Inferring the Historical Origin of Vegetation Patterns

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This talk can be downloaded from my web site

www.ma.hw.ac.uk/∼jas

Outline

1. Ecological Background
2. The Central Question
3. A Mathematical Model for Vegetation Pattern Formation
4. Answering the Central Question
5. Conclusions

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2 The Central Question
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Vegetation Patterns

Desert ecosystems provide a classic example of self-organised pattern formation.

W National Park, Niger

Average patch width is 50 m

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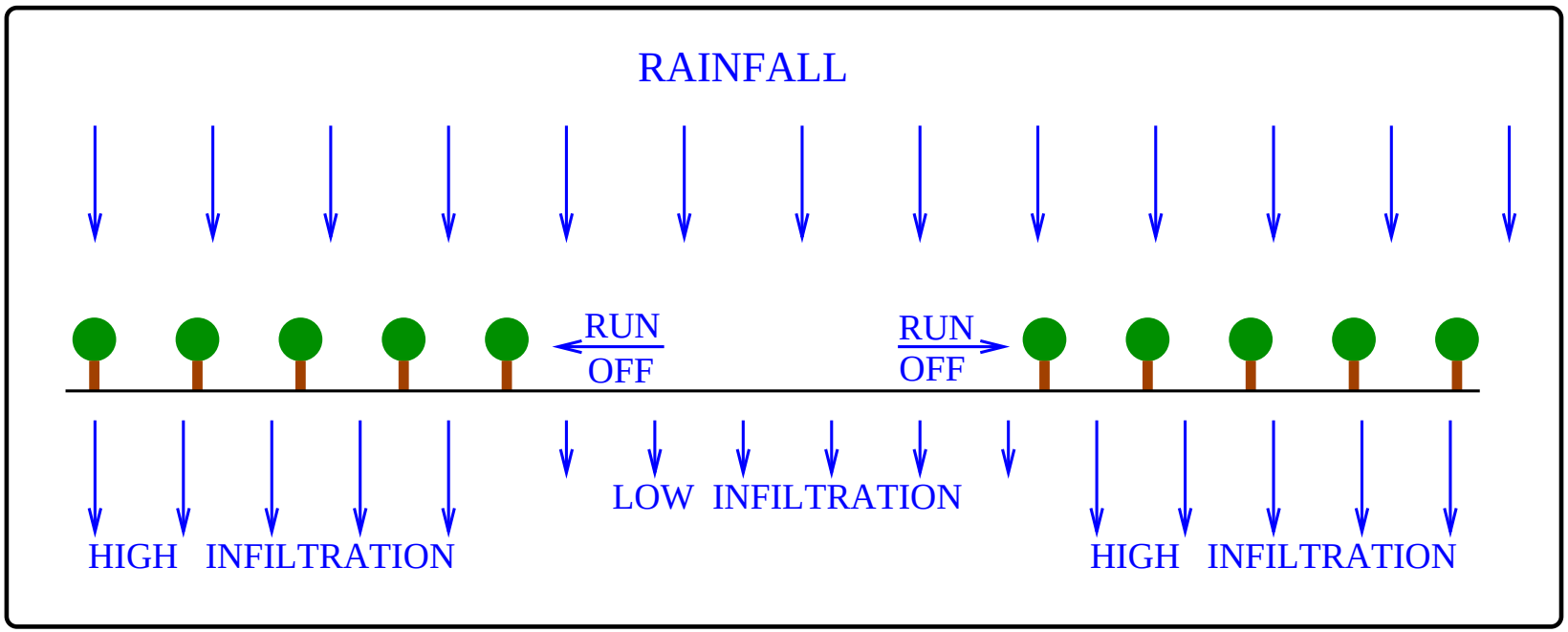

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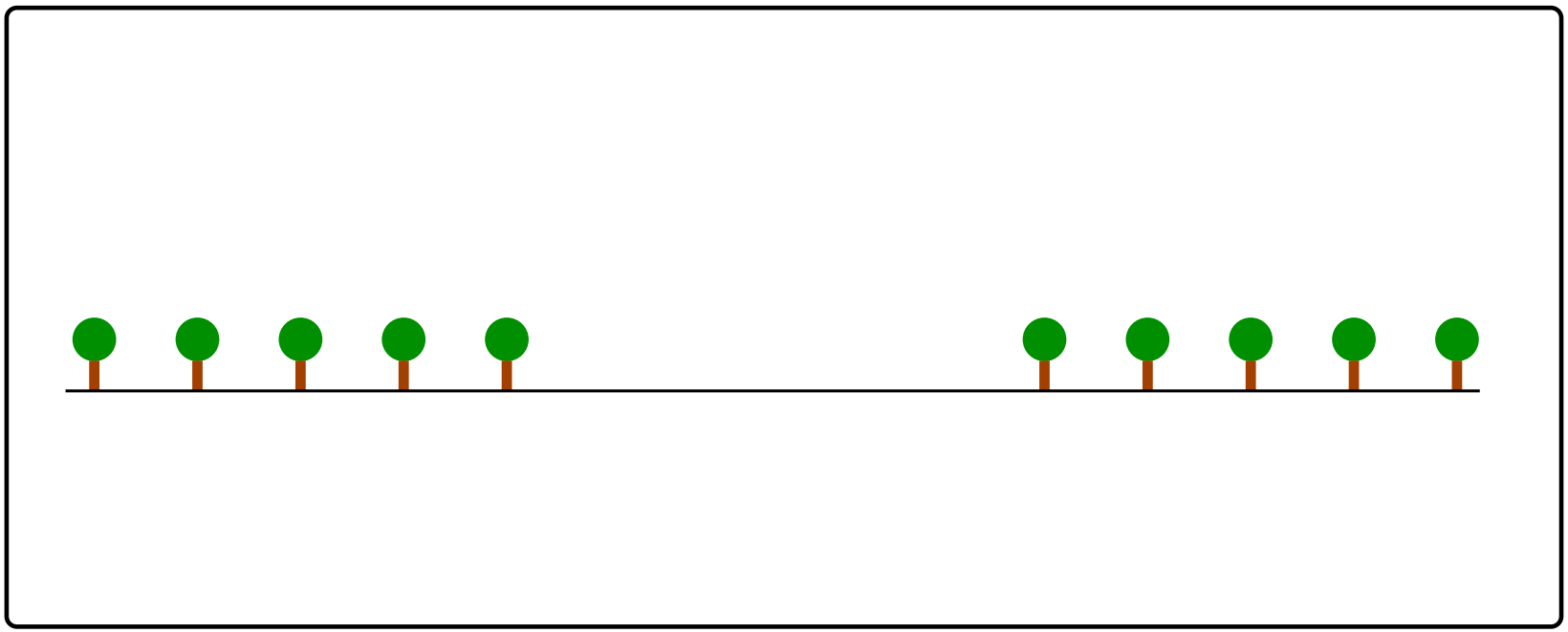

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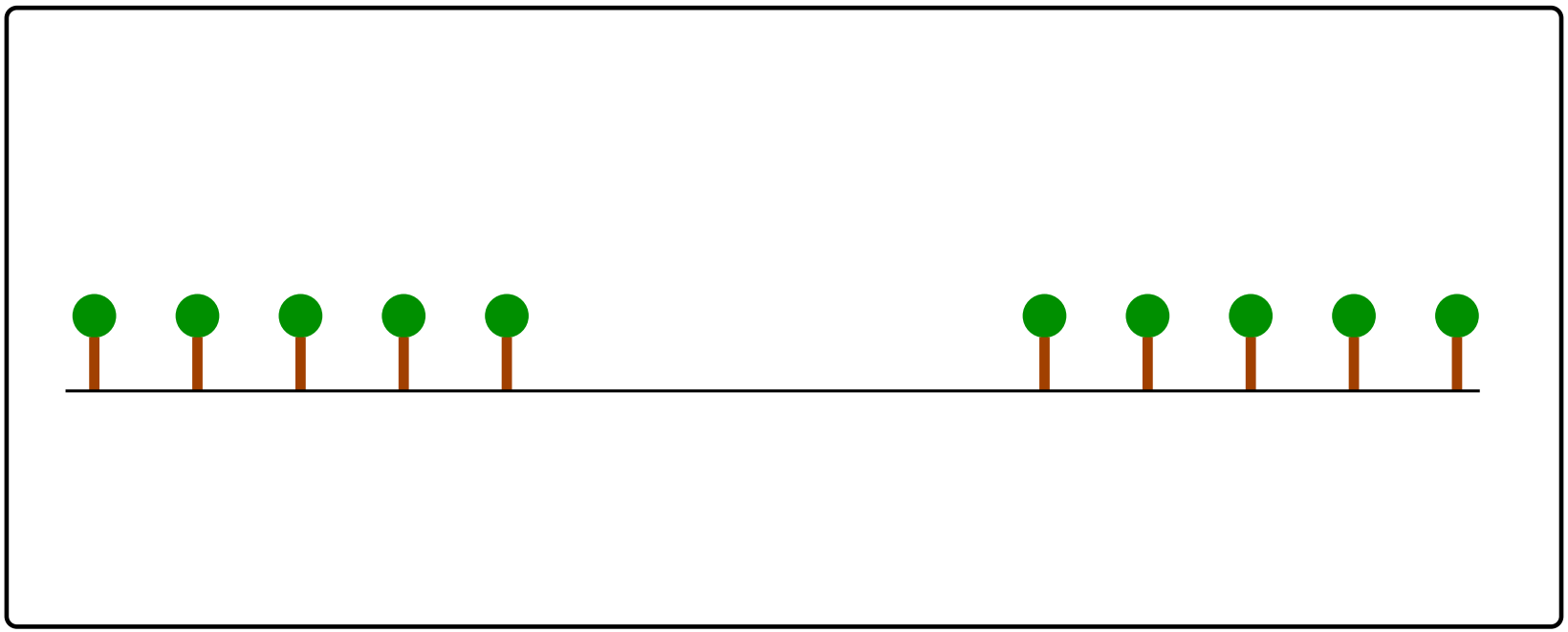

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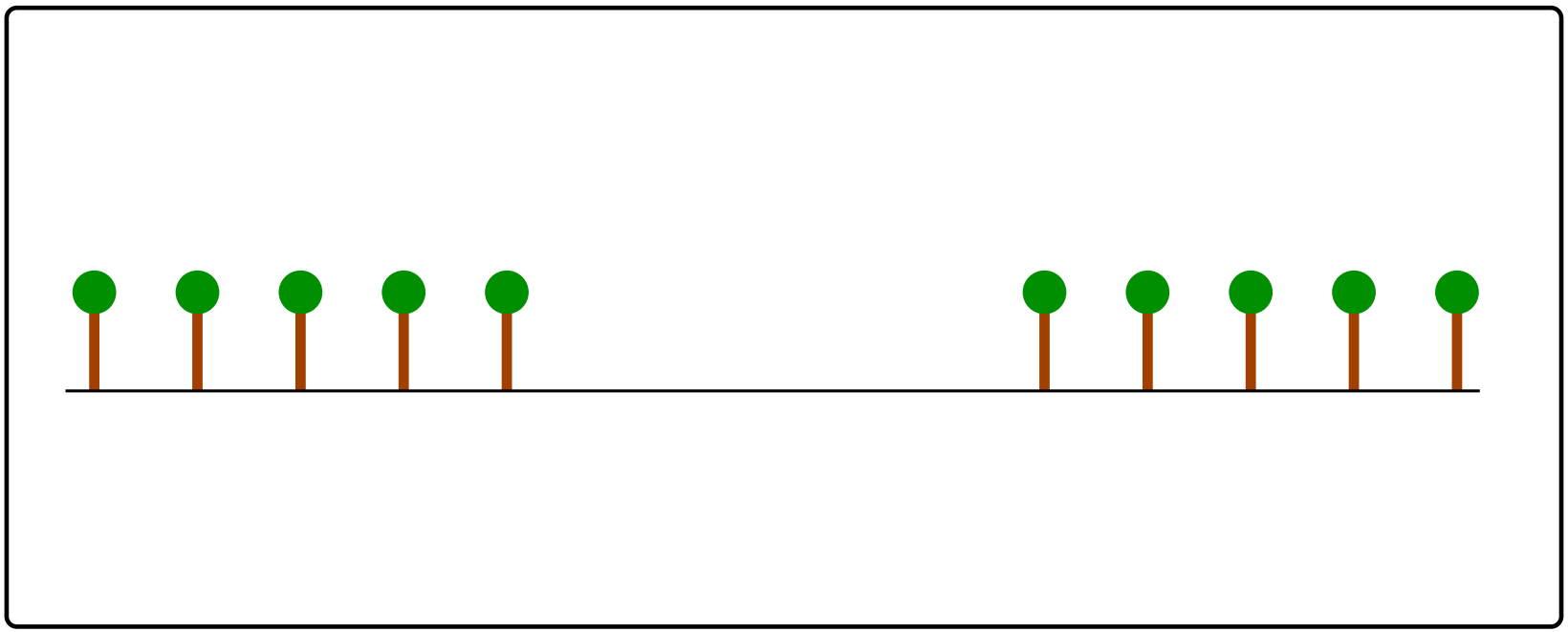

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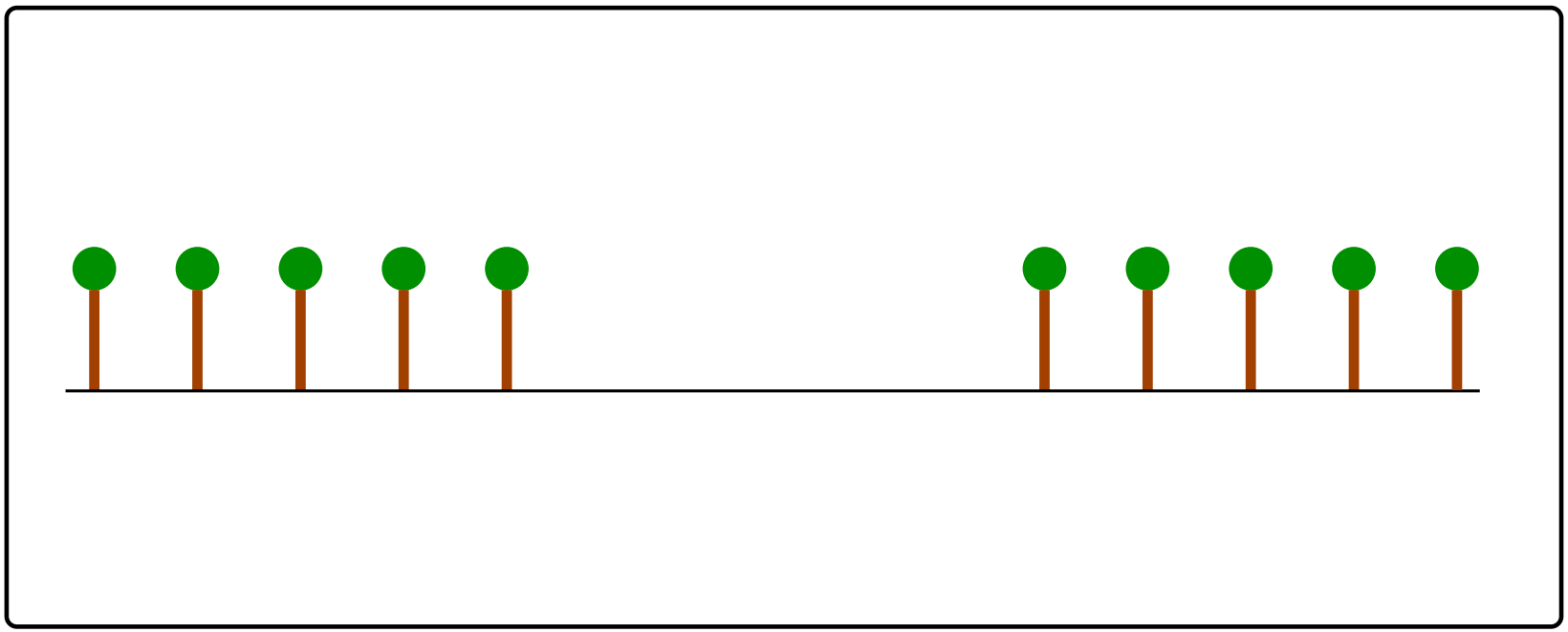

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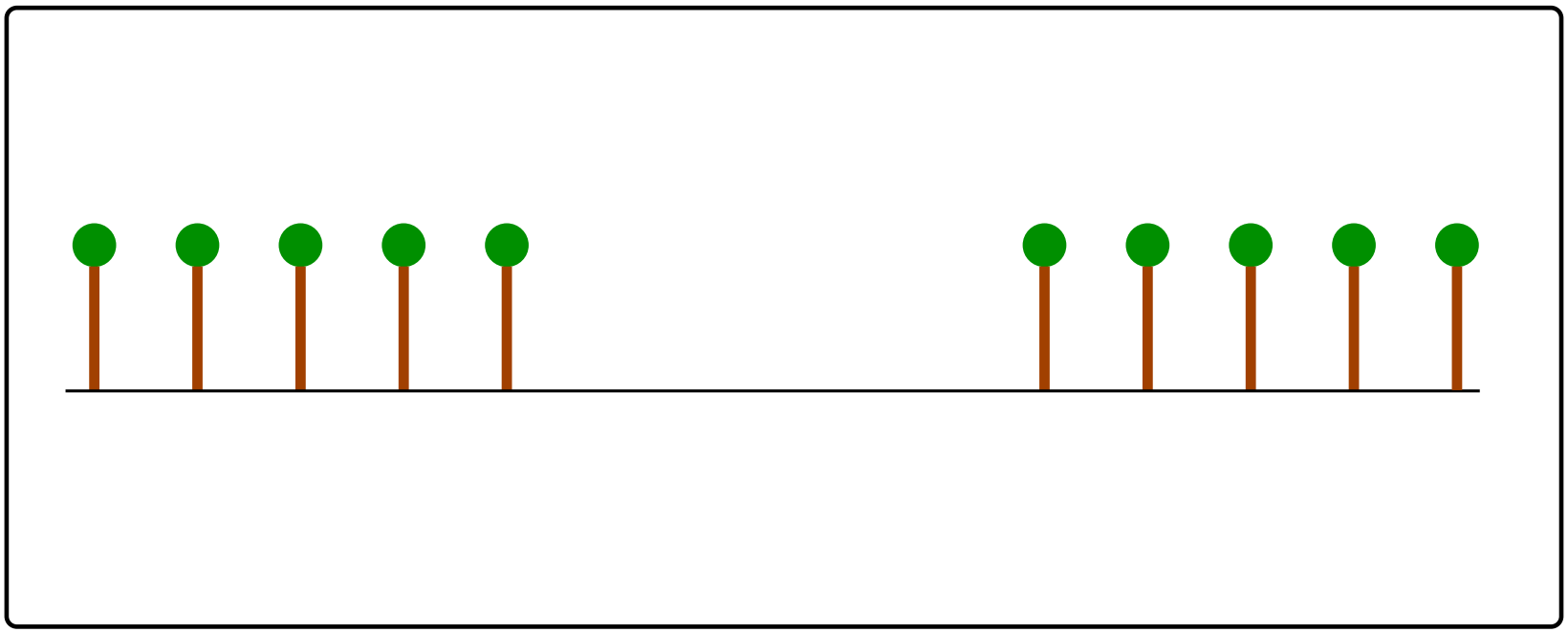

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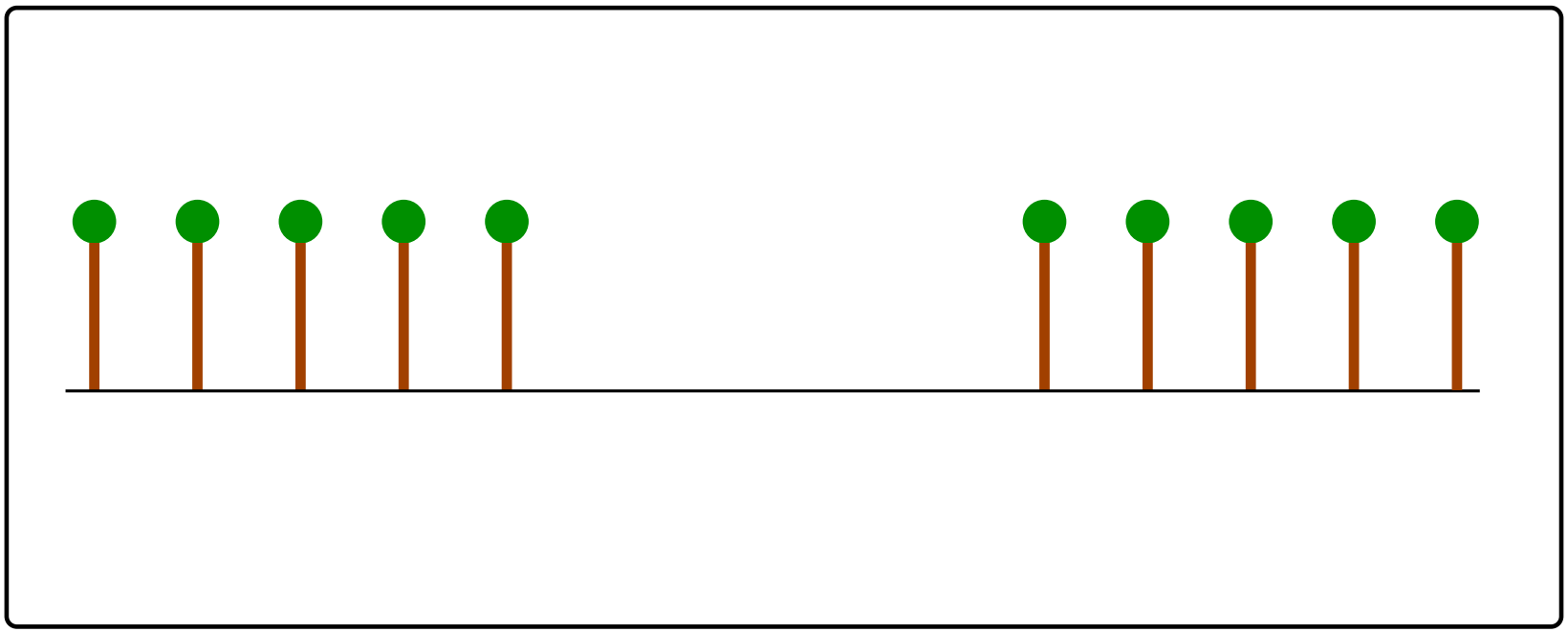

Banded Vegetation on Slopes

On slopes, run-off occurs in one direction only, giving striped patterns parallel to the contours.

Bushy vegetation in Niger

Mitchell grass in Australia

(Western New South Wales)

Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia, N America and Asia.

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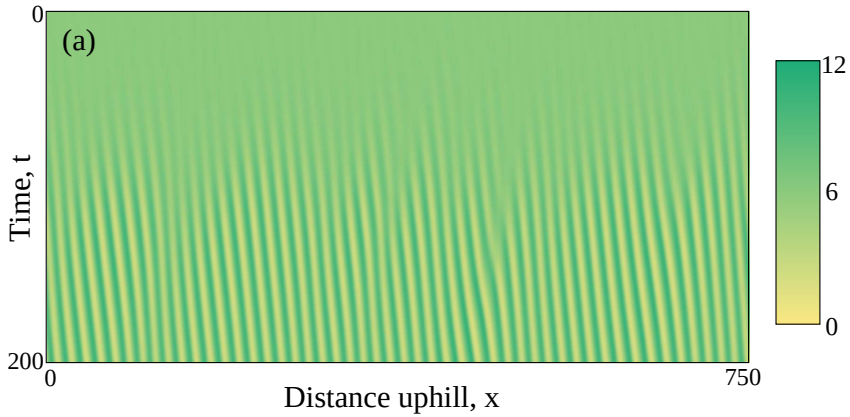
The Central Question

Bushy vegetation in Niger

For a given pattern, how can we determine its historical origin?

Possible Origins of Vegetation Patterns

Patterns can arise either via degradation of uniform vegetation



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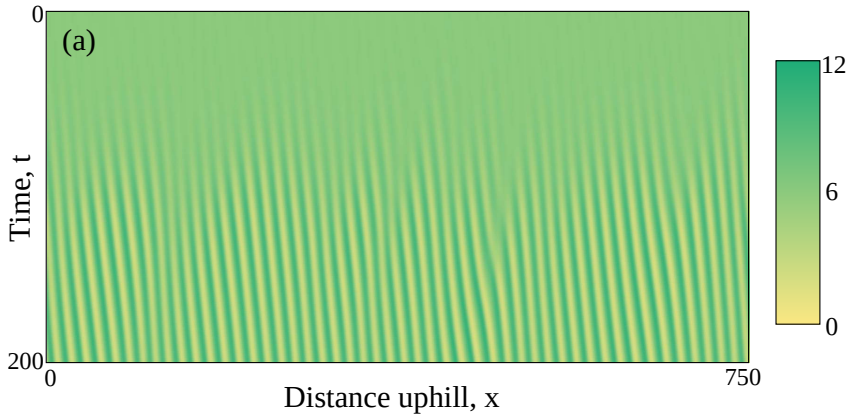
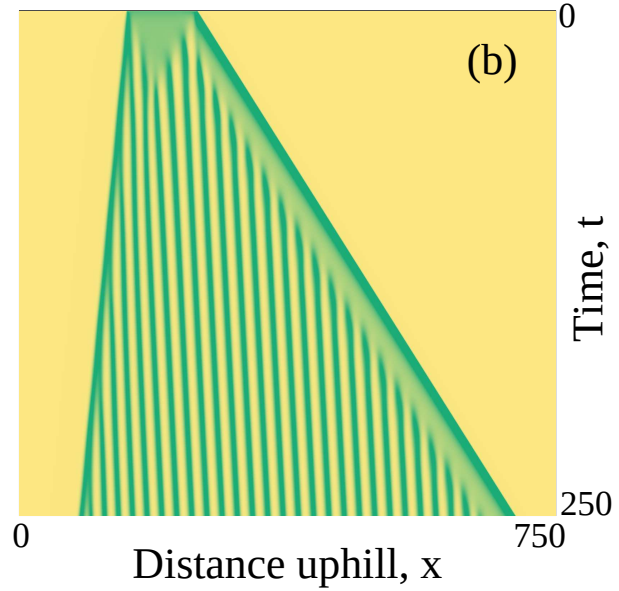


. . . or via colonisation of bare ground.



The Central Question

Bushy vegetation in Niger

For a given pattern, how can we determine its historical origin?

The Central Question

Bushy vegetation in Niger

For a given pattern, how can we determine its historical origin

. . . without any historical data?

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Mathematical Model of Klausmeier

$$\partial u/\partial t = \overbrace{wu^2}^{\text{plant growth}} - \overbrace{Bu}^{\text{plant loss}} + \overbrace{\partial^2 u/\partial x^2}^{\text{plant dispersal}}$$

$$\partial w/\partial t = \underbrace{A}_{\text{average rainfall}} - \underbrace{w}_{\text{evaporation \& drainage}} - \underbrace{wu^2}_{\text{uptake by plants}} + \underbrace{\nu\,\partial w/\partial x}_{\text{flow downhill}} + \underbrace{D\,\partial^2 w/\partial x^2}_{\text{diffusion of water}}$$

(Klausmeier, Science 284: 1826-8, 1999)

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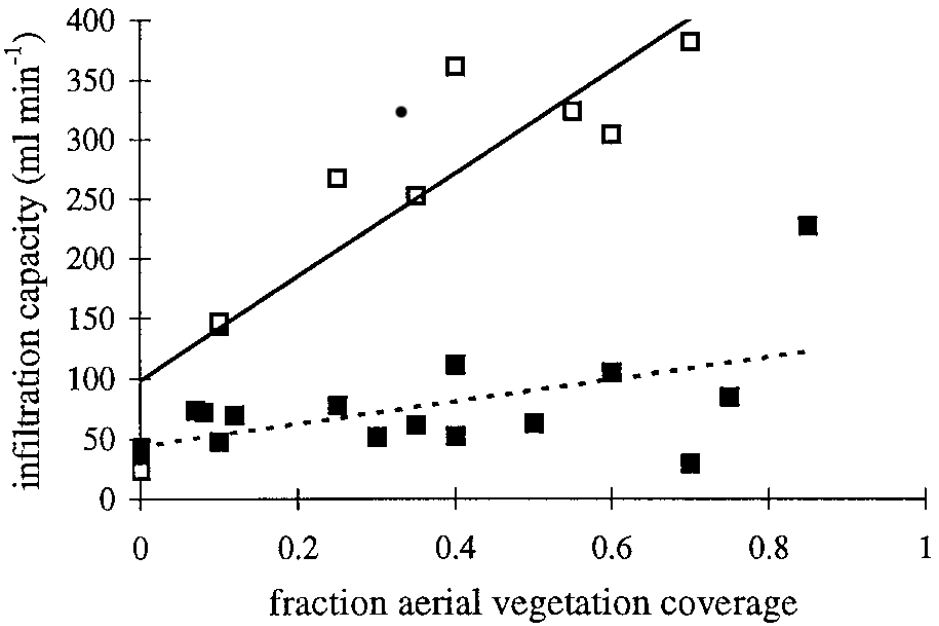
The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

Mathematical Model of Klausmeier



Water uptake=
Water density
$\times$ Plant density
$\times$ $\begin{pmatrix}\text{infiltration}\\ \text{rate}\end{pmatrix}$

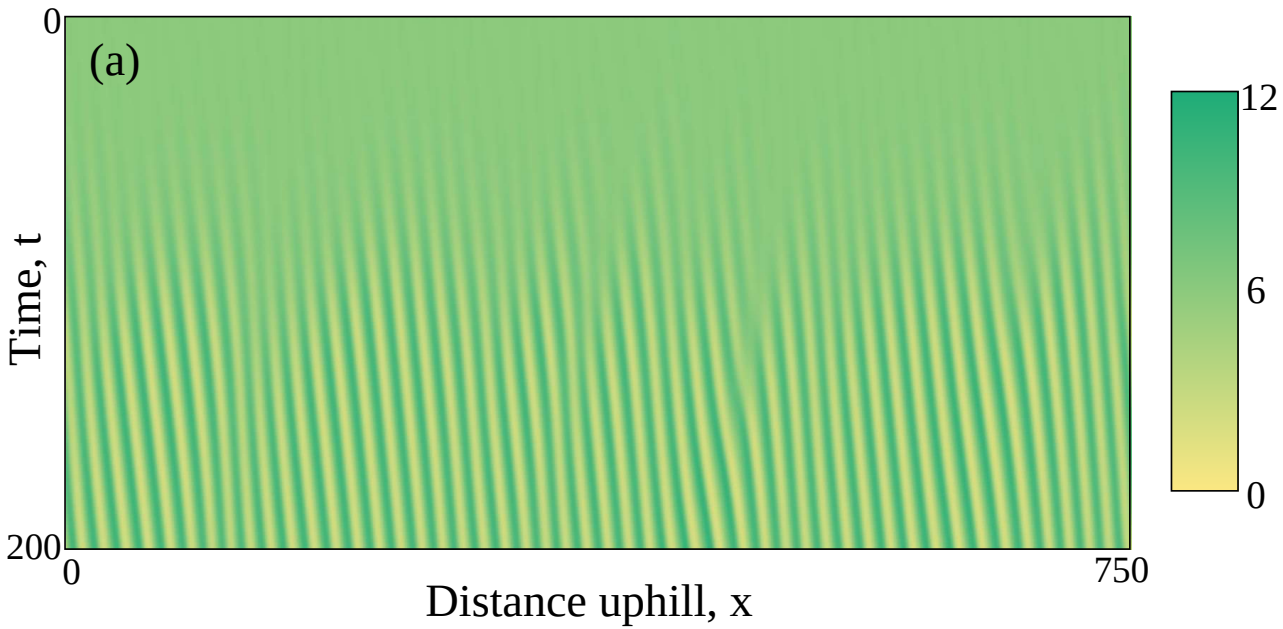
The nonlinearity in water uptake occurs because the presence of plants increases water infiltration into the soil.

Spatial Pattern Formation

The steady state (u_s, w_s) becomes unstable to spatially inhomogeneous perturbations at $A = A_{crit}$, giving patterns.

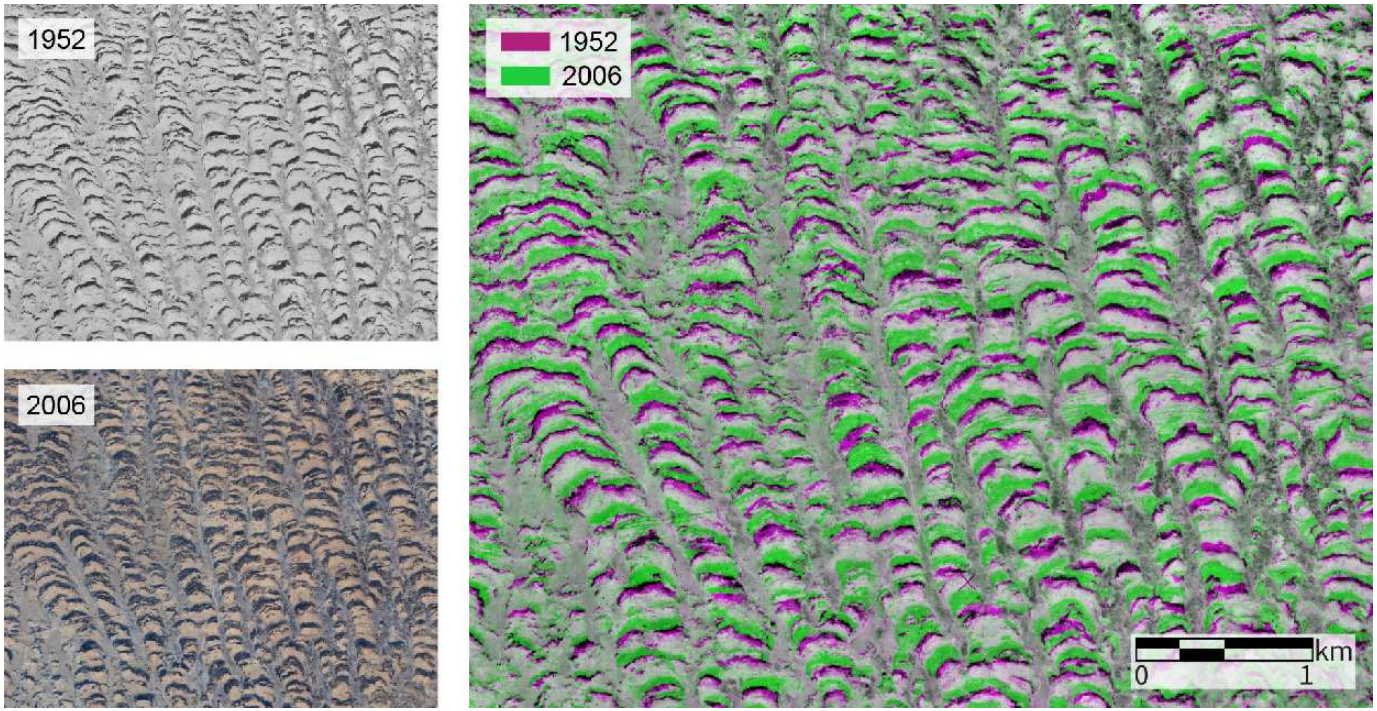
This is a Turing-Hopf bifurcation; the patterns move uphill (slowly).

A Simulation of Pattern Formation



Note that the pattern moves uphill (slowly).

Data on Pattern Migration



Vegetation near Dhahar, Somalia

(Gandhi et al, in “Dryland Ecology” ed. D’Odorico et al, Springer, 2019)

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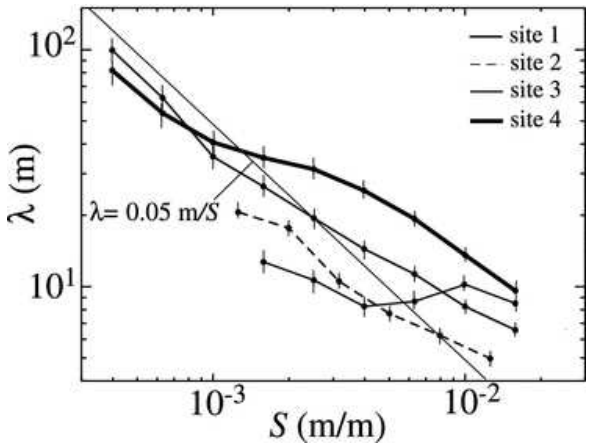
Wavelength vs Slope

To be most useful, a method for estimating historical origin should be based on remotely sensed data

- wavelength of pattern
- slope of hillside



Data from Nevada, USA (Pelletier et al, J. Geophys. Res. 117: F04026, 2012)

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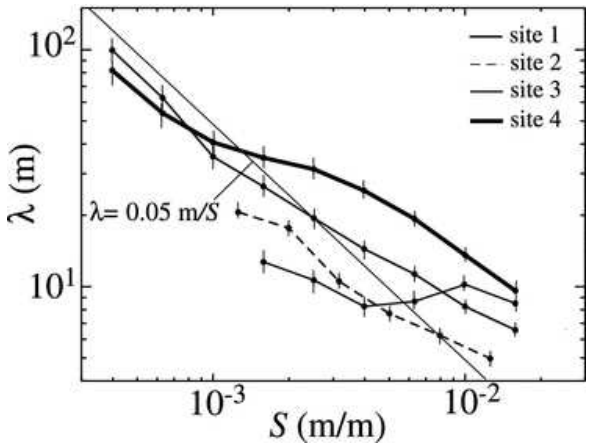

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What does the model predict for wavelength vs slope?

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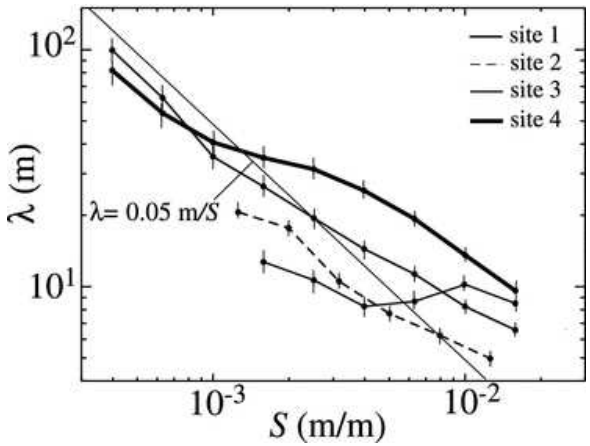

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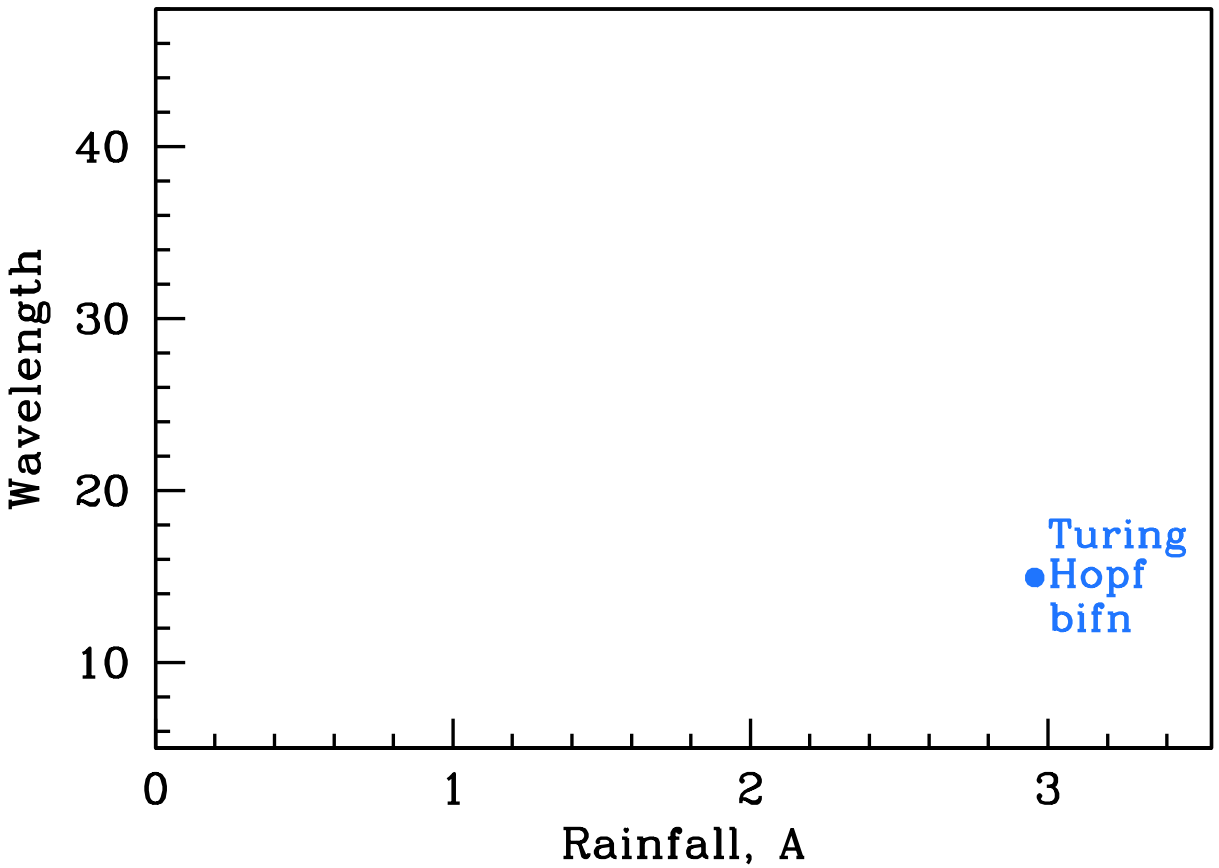
What does the model predict for wavelength vs slope?

Problem: in the model there is a range of possible pattern wavelengths

Existence and Stability of Patterns



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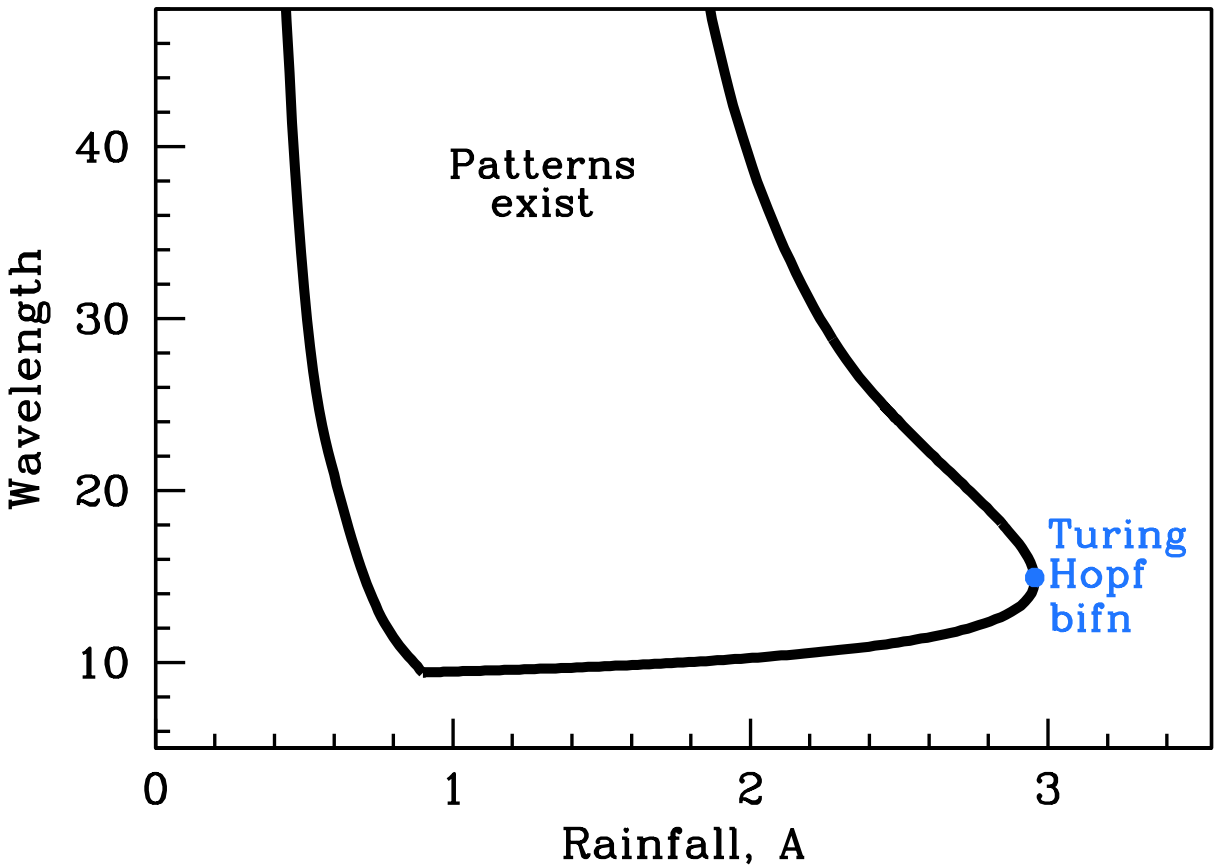

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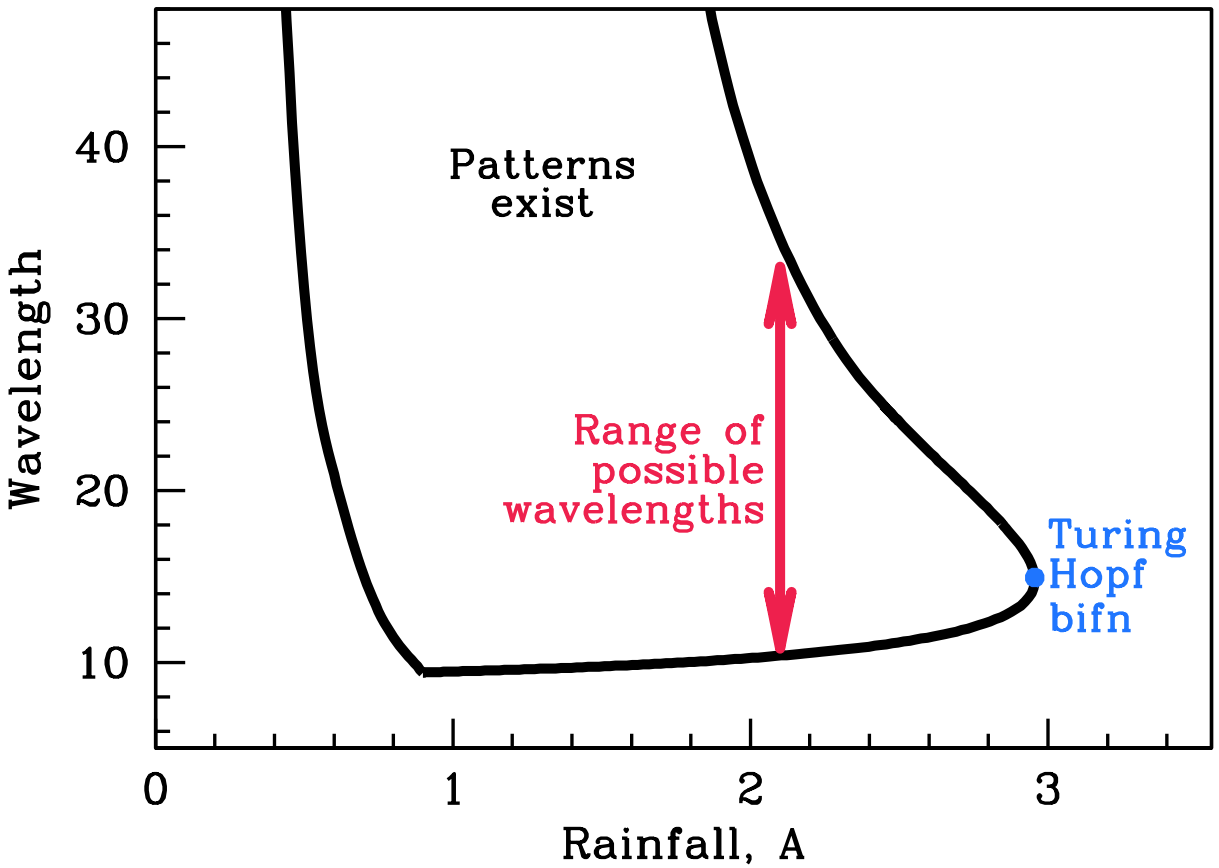

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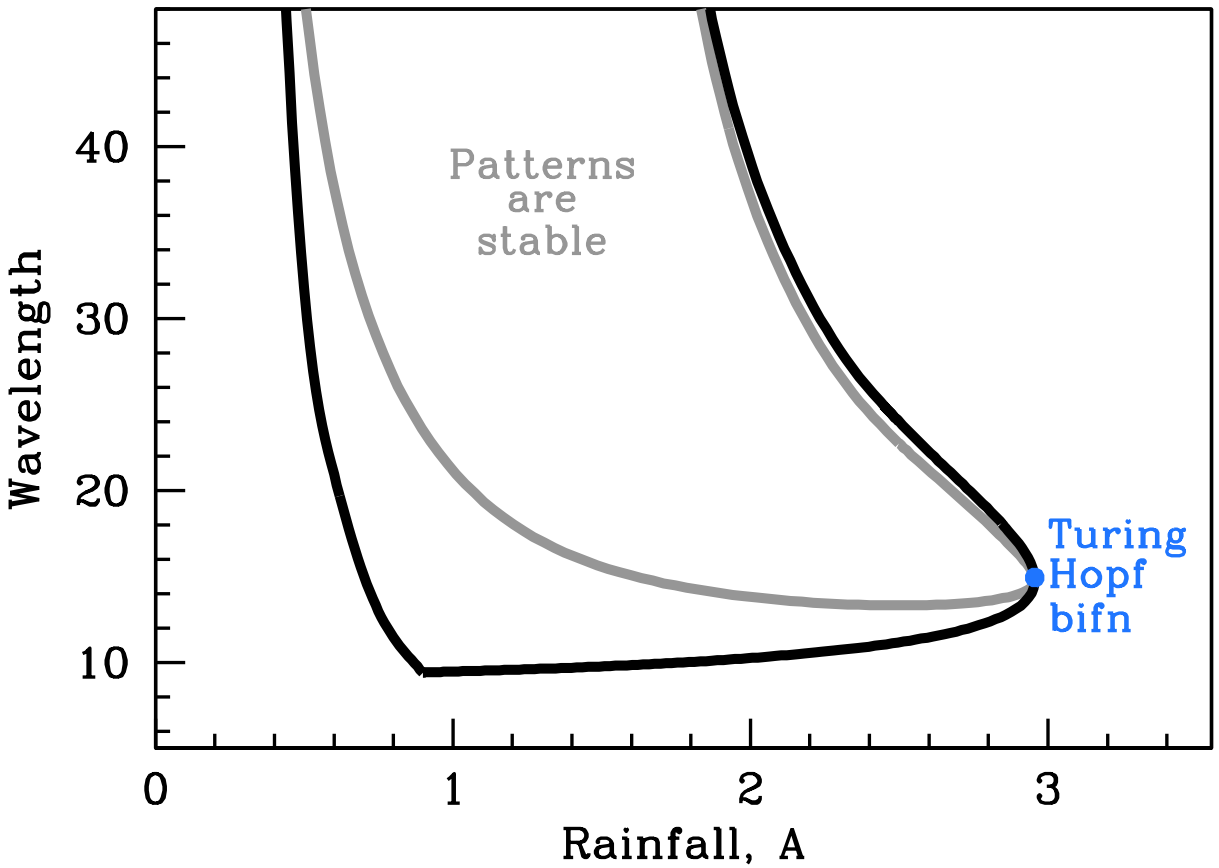

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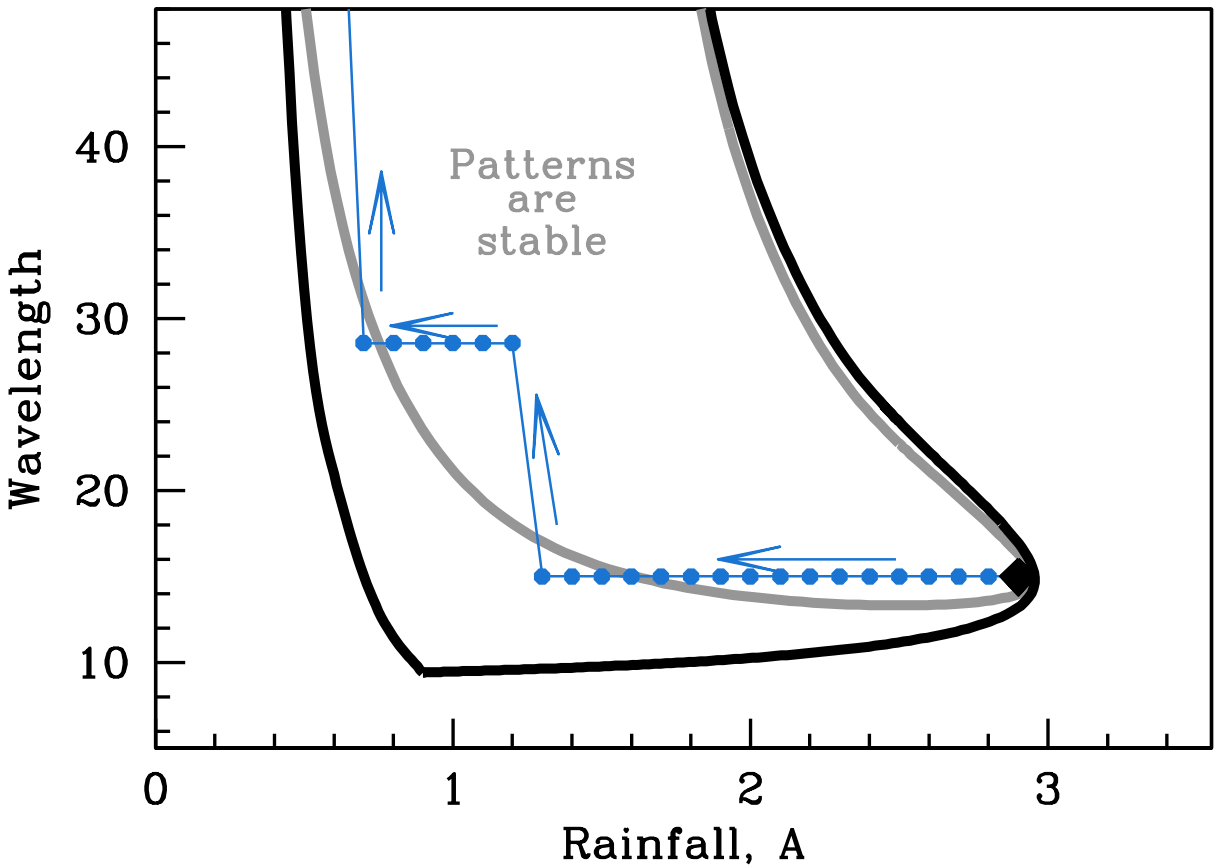
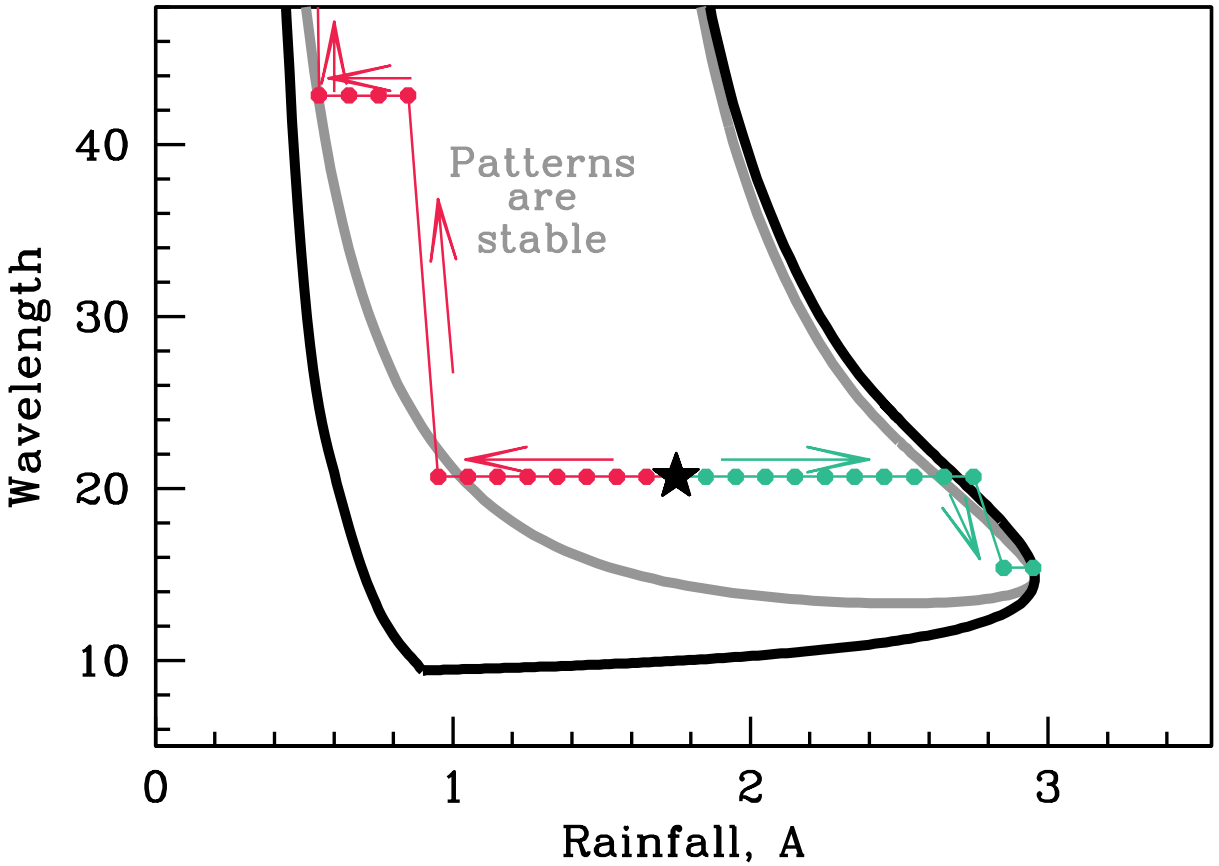


- Starting point: degradation of uniform vegetation

Existence and Stability of Patterns



★

Starting point:
colonisation
of bare ground

Existence and Stability of Patterns

Objective: what does the model predict for wavelength vs slope?

Problem: in the model there is a range of possible pattern wavelengths.

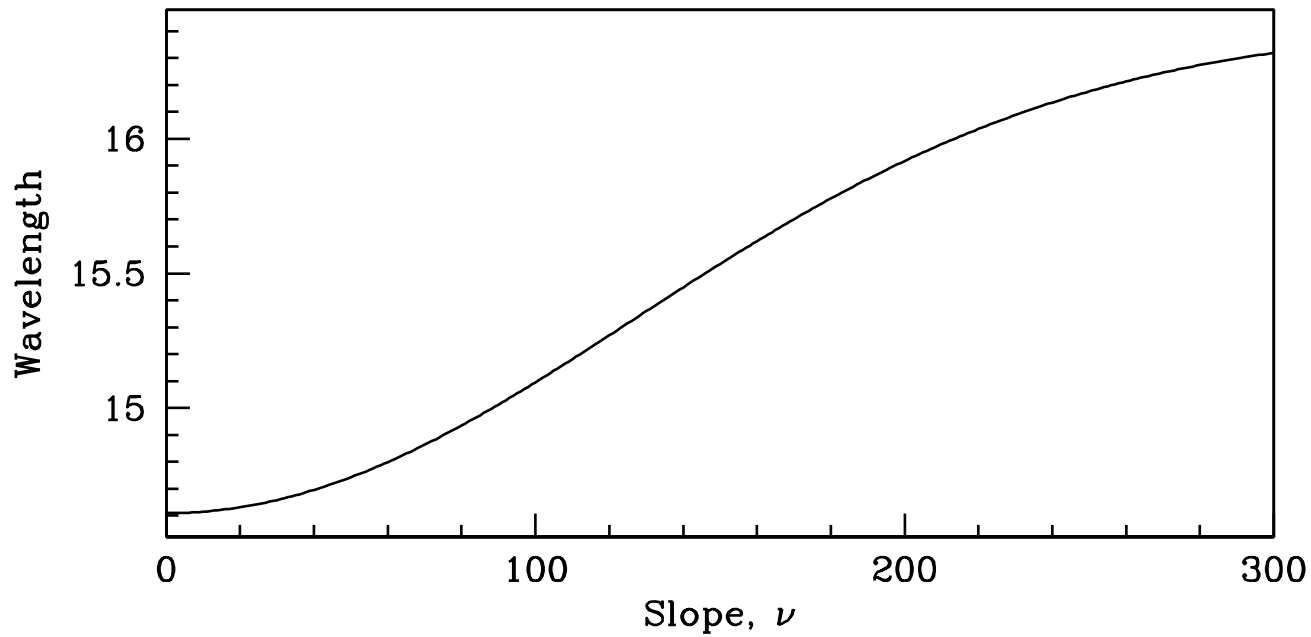
Resolution: for wavelength vs slope we must study the onset of patterning.

Case 1: degradation of uniform vegetation.

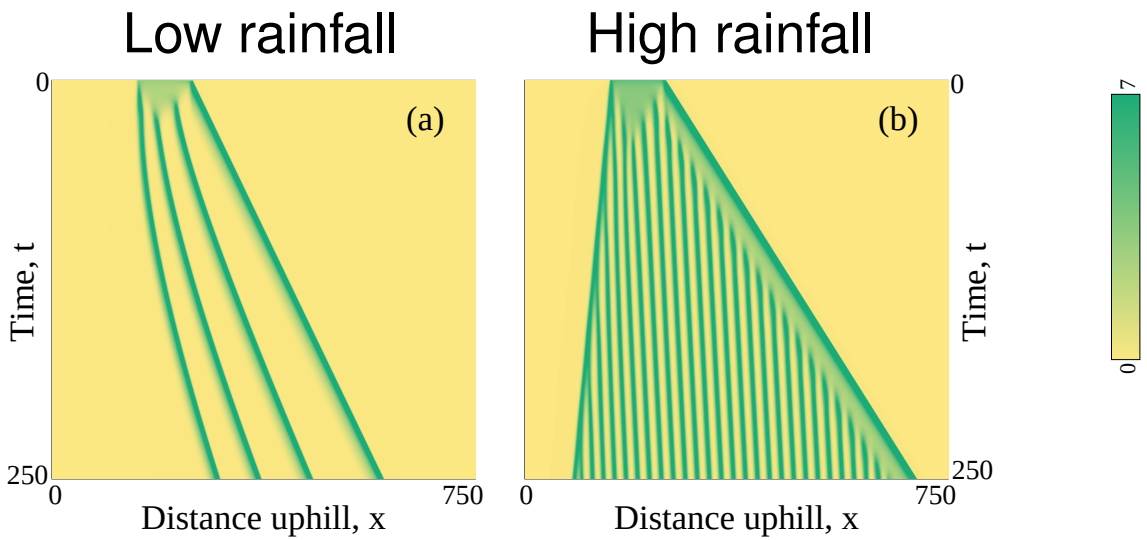
Case 2: colonisation of bare ground.

Wavelength vs Slope for Degradation of Uniform Vegetation

Linear stability analysis $\Rightarrow$ wavelength increases with slope at pattern onset from degradation of uniform vegetation.



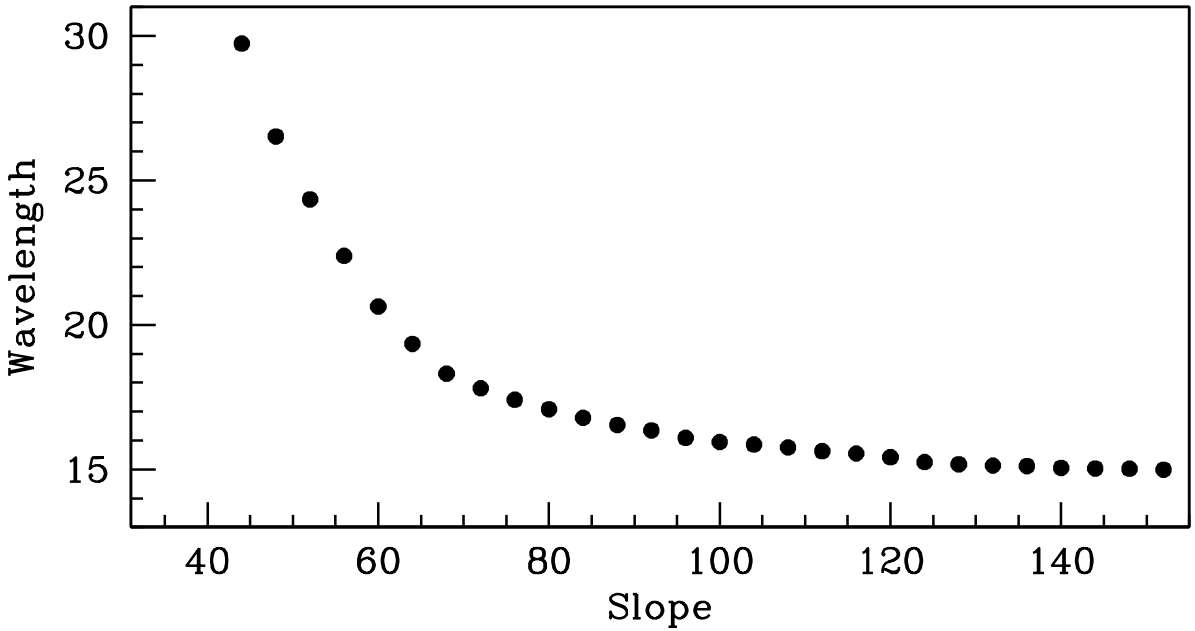
When Does Vegetation Colonise Bare Ground?



The critical rainfall for colonisation can be calculated numerically $\Rightarrow$ wavelength at pattern onset.

Wavelength vs Slope for Colonisation of Bare Ground

Wavelength decreases with slope at pattern onset from colonisation of bare ground.



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Conclusions

Wavelength is positively correlated with slope $\Rightarrow$ vegetation pattern originated by degradation of uniform vegetation

Wavelength is negatively correlated with slope $\Rightarrow$ vegetation pattern originated by colonisation of bare ground

Example: The African Sahel

- Patterned vegetation is widespread in the Sahel
- Several studies of banded vegetation show wavelength ↓ as slope ↑

Rainfall History in the Sahel

- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.

Rainfall History in the Sahel

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- The Sahel was relatively humid in the 16th and 17th centuries.

How do we know this?

- There is no direct data on rainfall before c. 1850.

- Proxy data: (i) lake levels, esp. Lake Chad; (ii) historical chronologies, e.g. Bornu Empire; (iii) memories of local peoples.

Rainfall History in the Sahel

- The Sahara and Sahel have been arid for about 5000 years, but the level of aridity has varied significantly.
- The Sahel was relatively humid in the 16th and 17th centuries.
- Reasonable assumption: areas with vegetation patterns today had uniform vegetation at the end of the 17th century.
- Since wavelength decreases with slope, my results imply that vegetation must have died out and then recolonised since the end of the 17th century.
- The most severe drought since 1700 was c. 1738-1756. So today's vegetation patterns result from recolonisation since 1760.

List of Frames

1 Ecological Background
- Vegetation Patterns
- Banded Vegetation on Slopes

2 The Central Question
- The Central Question
- Possible Origins of Vegetation Patterns

3 A Mathematical Model for Vegetation Pattern Formation
- Mathematical Model of Klausmeier
- Spatial Pattern Formation
- Data on Pattern Migration

4 Answering the Central Question
- Wavelength vs Slope
- Existence and Stability of Patterns
- Wavelength vs Slope for Degradation of Uniform Vegetation
- When Does Vegetation Colonise Bare Ground?
- Wavelength vs Slope for Colonisation of Bare Ground

5 Conclusions
- Conclusions
- Example: The African Sahel
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