Vegetation Patterns in Semi-Deserts

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This talk can be downloaded from my web site

www.ma.hw.ac.uk/∼jas

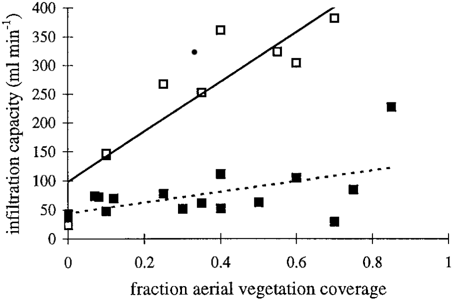
Vegetation Patterns

Bushy vegetation in Niger

Mitchell grass in Australia
(Western New South Wales)

- Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia and Mexico
- Plants vary from grasses to shrubs and trees
- Typical wavelength 1km for shrubs and trees

Why Do Plants Form Patterns?



Data from Burkina Faso

Rietkerk et al
Plant Ecology 148: 207-224, 2000

More plants $\Rightarrow$ more roots and organic matter in soil
$\Rightarrow$ more infiltration of rainwater

Banded Patterns on Slopes

- On slopes, water flow downhill causes stripes which move uphill

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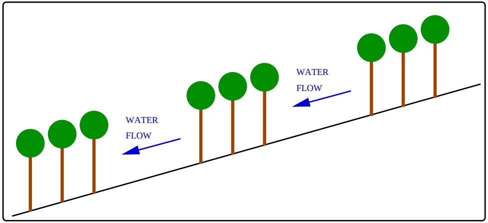

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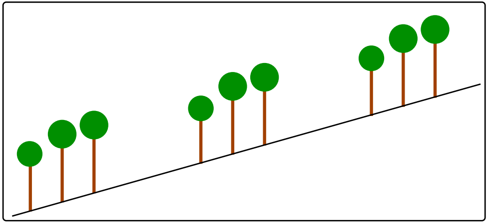

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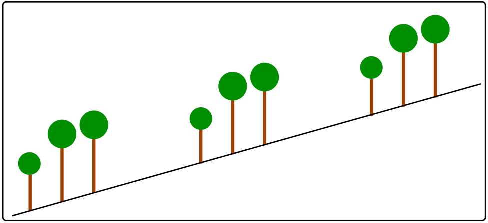

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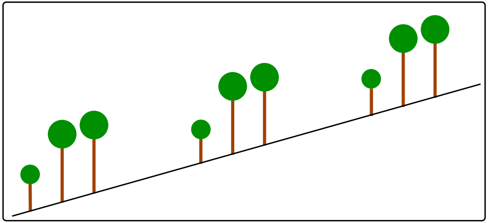

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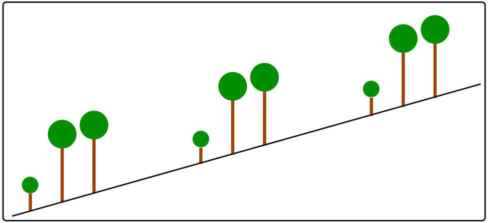

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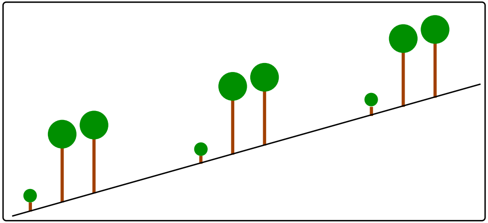

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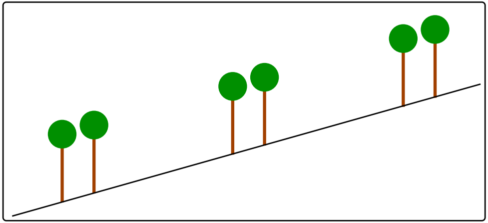

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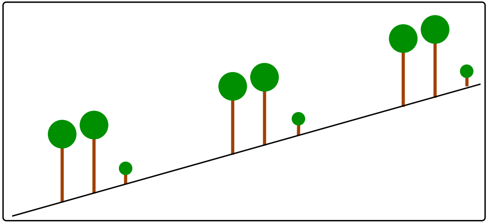

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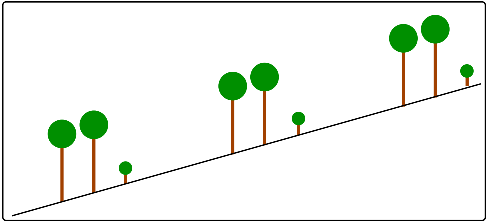

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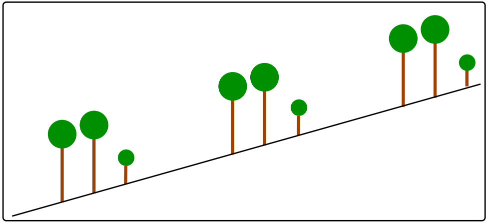

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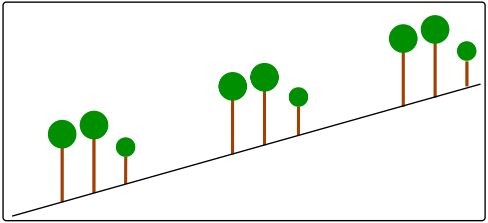

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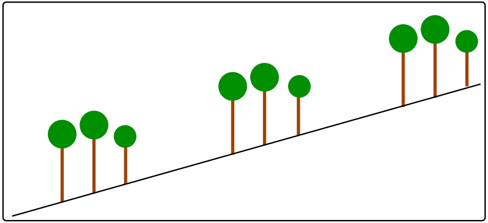

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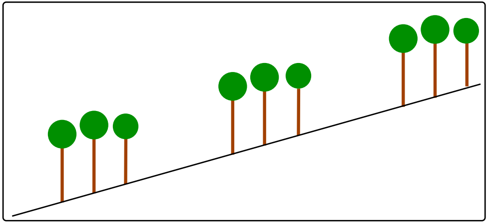

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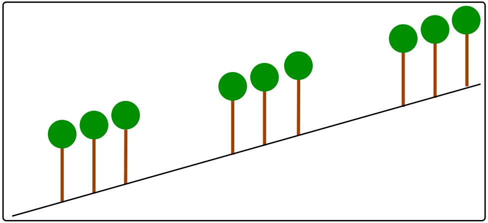

Key Ecological Questions

- At what rainfall level is there a switch from uniform vegetation to patterns?
- At what rainfall level is there a transition to desert?
- How does the spacing of the vegetation bands depend on rainfall, herbivory and slope?

Mathematical Model of Klausmeier

Rate of change of water = Rainfall – Evaporation – Uptake by plants + Flow downhill

Rate of change plant biomass = Growth, proportional to water uptake – Mortality + Random dispersal

$$\partial w/\partial t = A - w - wu^2 + \nu\partial w/\partial x$$

$$\partial u/\partial t = wu^2 - Bu + \partial^2 u/\partial x^2$$

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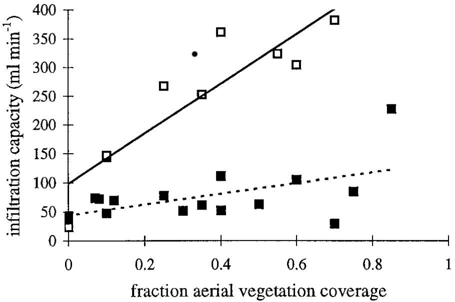

$$wu^2 = w \cdot u \cdot \begin{pmatrix}\text{infiltration} \\ \text{rate}\end{pmatrix}$$

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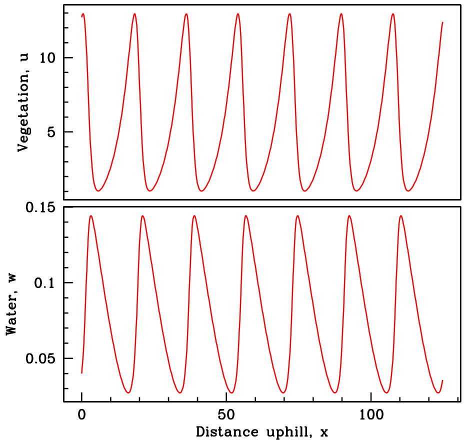
Typical Solution of the Model



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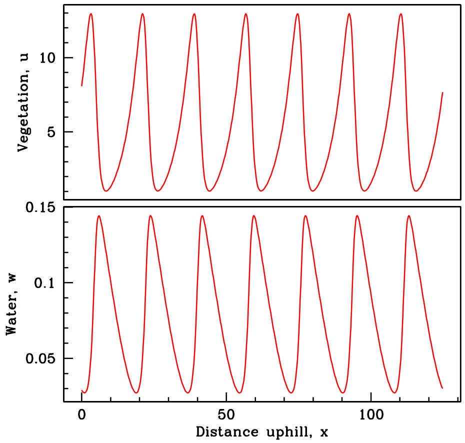

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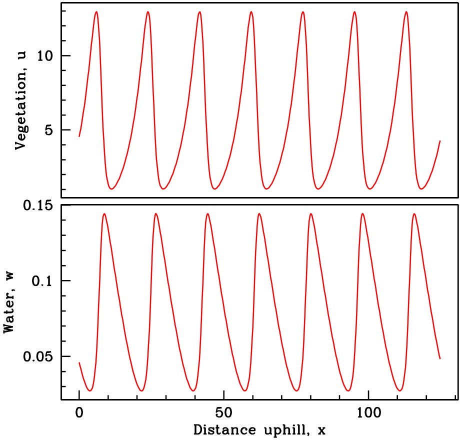

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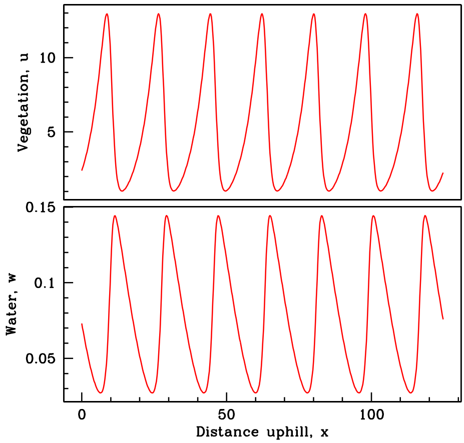

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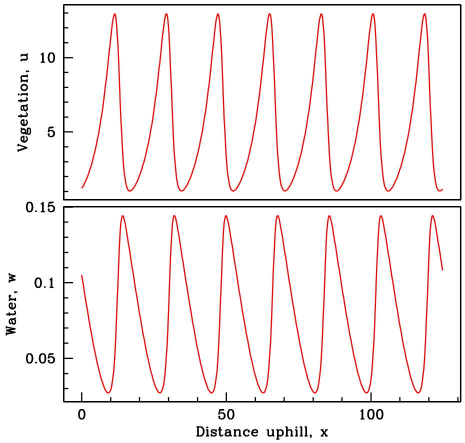

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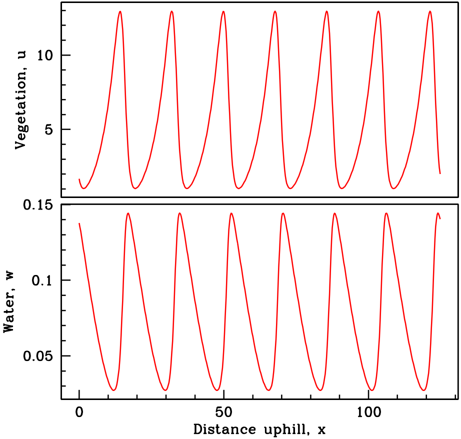

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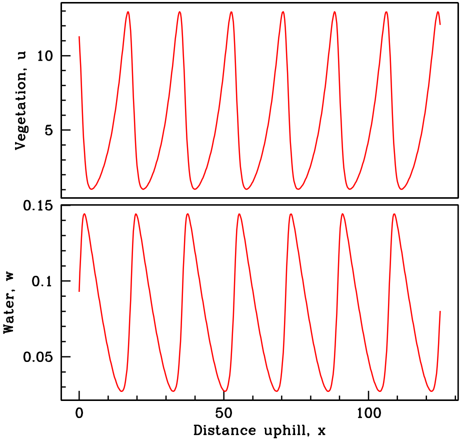

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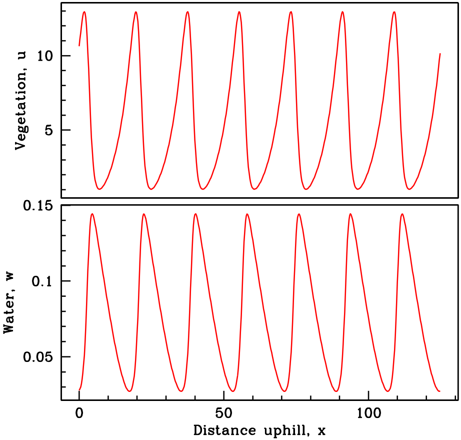

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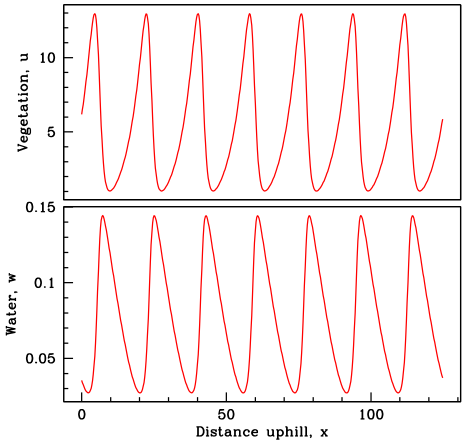

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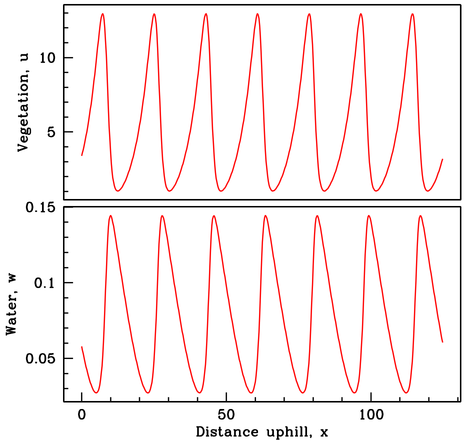

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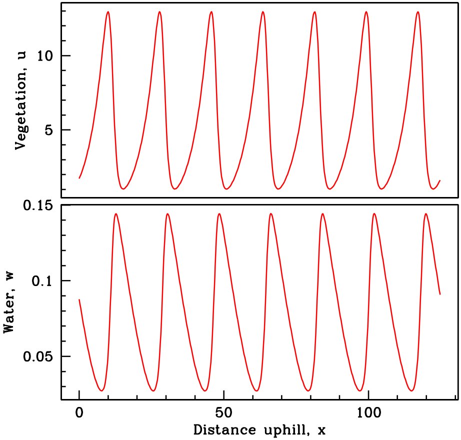

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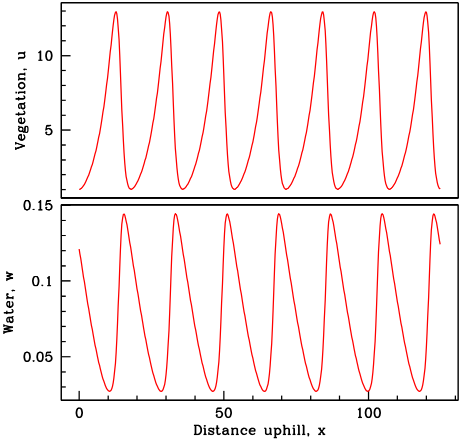

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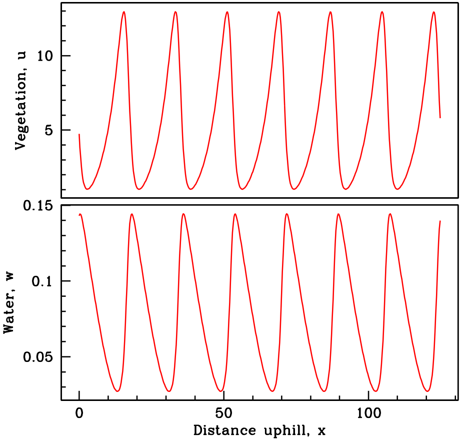

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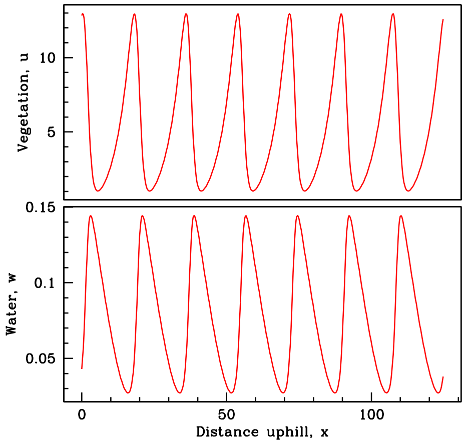

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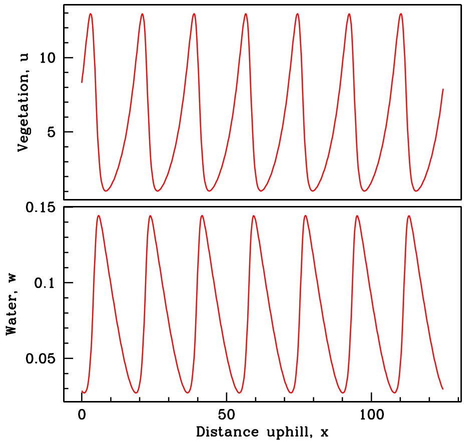

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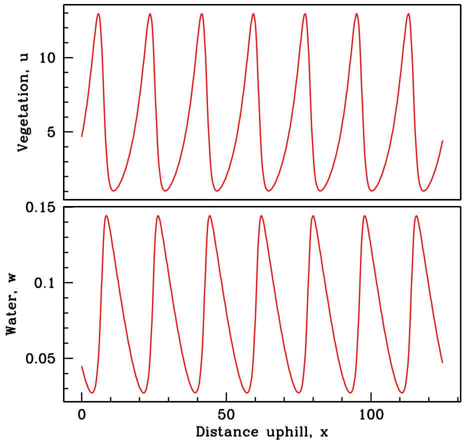

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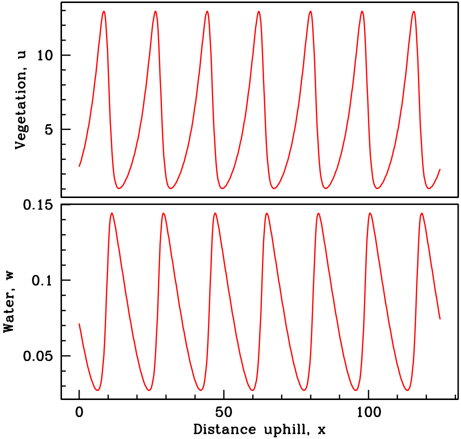

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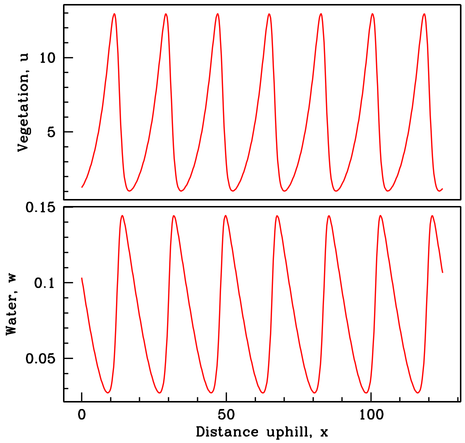

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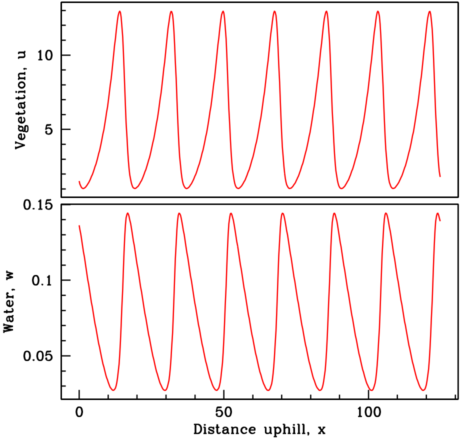

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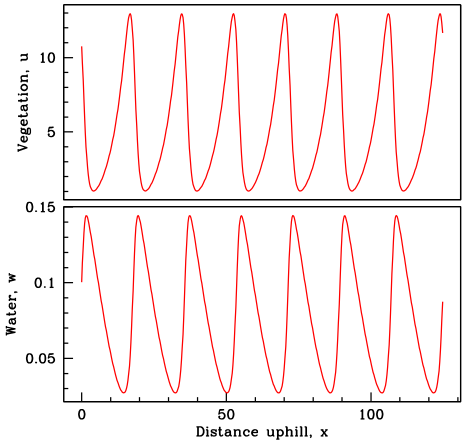

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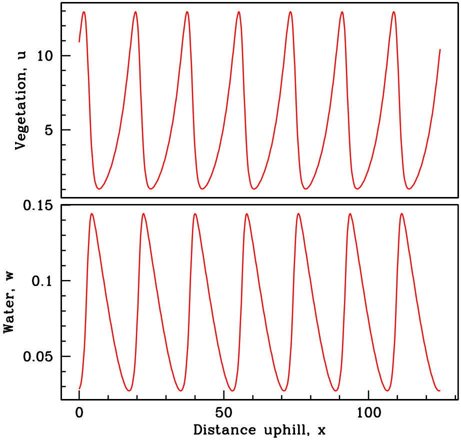

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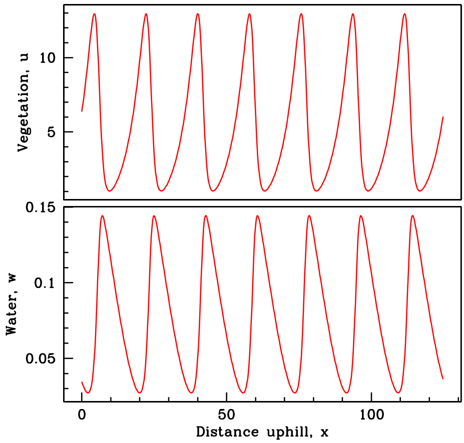

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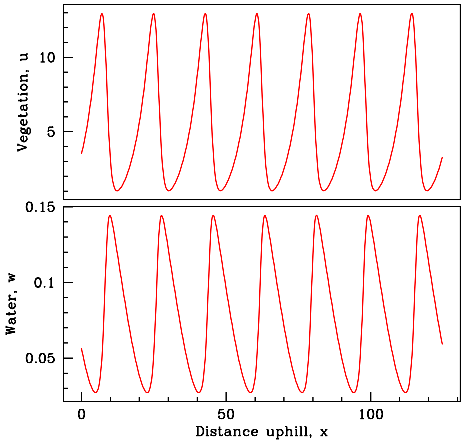

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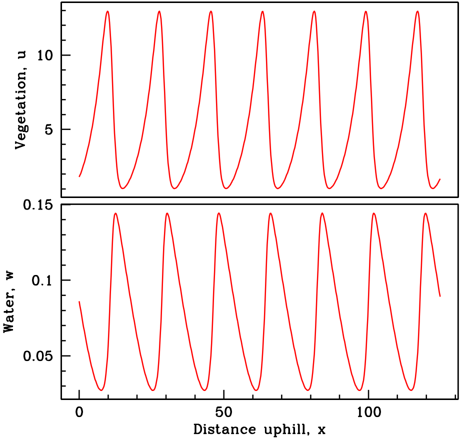

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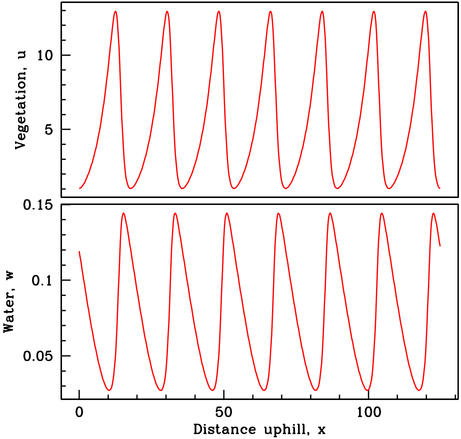

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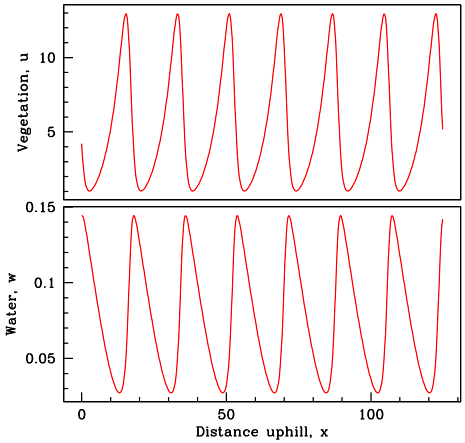

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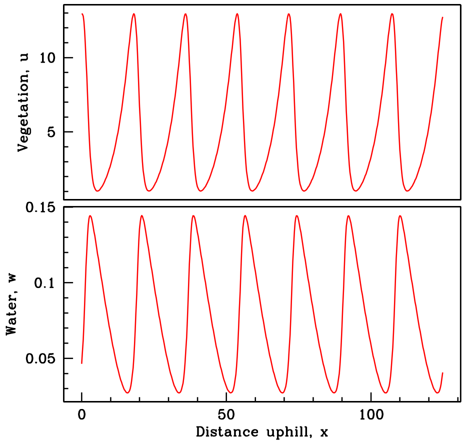

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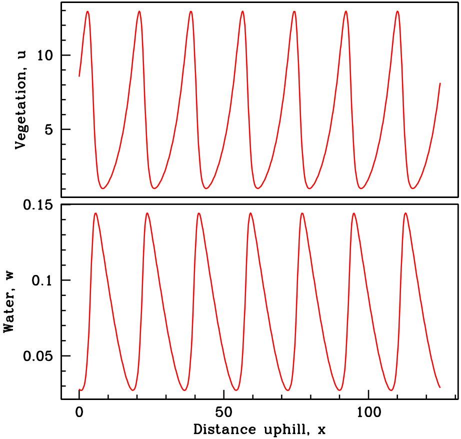

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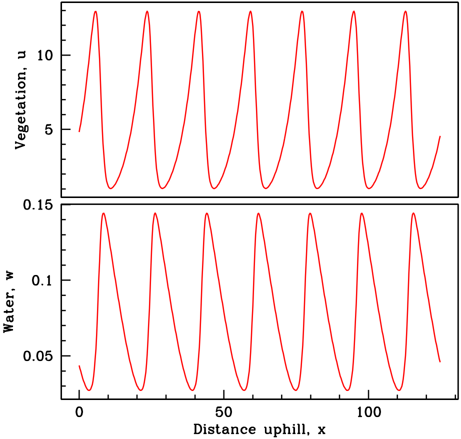

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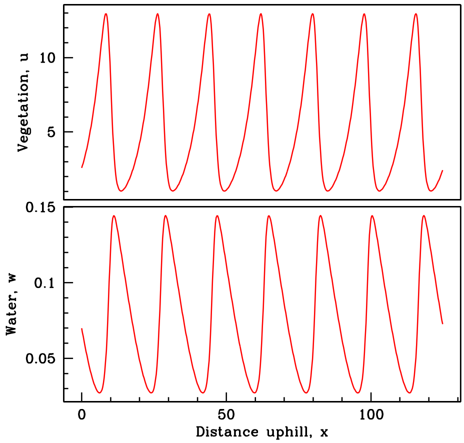

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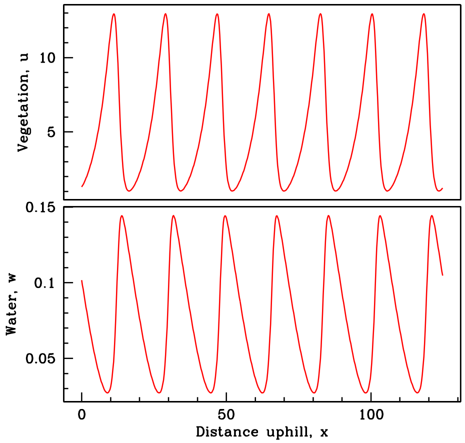

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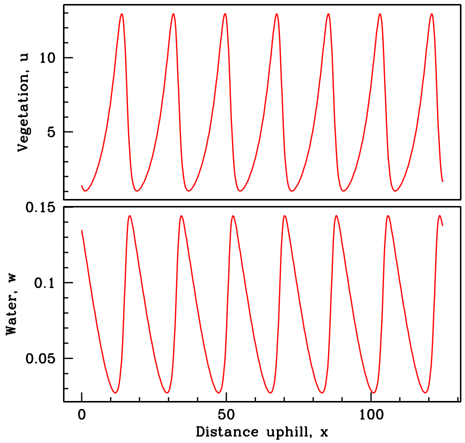

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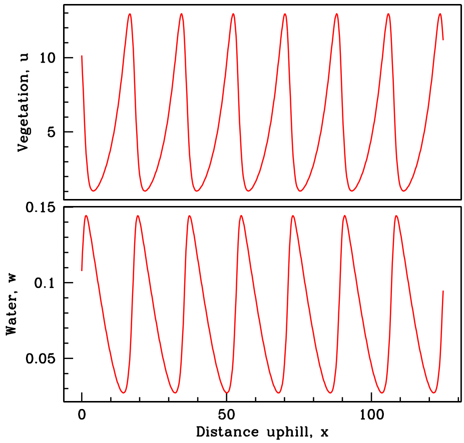

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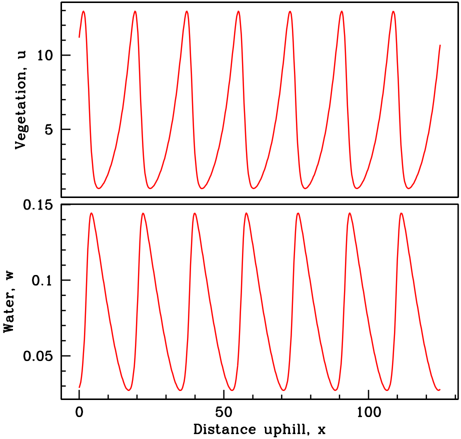

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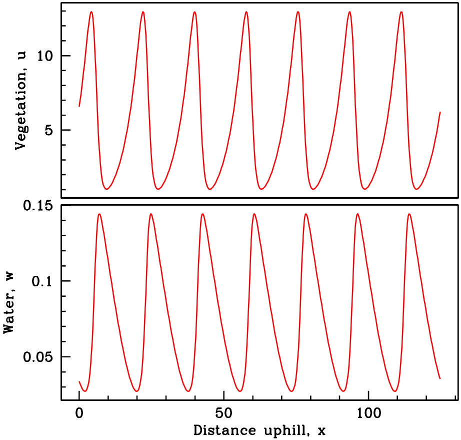

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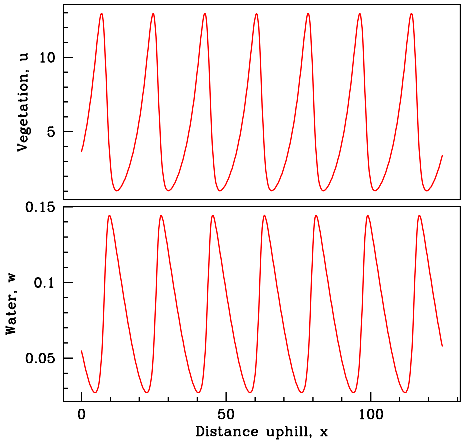

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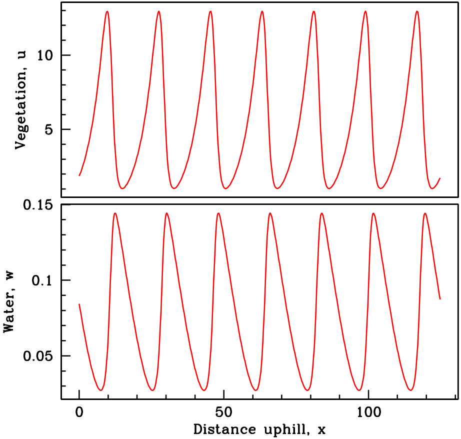

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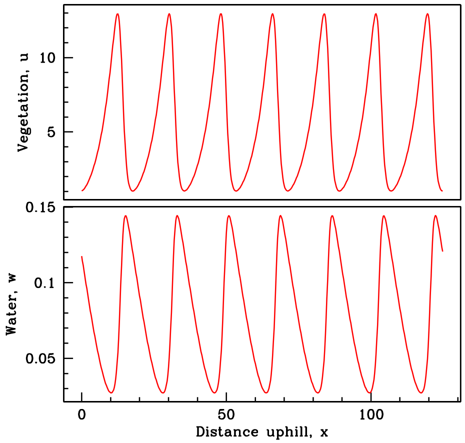

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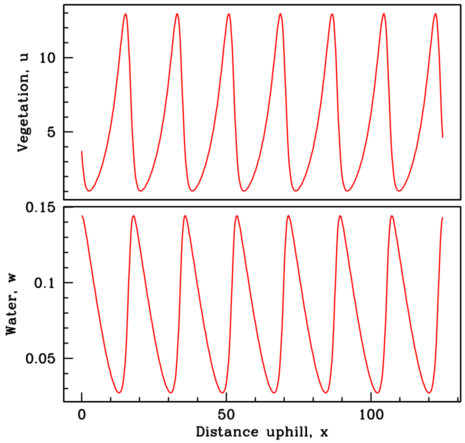

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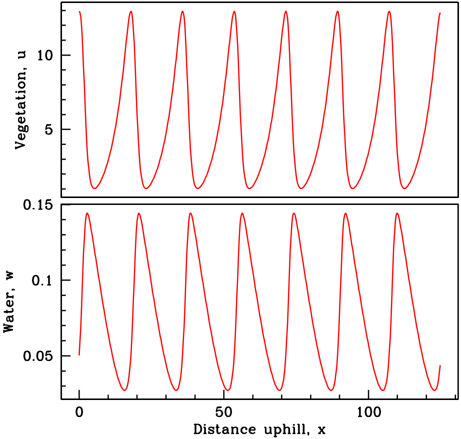
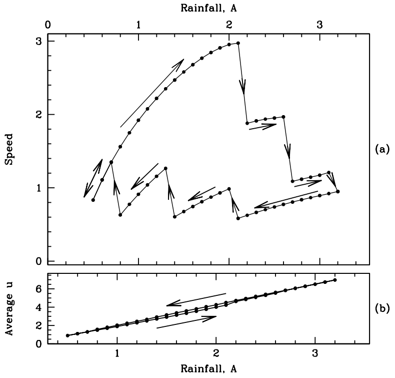
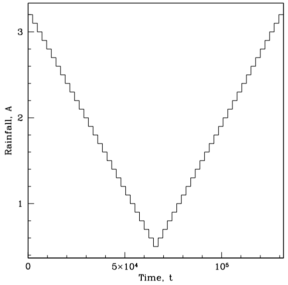


Variations in Rainfall: Simulations

Numerical simulations of patterns with varying rainfall show sudden changes and hysteresis.

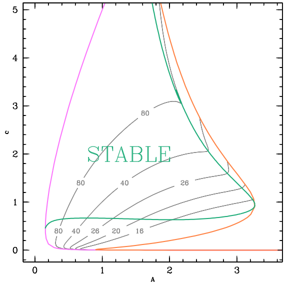




Domain length 150, periodic bc's

Pattern Existence and Stability

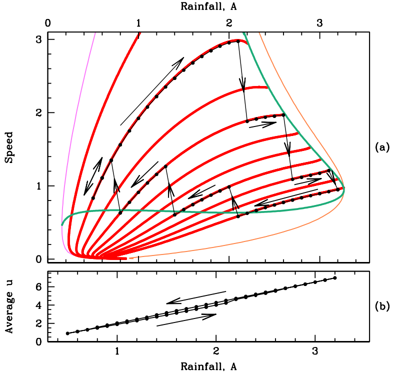
Detailed study using numerical continuation enables calculation of the region of parameter space in which patterns exist, and the sub-region in which they are stable.

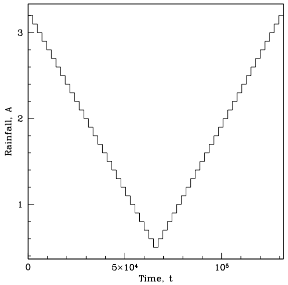


Software for this type of calculation is available at
www.ma.hw.ac.uk/wavetrain

Variations in Rainfall: Explanation

The stability region explains the sudden jumps and hysteresis.

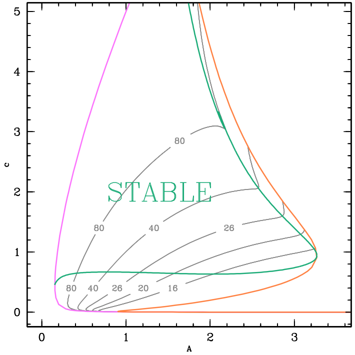




Domain length 150, periodic bc's

Tipping Points for Patterns

The parameter region for pattern existence/stability indicates the tipping points for pattern emergence and for desertification.



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J.A. Sherratt: History-dependent patterns of whole ecosystems. *Ecological Complexity* 14, 8-20 (2013).

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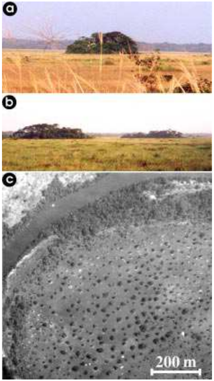
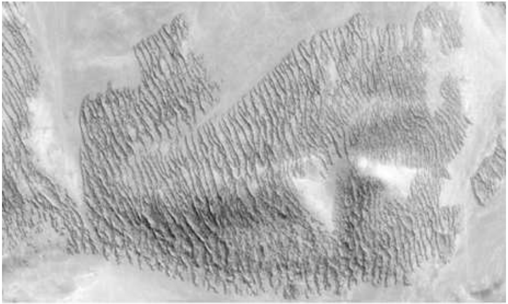
Tree Patches in Savannah Grasslands



(Olivier Lejeune et al, Phys. Rev. E 66: 010901, 2002)

Pattern of Fog-Dependent Vegetation in Chile



Aerial photo over Atacama Desert, Northern Chile

(Borthagaray et al, J. Theor. Biol. 265: 18-26, 2010)

Tillandsia landbeckii

Ribbon Forest in Colorado, USA

Photo taken by David Buckner

Mudflat Pattern in The Netherlands

(Weerman et al, Am. Nat. 176: E15-E32, 2010)

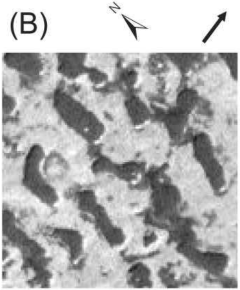
Mussel Bed Pattern in the Wadden Sea

In the Wadden Sea, mussel beds self-organise into striped patterns

Aerial photo of a mussel bed

Mussel Bed Pattern in the Wadden Sea

In the Wadden Sea, mussel beds self-organise into striped patterns



25 m by 25 m

Aerial photo of a mussel bed

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