Vegetation Patterns in Semi-Deserts

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This talk can be downloaded from my web site
www.ma.hw.ac.uk/~jas
Vegetation Patterns

Bushy vegetation in Niger

Mitchell grass in Australia
(Western New South Wales)

- Banded vegetation patterns are found on gentle slopes in semi-arid areas of Africa, Australia and Mexico
- Plants vary from grasses to shrubs and trees
- Typical wavelength 1km for shrubs and trees
Why Do Plants Form Patterns?

More plants ⇒ more roots and organic matter in soil
⇒ more infiltration of rainwater

Data from Burkina Faso
Rietkerk et al
Plant Ecology 148: 207-224, 2000

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Vegetation Patterns in Semi-Deserts
Banded Patterns on Slopes

On slopes, water flow downhill causes stripes which move uphill.
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Banded Patterns on Slopes

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Key Ecological Questions

- At what rainfall level is there a switch from uniform vegetation to patterns?
- At what rainfall level is there a transition to desert?
- How does the spacing of the vegetation bands depend on rainfall, herbivory and slope?
Mathematical Model of Klausmeier

Rate of change = Rainfall – Evaporation – Uptake by plants + Flow of water downhill

Rate of change = Growth, proportional to water uptake – Mortality + Random dispersal

\[ \frac{\partial w}{\partial t} = A - w - wu^2 + \nu \frac{\partial w}{\partial x} \]

\[ \frac{\partial u}{\partial t} = wu^2 - Bu + \frac{\partial^2 u}{\partial x^2} \]
Mathematical Model of Klausmeier

Rate of change = Rainfall – Evaporation – Uptake by + Flow of water
by plants downhill

Rate of change = Growth, proportional to water uptake
– Mortality + Random dispersal

\[ \frac{\partial w}{\partial t} = A - w - wu^2 + \nu \frac{\partial w}{\partial x} \]

\[ \frac{\partial u}{\partial t} = wu^2 - Bu + \frac{\partial^2 u}{\partial x^2} \]

The nonlinearity in \( wu^2 \) arises because the presence of plants increases water infiltration into the soil.
The nonlinearity in $wu^2$ arises because the presence of plants increases water infiltration into the soil.
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance](image-url)
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

![Diagram showing vegetation and water distribution over distance uphill, x.](attachment:image.png)
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill]

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill, x.](image-url)
Typical Solution of the Model

![Graph showing vegetation patterns in semi-deserts](image-url)
Typical Solution of the Model
Typical Solution of the Model

![Graph showing the typical solution of the model with vegetation and water levels as functions of distance uphill.](image)
Typical Solution of the Model

![Graph showing vegetation and water profiles over distance downhill, x.](image-url)
Typical Solution of the Model

- Vegetation, $u$
- Water, $w$
- Distance uphill, $x$
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill, x.](graph_image)
Typical Solution of the Model
Typical Solution of the Model

![Graph showing vegetation and water distribution across distance uphill.](image-url)
Typical Solution of the Model

![Graph showing vegetation and water patterns](image-url)
Typical Solution of the Model

![Graph showing the typical solution of the model](image-url)
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

- Vegetation, $u$
- Water, $w$

Distance uphill, $x$
Typical Solution of the Model

Vegetation patterns in semi-deserts can be described using mathematical models. The figure illustrates the typical solution of the model proposed by Klausmeier, showing oscillations in vegetation and water content as a function of distance uphill. The graphs depict the change in vegetation density and water content over distance, highlighting the periodic patterns characteristic of such ecosystems.
Typical Solution of the Model
Typical Solution of the Model

![Graph showing typical solution of the model with axes labeled Vegetation, u and Water, w against Distance uphill, x.]
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

![Graph showing vegetation distribution over distance uphill.](graph.png)
Typical Solution of the Model

![Graph showing vegetation and water patterns](image-url)

- Vegetation, $u$
- Water, $w$
- Distance uphill, $x$

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Vegetation Patterns in Semi-Deserts
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill.](image)

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill](image-url)
Typical Solution of the Model

![Graph showing vegetation and water distribution over distance uphill]

Vegetation Patterns in Semi-Deserts

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Mathematical Model of Klausmeier
Variations in Rainfall
References
Other Examples of Landscape-Scale Patterns
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

Vegetation, $u$

Water, $w$

Distance uphill, $x$
Typical Solution of the Model

![Graph of vegetation and water distribution](image-url)
Typical Solution of the Model

![Graph showing vegetation and water dynamics over distance uphill](attachment:image.png)
Typical Solution of the Model

Vegetation, \( u \)

Water, \( w \)

Distance uphill, \( x \)
Typical Solution of the Model

![Graph showing vegetation and water levels over distance uphill](image-url)
Typical Solution of the Model

The graph shows the typical solution of the Mathematical Model of Klausmeier, with oscillations in vegetation density and water content as a function of distance uphill (x). The upper graph represents vegetation density (u), while the lower graph represents water content (w). The patterns demonstrate the self-organized structure characteristic of semi-desert landscapes.
Typical Solution of the Model

![Graph showing vegetation and water patterns over distance](image-url)

Vegetation Patterns in Semi-Deserts

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Numerical simulations of patterns with varying rainfall show sudden changes and hysteresis.

Domain length 150, periodic bc’s
Pattern Existence and Stability

Detailed study using numerical continuation enables calculation of the region of parameter space in which patterns exist, and the sub-region in which they are stable.

Software for this type of calculation is available at www.ma.hw.ac.uk/wavetrain
The stability region explains the sudden jumps and hysteresis.

Domain length 150, periodic bc’s
The parameter region for pattern existence/stability indicates the tipping points for pattern emergence and for desertification.
References


Tree Patches in Savannah Grasslands

Pattern of Fog-Dependent Vegetation in Chile

Aerial photo over Atacama Desert, Northern Chile

Tillandsia landbeckii
Ribbon Forest in Colorado, USA

Photo taken by David Buckner
Mudflat Pattern in The Netherlands

B  A patterned mudflat

Mussel Bed Pattern in the Wadden Sea

In the Wadden Sea, mussel beds self-organise into striped patterns

Aerial photo of a mussel bed
Mussel Bed Pattern in the Wadden Sea

In the Wadden Sea, mussel beds self-organise into striped patterns

25 m by 25 m
Ecological Background

Why Do Plants Form Patterns?
Banded Patterns on Slopes
Key Ecological Questions

A Simple Mathematical Model

Mathematical Model of Klausmeier
Typical Solution of the Model

Variations in Rainfall

Variations in Rainfall: Simulations
Pattern Existence and Stability
Variations in Rainfall: Explanation
Tipping Points for Patterns

References

Other Examples of Landscape-Scale Patterns

Photo Gallery of Landscape-Scale Patterns