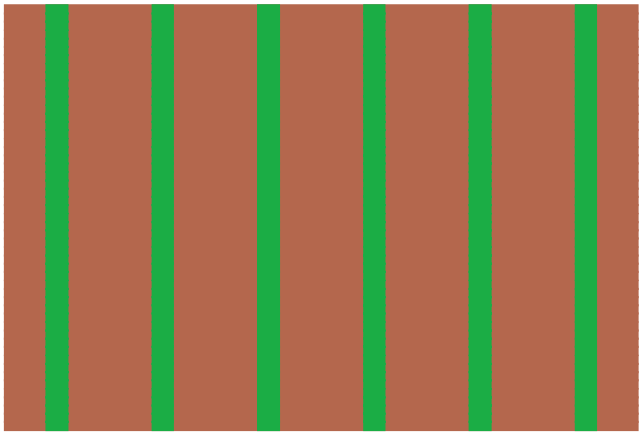

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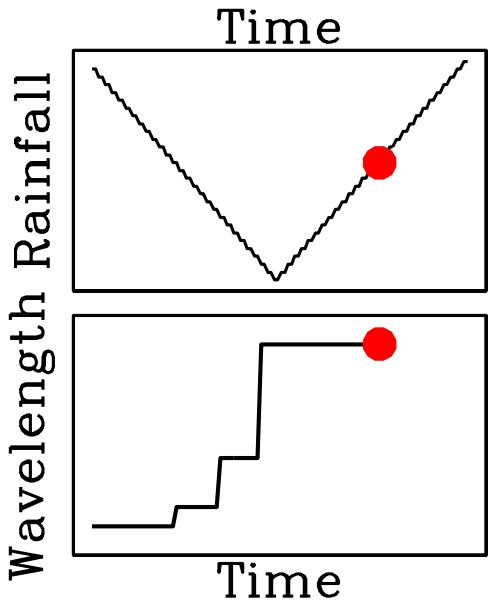


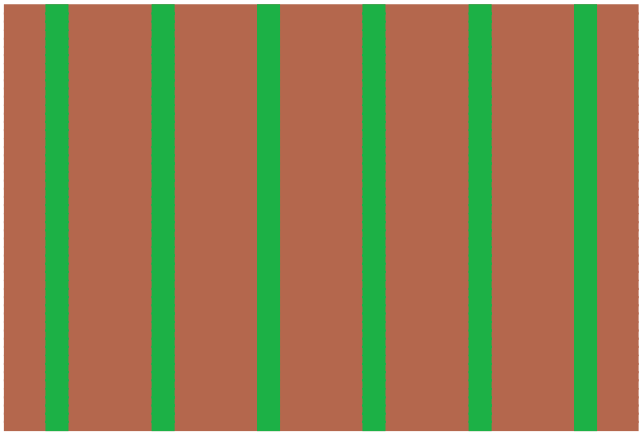

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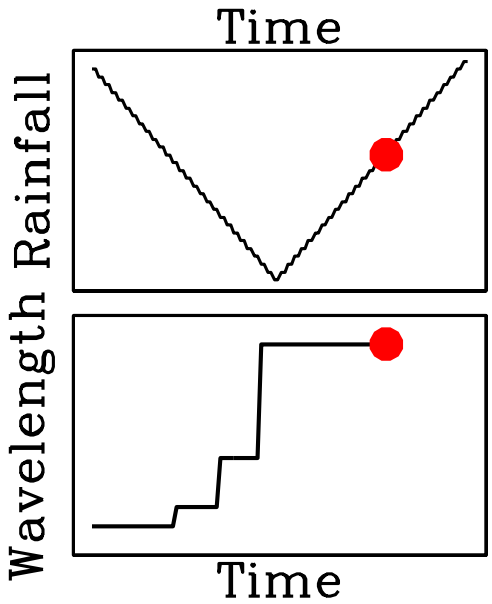


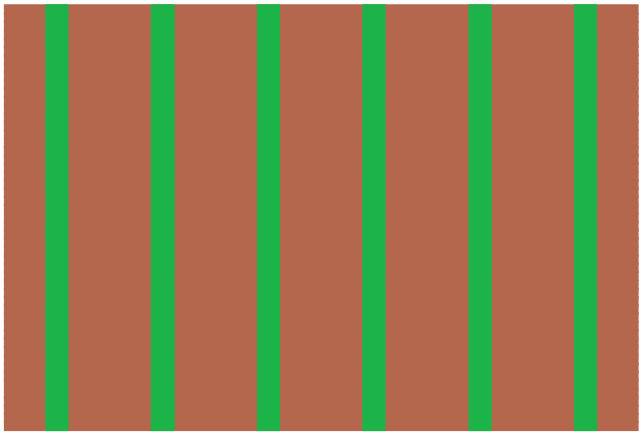

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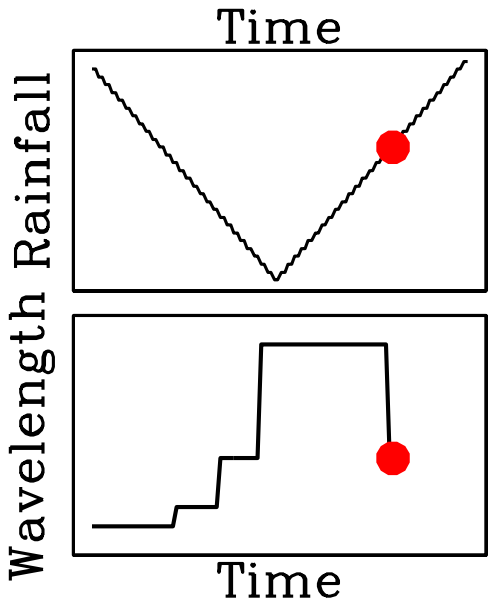


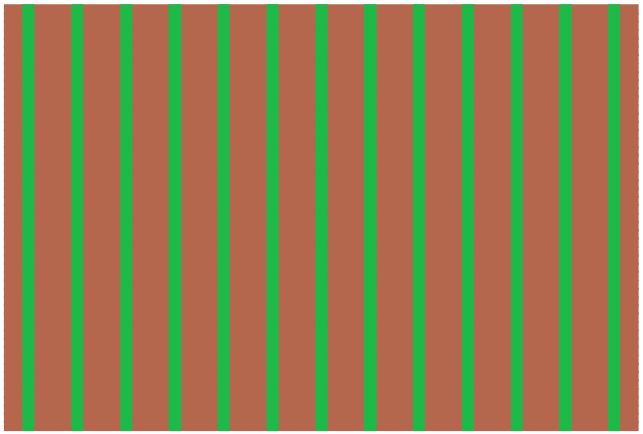

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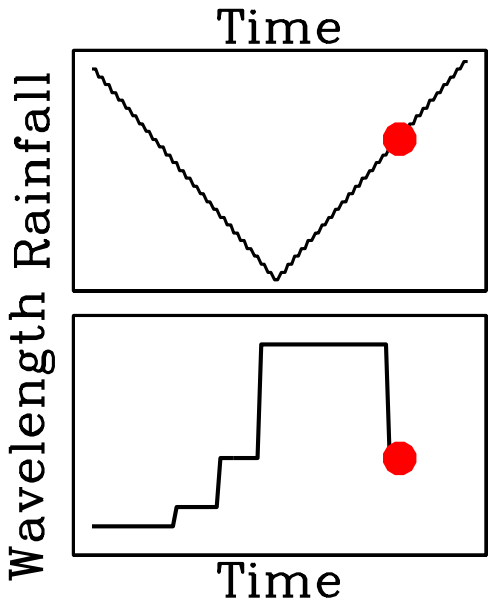


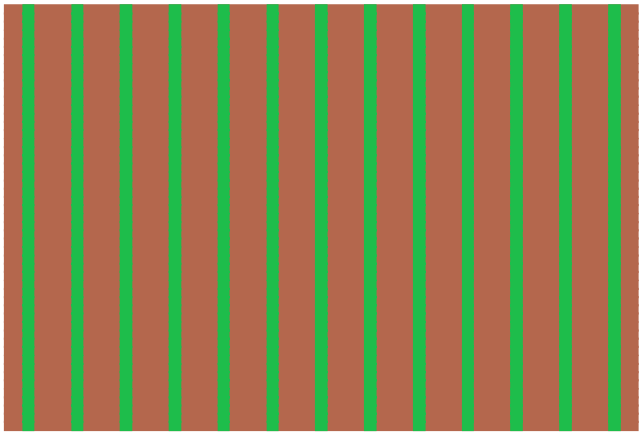

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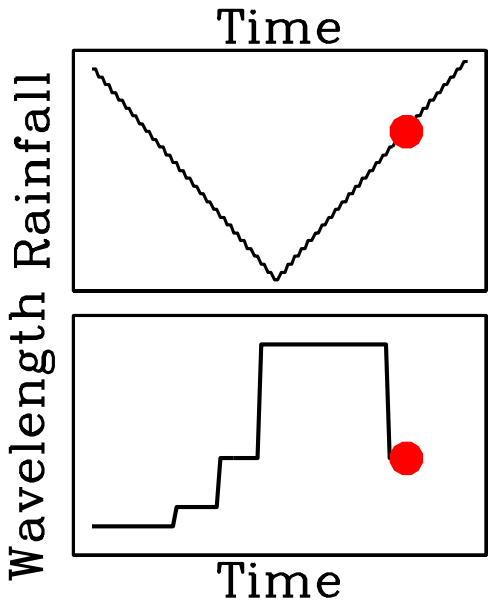


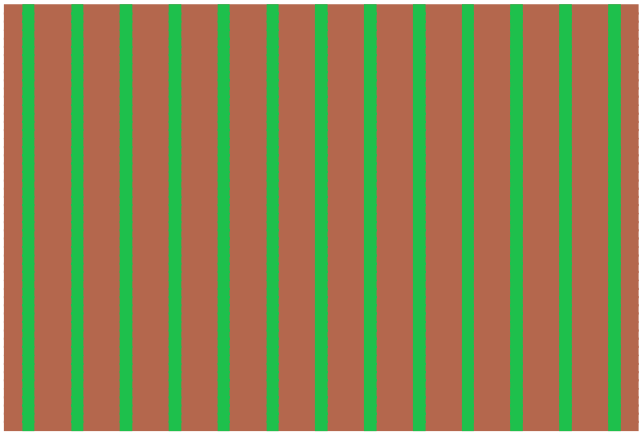

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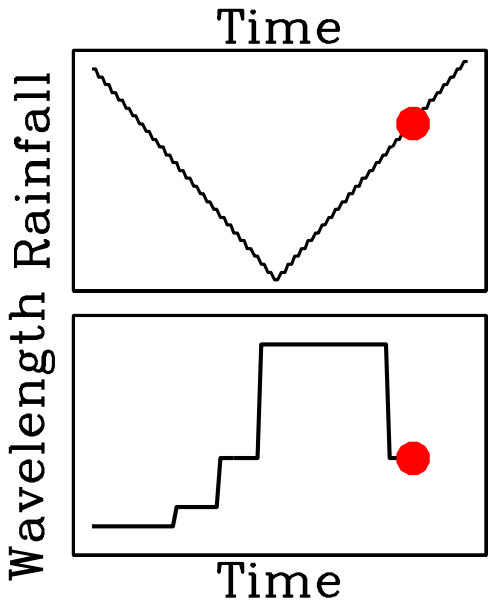


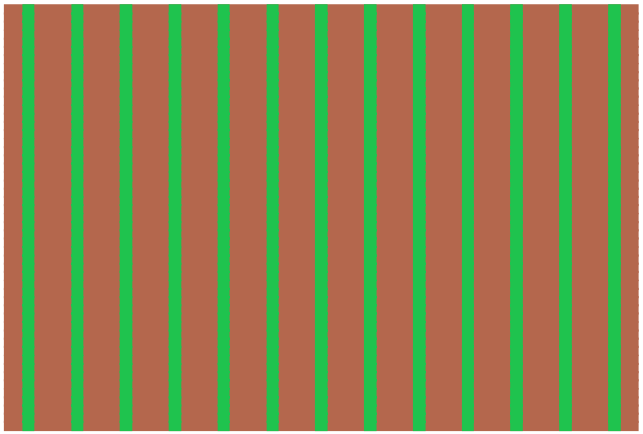

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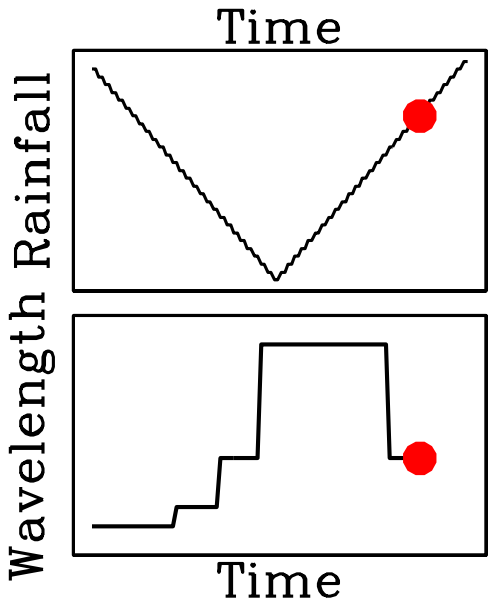


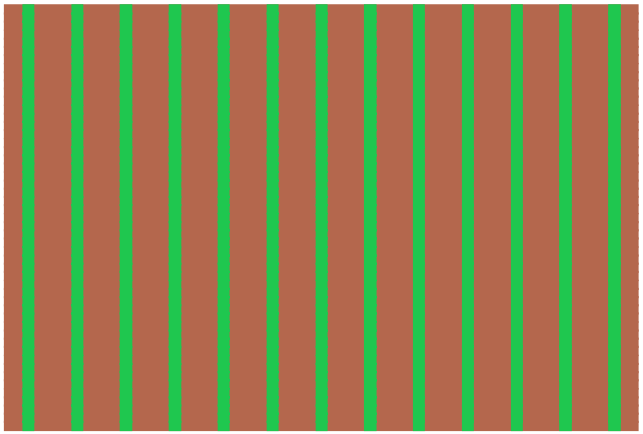

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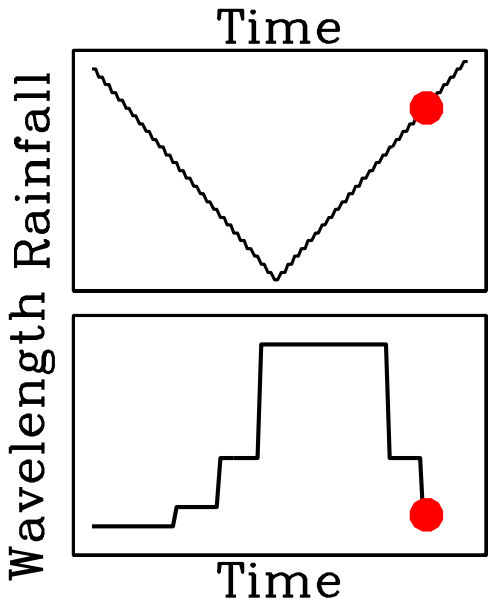


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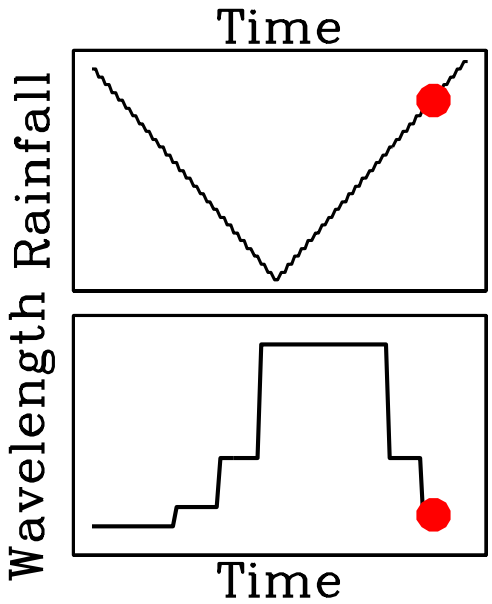


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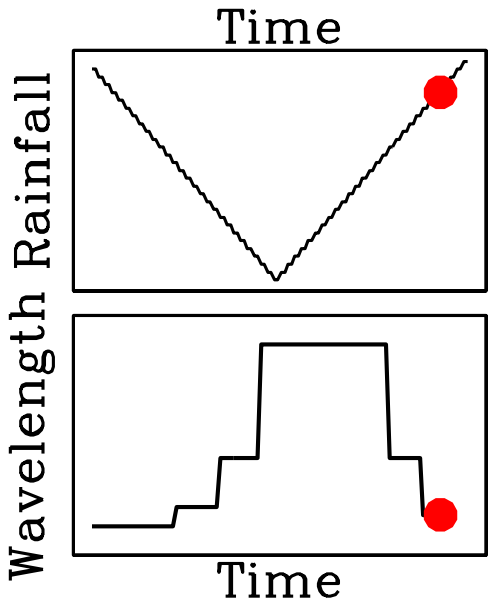


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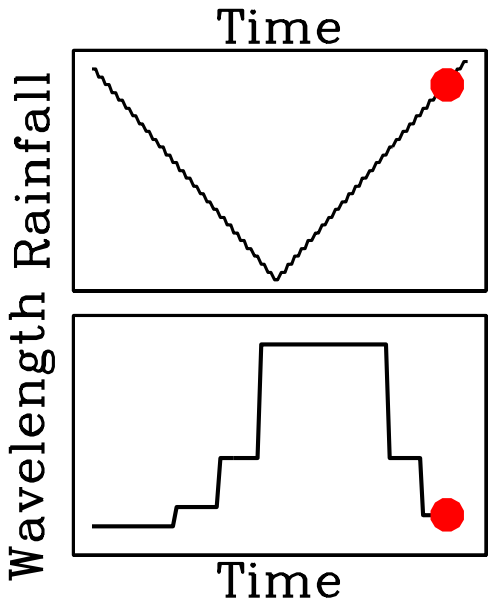


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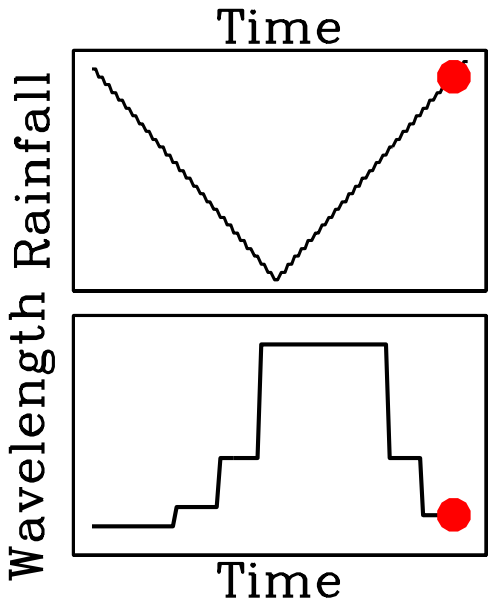


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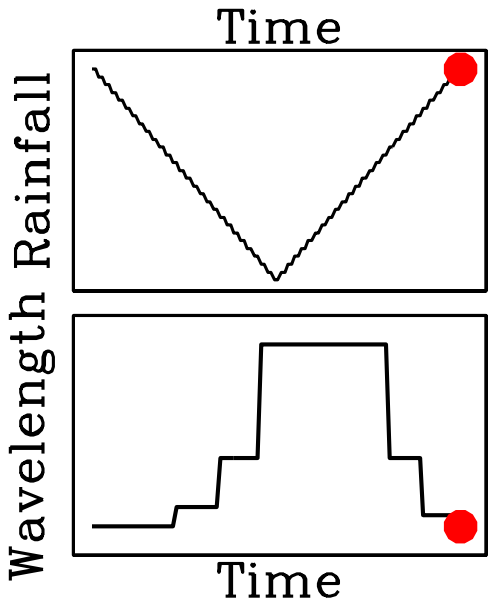


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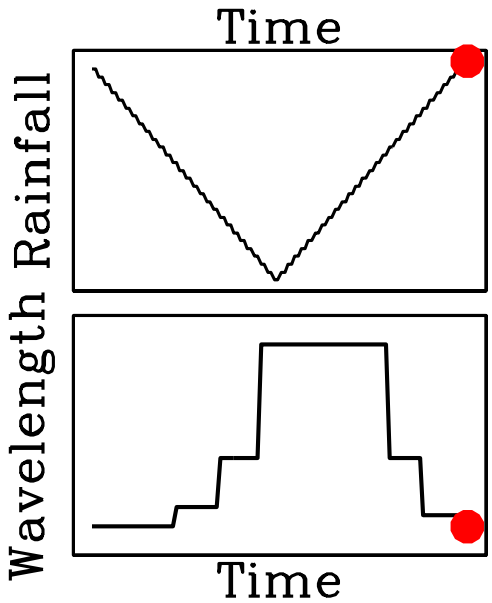


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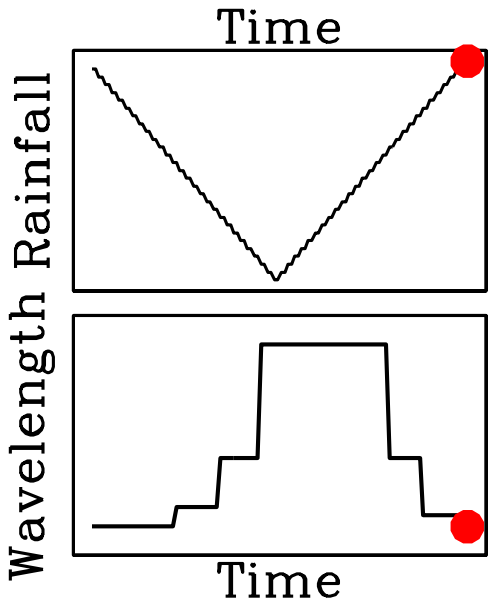
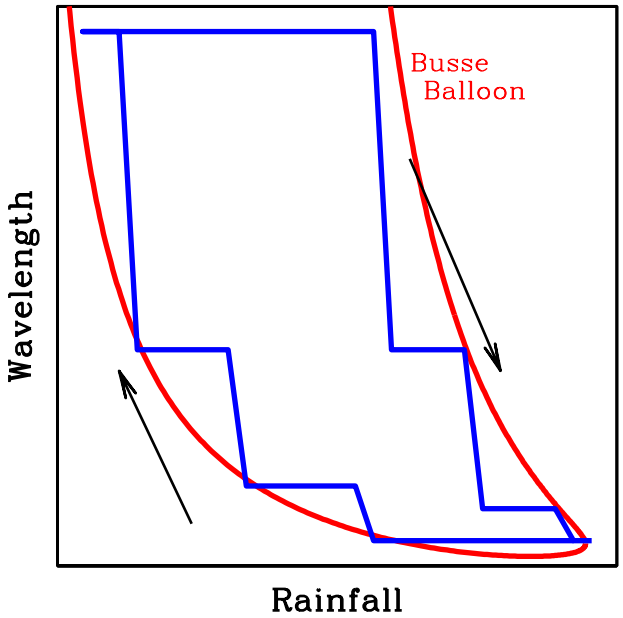



# Variations in Rainfall: Hysteresis

Wavelength changes abruptly at the edge of the Busse Balloon.



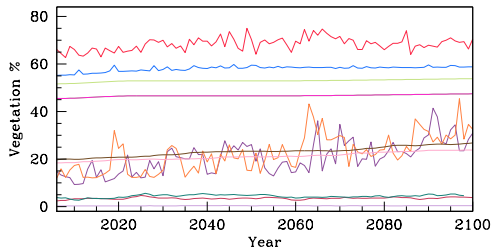

(work of JAS, Koen Siteur, Eric Siero, Arjen Doelman, Max Rietkerk)

# Outline

**Question:** How will vegetation levels in the Sahel region of Africa change over the remainder of the century?

# Global Climate Models

- Prediction of future climate is an active research area: 60 models in CMIP5, results of CMIP6 due in 2020
- Some of these models include "dynamic vegetation" (12/60 in CMIP5)
- But: spatial grid cells ($\sim$100km) are too large to deal effectively with patterned vegetation
- This is demonstrated by the huge variability in predictions of future vegetation levels in the Sahel.




- So: a different approach is needed

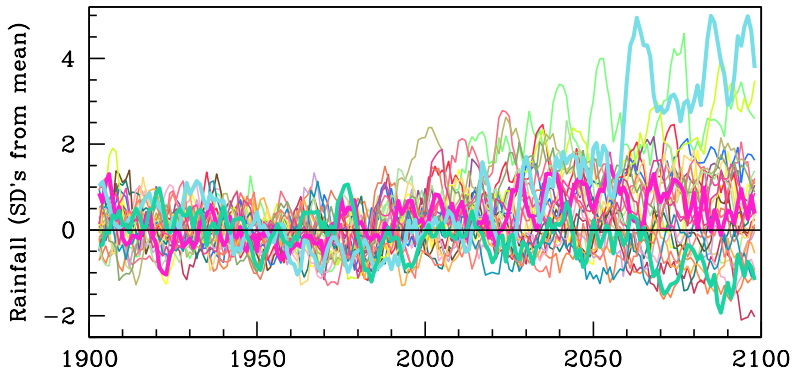
# Approaches to Predicting Future Vegetation

1. Improve the spatial resolution in global climate models: in progress but a resolution suitable for patterned vegetation lies well in the future.
2. Improve models for patterned vegetation, to include some climate data or feedbacks
   (e.g. work of Mara Baudena & Max Rietkerk)

**My approach:** use predictions of future rainfall from global climate models (CMIP5) as a forcing term in a simple model for semi-arid vegetation.

# Rainfall Predictions for the Sahel

- Predictions of future rainfall for the Sahel are highly variable.

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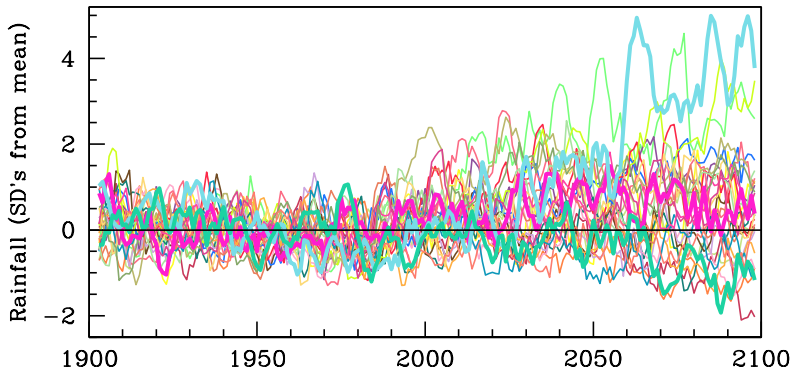
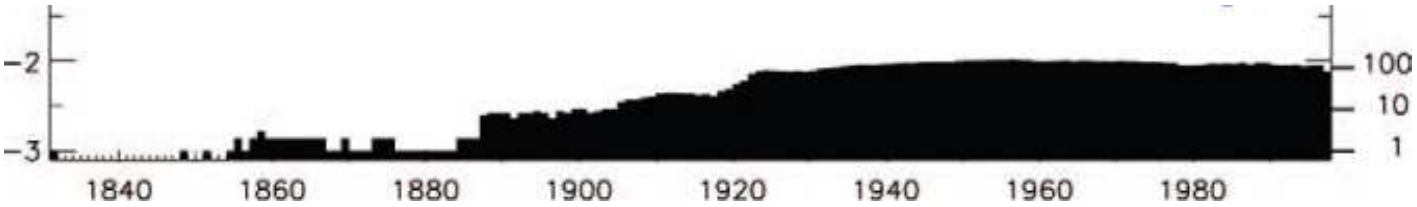



- In view of this, is it possible to make meaningful predictions of future vegetation?
- Another complication: the history-dependence of vegetation patterns means that historical data is needed to predict future behaviour.

# Rainfall History in the Sahel

- A very severe drought occurred c. 1738-1756: a reasonable starting point for simulations is zero vegetation in 1750.

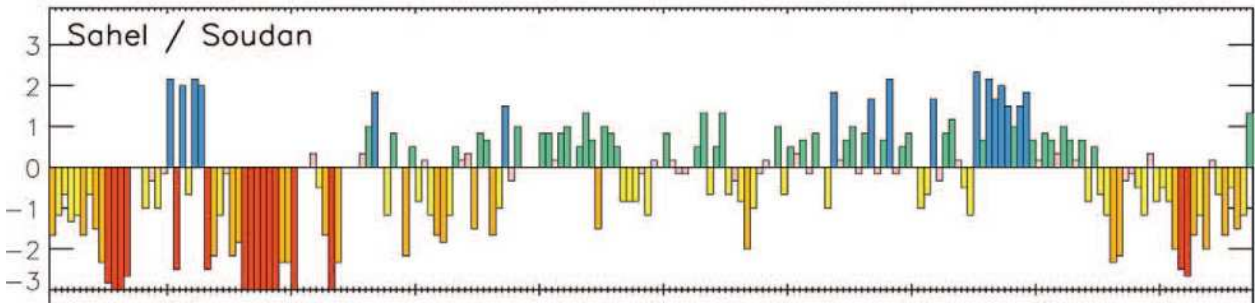



- There is very limited direct data on rainfall before 1900.
- Proxy data: (i) lake levels, esp. Lake Chad; (ii) historical chronologies, e.g. Bornu Empire; (iii) memories of local peoples.

# Historical Rainfall Data Set

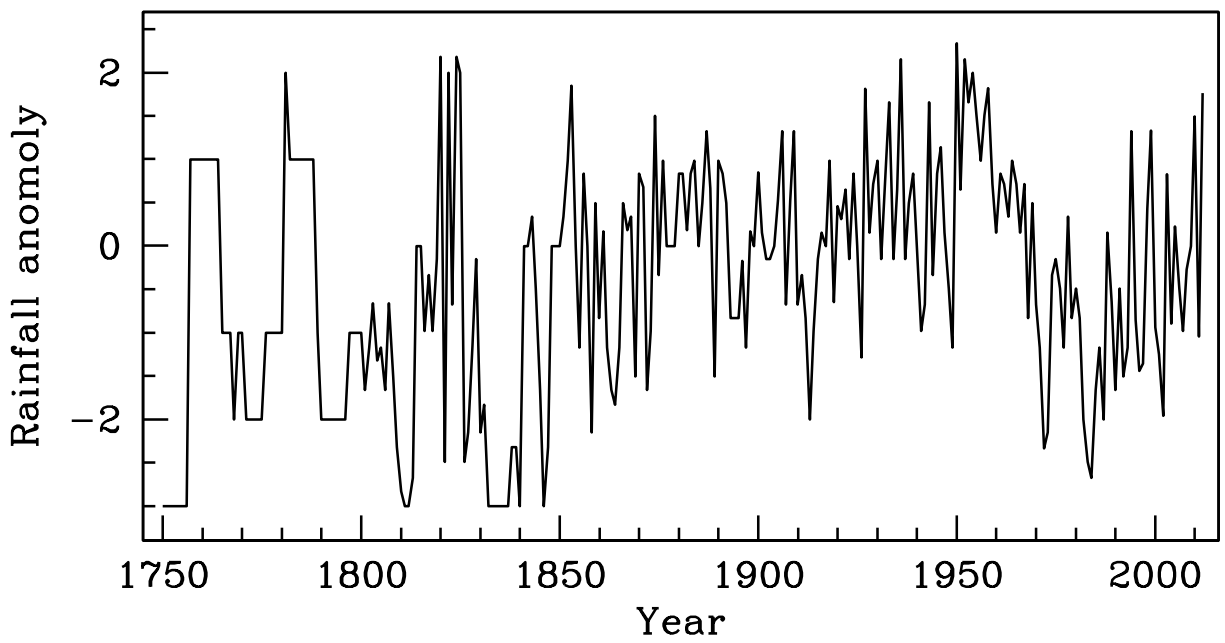
I base my historical data set on work by Sharon Nicholson (Florida State U) on rainfall history in the Sahel.

Sahel "wetness index" 1800-2000




- Extension back to 1750 is based on historical work of Stefan Norrgård (Turku)
- Extension forwards to present is based on recent rain gauge data
- I use linear correlation of data for overlapping years to combine the data sets

# Historical Rainfall Data Set



# Outline

# Simulation Approach

$$\partial u/\partial t = \overbrace{wu^2}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\partial^2 u/\partial x^2}^{\text{plant dispersal}}$$

$$\partial w/\partial t = \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{D\,\partial^2 w/\partial x^2}_{\text{diffusion of water}}$$

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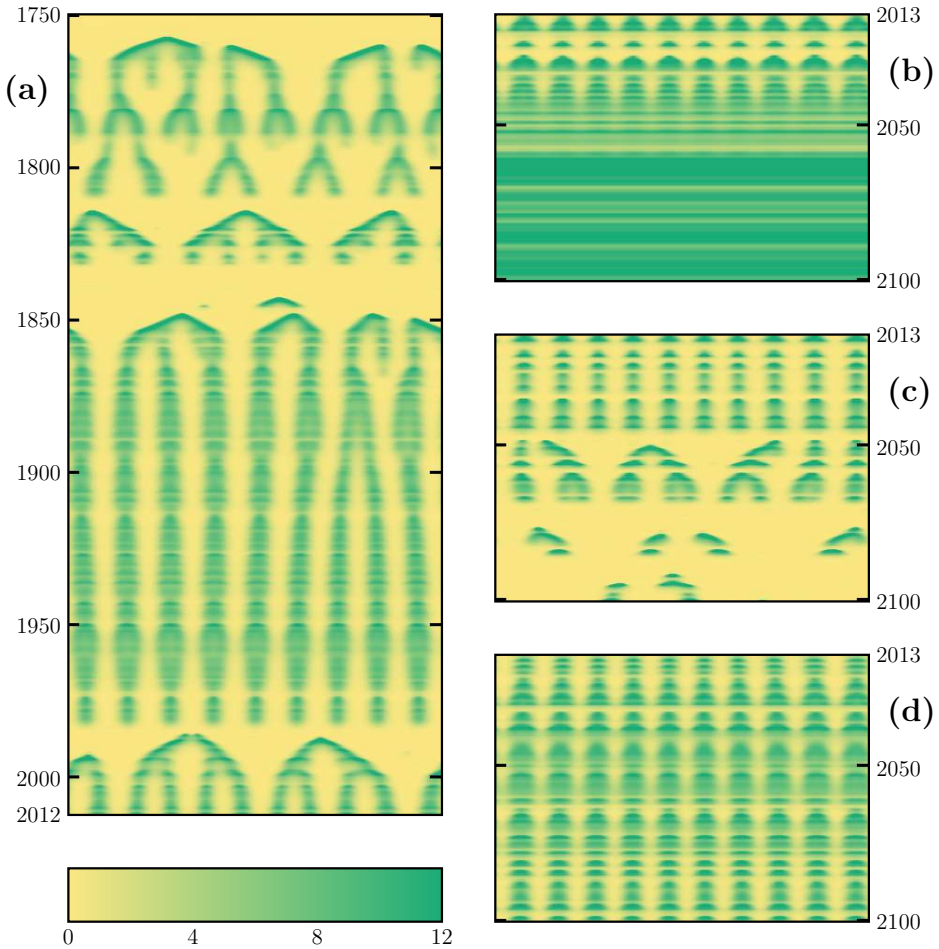
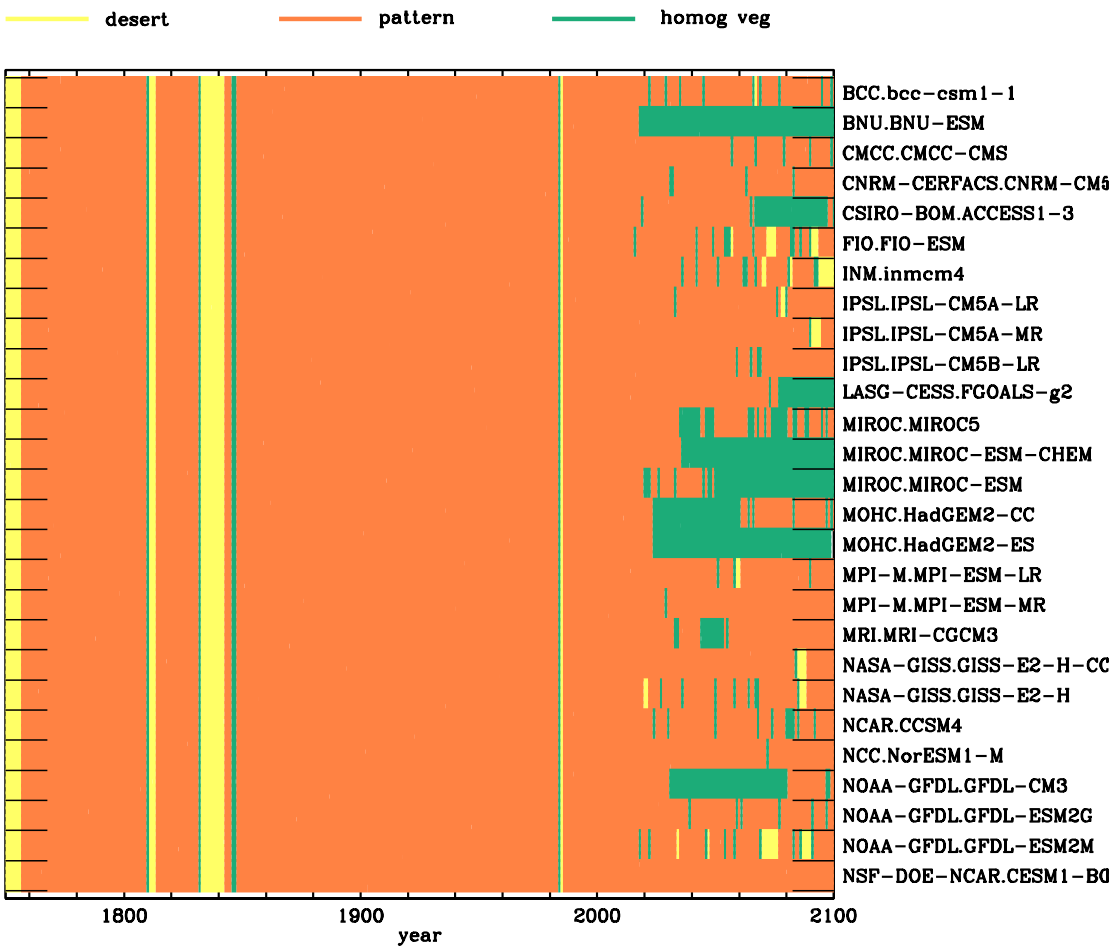
- I run simulations from 1750-2100
- $A$ varies over time to reflect the historical rainfall data set, and predictions of future rainfall levels (CMIP5)
- I vary parameter values and include various levels of noise; all runs done for 27 CMIP5 datasets
  – a total of 46 000 simulations

# Example Simulations



# Classification of Vegetation



# Predictions on Desertification

Percentage of years with
(almost) no vegetation
Historical (1750-2012): 10%
Future (2013-2100): 3.5%

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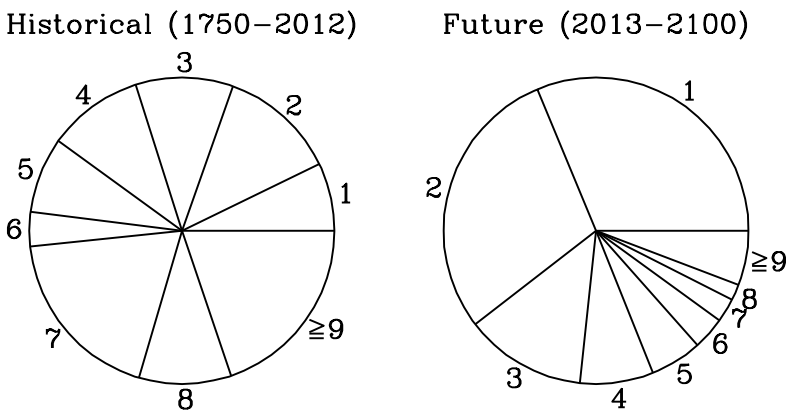

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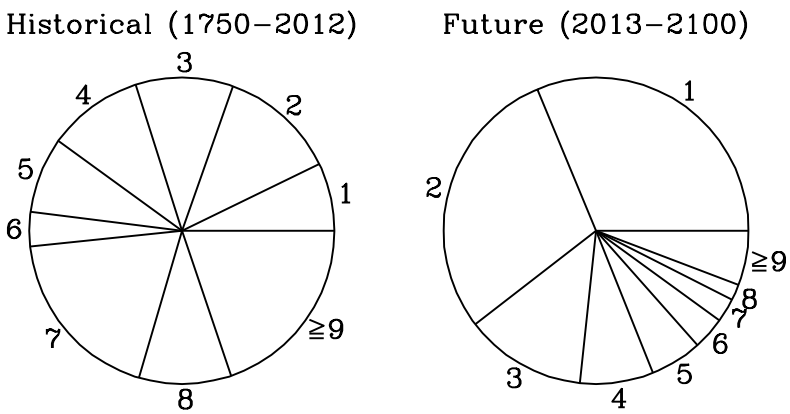


Conclusion: the vast majority of simulations imply relatively high vegetation levels throughout the 21st century, with much lower levels of desertification than for the previous 2.5 centuries.

# Outline

# Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.

Bushy vegetation in Niger

Mitchell grass in Australia

(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, Mexico and S-W USA.

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Bushy vegetation in Niger

Mitchell grass in Australia

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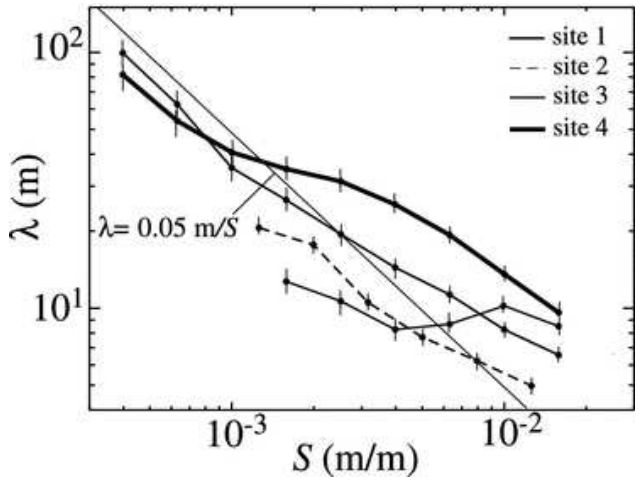
Wavelength can be measured via remote sensing.

# Data on Wavelength vs Slope

I will show that the relationship between pattern wavelength and slope provides valuable historical insights.




Data from Nevada, USA (Pelletier et al, J. Geophys. Res. 117: F04026, 2012)

# The Origin of Vegetation Patterns

Vegetation patterns develop via

either degradation of uniform vegetation

or colonisation of bare ground

# Mathematical Model on a Slope

$$\partial u/\partial t = \overbrace{wu^2}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\partial^2 u/\partial x^2}^{\text{plant dispersal}}$$

$$\partial w/\partial t = \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{\nu\,\partial w/\partial x}_{\text{flow downhill}} + \underbrace{D\,\partial^2 w/\partial x^2}_{\text{diffusion of water}}$$

# How to Predict Pattern Wavelength

Pattern wavelength

is history-dependent

$\Downarrow$

We must focus on the

onset of patterning

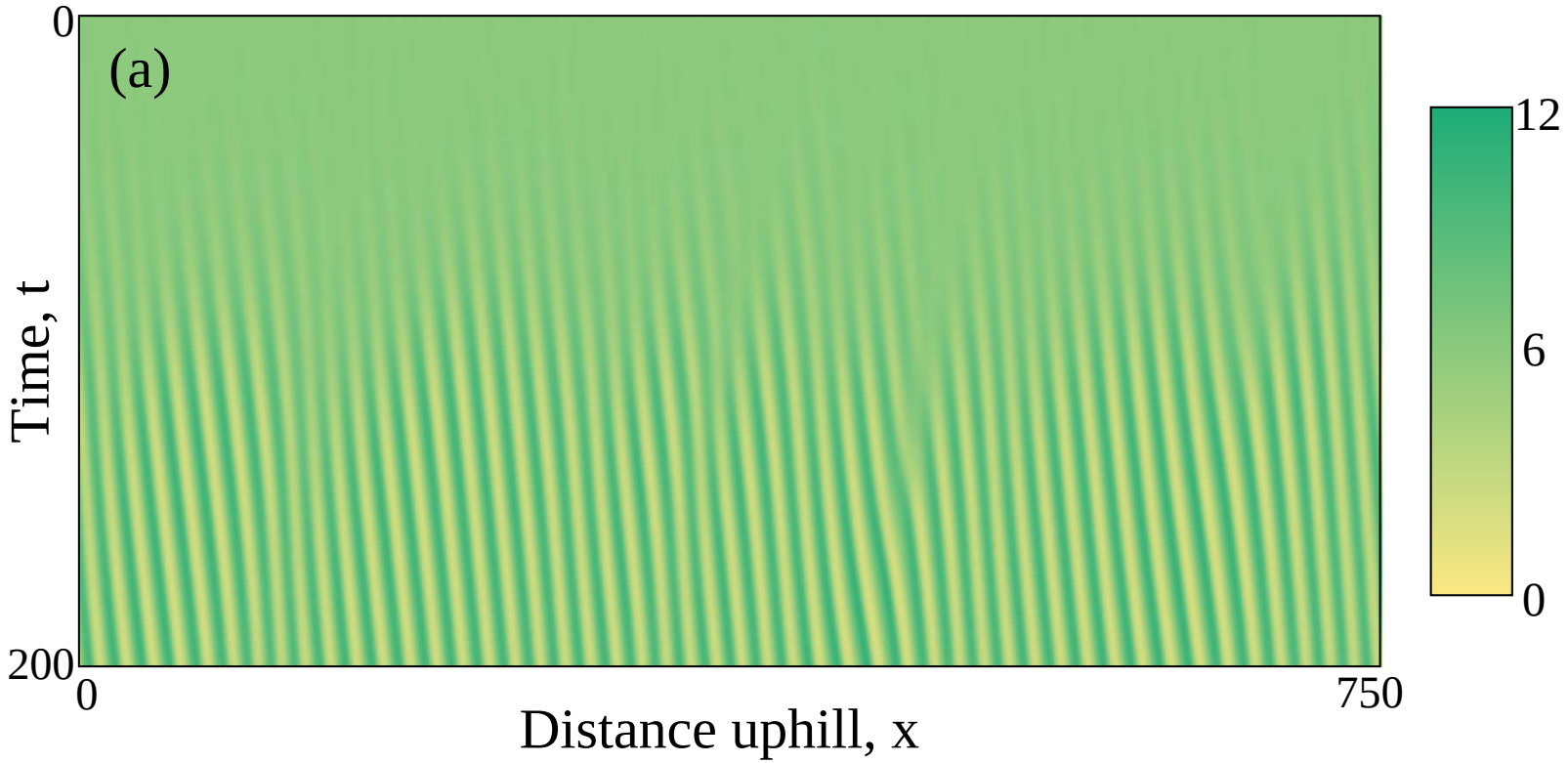
Degradation of

uniform vegetation

Colonisation

of bare ground

# Wavelength vs Slope for Degradation of Uniform Vegetation



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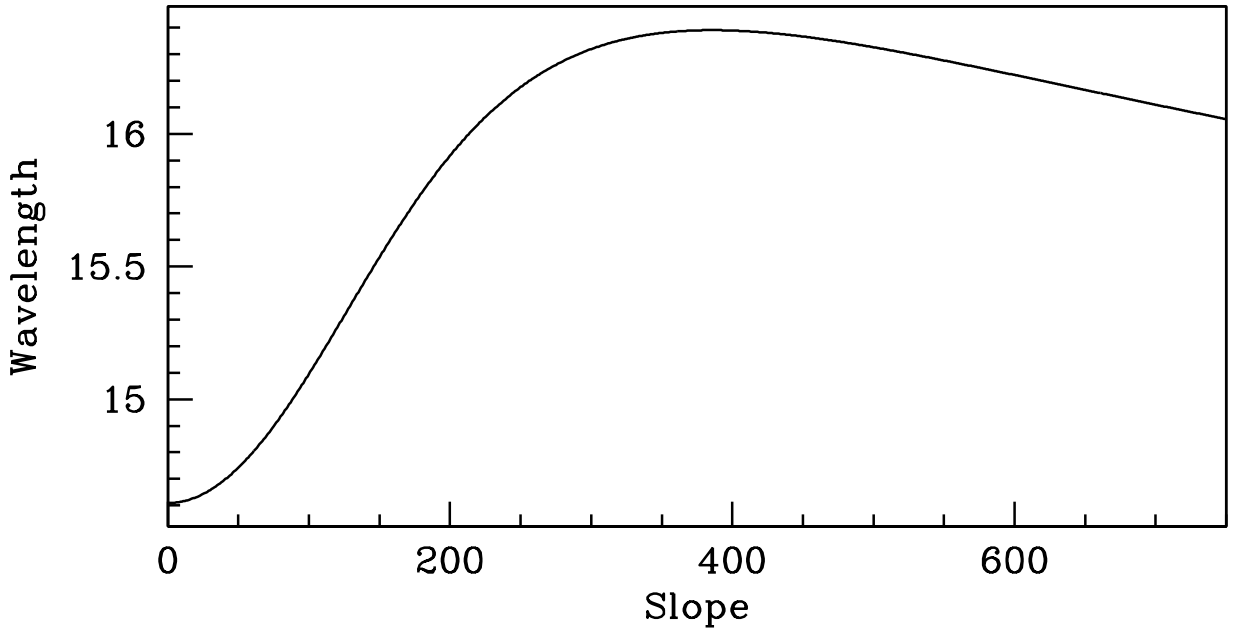

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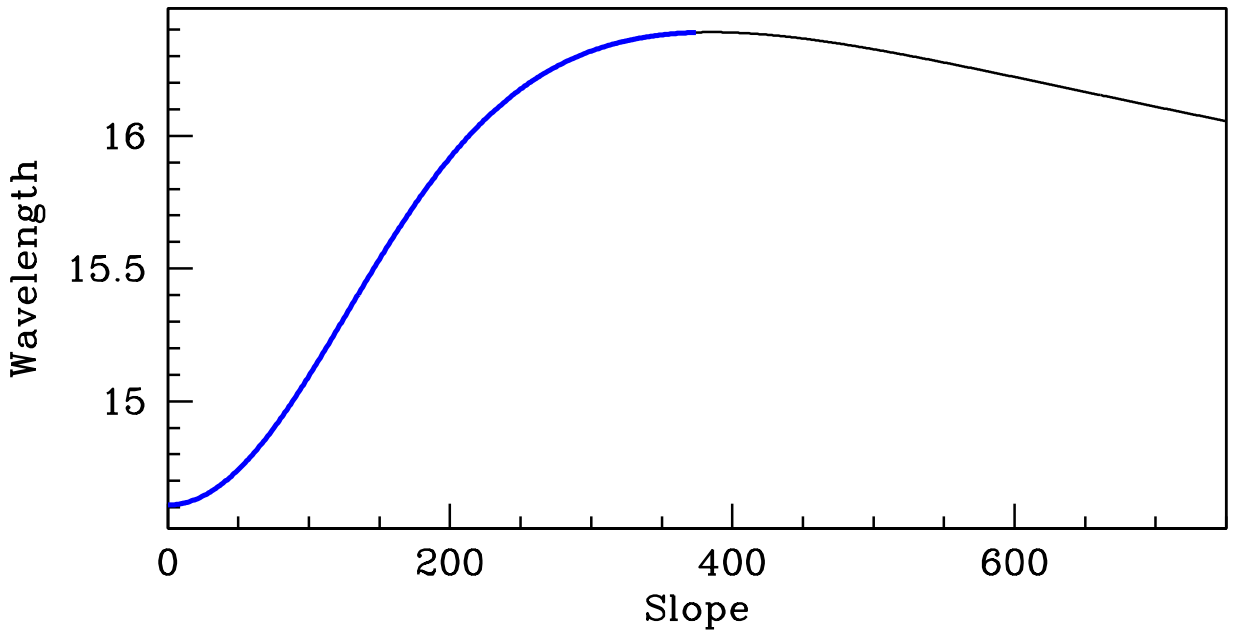
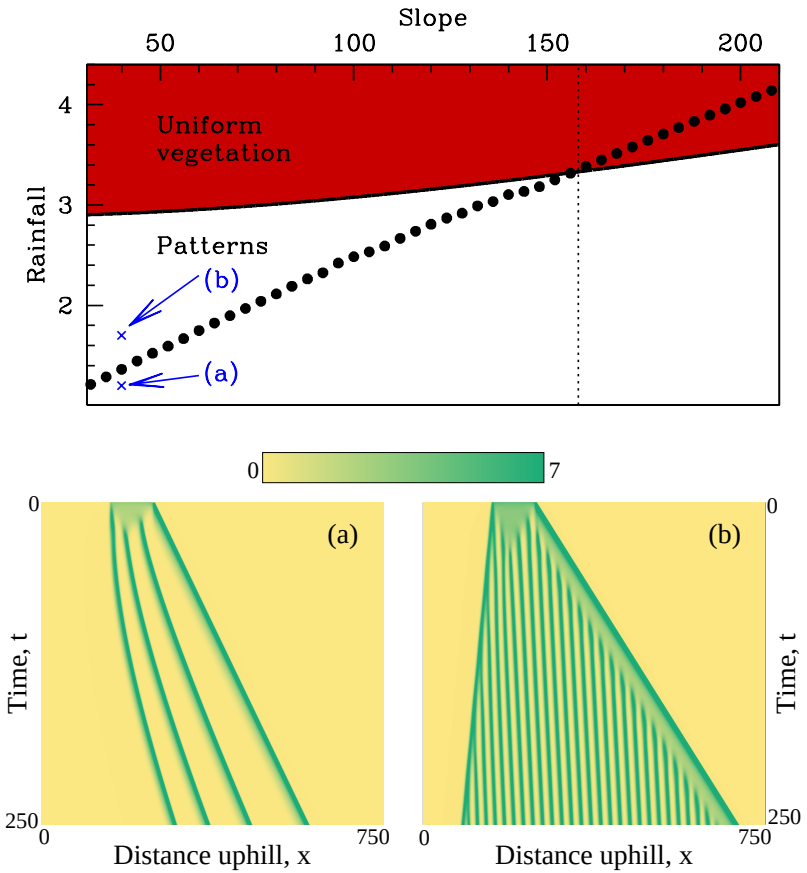



For realistic parameters, wavelength increases with slope

# Wavelength vs Slope for Colonisation



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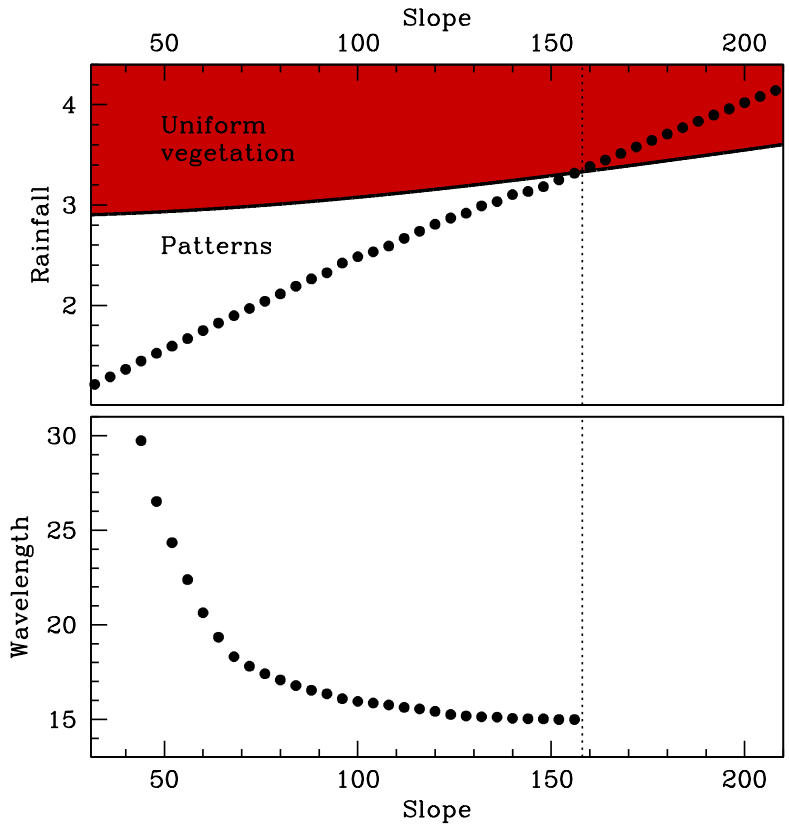


Wavelength decreases with slope

# Conclusions

Wavelength is positively correlated with slope $\Rightarrow$ vegetation pattern originated by degradation of uniform vegetation

Wavelength is negatively correlated with slope $\Rightarrow$ vegetation pattern originated by colonisation of bare ground

Main message: combined wavelength–slope data is much more valuable than wavelength data alone.

# Example: The African Sahel

- Patterned vegetation is widespread in the Sahel
- Several studies of banded vegetation show wavelength ↓ as slope ↑

# Rainfall History in the Sahel

- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.
- Reasonable assumption: areas with vegetation patterns today had uniform vegetation at the end of the 17th century.
- Since wavelength decreases with slope, my results imply that vegetation must have died out and then recolonised since the end of the 17th century.
- The most severe drought since 1700 was c. 1738-1756. So today's vegetation patterns result from recolonisation since 1760.

# References

J.A. Sherratt: History-dependent patterns of whole ecosystems. *Ecological Complexity* 14, 8-20 (2013).

A.S. Dagbovie, J.A. Sherratt: Pattern selection and hysteresis in the Rietkerk model for banded vegetation in semi-arid environments. *J. R. Soc. Interface* 11: 20140465 (2014).

J.A. Sherratt: Using wavelength and slope to infer the historical origin of semi-arid vegetation bands. *PNAS USA* 112: 4202-4207 (2015).

J.A. Sherratt: When does colonisation of a semi-arid hillslope generate vegetation patterns? *J. Math. Biol.* 73: 199-226 (2016).

J.A. Sherratt: Using History to Predict the Future of Vegetation in the African Sahel. Submitted.

# List of Frames