Equational Reasoning for Linking with First-Class Primitive Modules

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Overview

- Motivation.
- The m-Calculus.
- An m-Calculus Toolkit.
- Module Systems via m-Calculus.
- Confluence.
- Related/Future Work and Conclusion.

Some Desirable Goals for a Module Formalism

- Reuse (linked in different ways) without source code modification or access. "Black-box reuse".
- Intra-language modules and linking.
- First-class modules.
- Formalism should be reasonably close to real software practice. It should support such things as mutual dependencies and hierarchical namespaces.
- Formalism should have well defined semantics and support for showing correctness of transformations.

Existing Practice: C

- The "module system" of C extensively involves features that are not part of the C language. Linking is specified with makefiles. Generic modules are frequently implemented via some preprocessor.

- Namespace of modules is the filesystem. All components exported from modules appear in a single top-level C-language namespace. Inter-module connections are hard-wired to component names. There is no support for renaming components.

- C modules do not support black-box reuse. There is limited (extra-linguistic) support for first-class modules. A hierarchical namespace is not directly supported by C modules.

Existing Practice: Packages

- Languages like Ada, Modula-3, and Haskell have a kind of module we call *packages*.

- Two-layer namespace — a top level of modules and within each module a flat layer of component names. Inter-module connections are hard-wired to module/component name pairs.

- Packages do not support black-box reuse or a hierarchical namespace. Packages are not first-class.

Existing Practice: SML

- Module namespace is fully hierarchical.

- Two kinds of inter-module connections:
  - There is a linear order of module declarations. Later modules can use components of earlier ones by name.
  - A *functor* abstracts a module (a *structure*) over references to other modules and can then be applied to different argument modules later.

- ML supports limited black-box reuse via functors.

- Unlike other module systems, mutually dependent modules are forbidden. Both kinds of connections are asymmetric.

A Generalization: Incomplete Modules and Linking

$M_1 =$

```
(module
  exported f = N(f,g,i)
  deferred g
  deferred h
  private i = O(h))
```

$M_2 =$

```
(module
  deferred f
  exported g = P(f,i)
  deferred h
  private i = Q)
```

$M_1 \oplus M_2 =$

```
(module
  exported f = N(f,g,i)
  exported g = P(f,i')
  deferred h
  private i = O(h)
  private i' = Q)
```

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The m-Calculus: Syntax

- The main constructor is the *module* (more on next page):

$$\{F_1 \triangleright x_1 = B_1,\ \ldots,\ F_n \triangleright x_n = B_n\}$$

- The operations are *linking*, *selection*, *hiding*, and *sieving*:

$$M_1 \oplus M_2, \quad M_1.f, \quad M\backslash f, \quad M \backslash\!- \{f_1, \ldots, f_n\}$$

- Although it is essentially definable, for convenience the auxiliary *letrec* constructor is included:

$$\langle M \mid x_1 = M_1,\ \ldots,\ x_n = M_1\rangle$$

Terminology for m-Calculus Modules

external names *empty body* *anonymous marker*

$$\{\underbrace{\underbrace{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \_ = \mathrm{N},}_{\textit{exported}}\ \underbrace{\mathrm{h} \triangleright \mathrm{y} = \bullet,}_{\textit{deferred}}\ \underbrace{\_ \triangleright \mathrm{z} = \mathrm{O}}_{\textit{private}}}_{\textit{components}}\}$$

- The *internal* name "_" abbreviates a variable occurring nowhere else.
- A module has features of a λ-abstraction, a record, and an environment.

The m-Calculus: Semantics

- The first major operation is *linking*:

$$\{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \mathrm{z} = \mathrm{N},\ \mathrm{h} \triangleright \mathrm{y} = \bullet\} \oplus \{\mathrm{g} \triangleright \mathrm{z} = \bullet,\ \mathrm{h} \triangleright \mathrm{y} = \mathrm{O},\ \_ \triangleright \mathrm{w} = \mathrm{P}\}$$

$$\downarrow\ (\textbf{link})$$

$$\{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \mathrm{z} = \mathrm{N},\ \mathrm{h} \triangleright \mathrm{y} = \mathrm{O},\ \_ \triangleright \mathrm{w} = \mathrm{P}\}$$

- The second major operation is *selection*:

$$\{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \mathrm{z} = \mathrm{N},\ \mathrm{h} \triangleright \mathrm{y} = \mathrm{O},\ \_ \triangleright \mathrm{w} = \mathrm{P}\}.\mathrm{g}$$

$$\downarrow\ (\textbf{select})$$

$$\langle \mathrm{z} \mid \mathrm{x} = \mathrm{M},\ \mathrm{z} = \mathrm{N},\ \mathrm{y} = \mathrm{O},\ \mathrm{w} = \mathrm{P}\rangle$$

Substitution in the m-Calculus

- The m-calculus does substitution *one* target at a time:

$$\{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \_ = \underline{\mathrm{x}}\} \longrightarrow \{\mathrm{f} \triangleright \mathrm{x} = \mathrm{M},\ \mathrm{g} \triangleright \_ = \mathrm{M}\}$$
$$\langle \underline{\mathrm{x}} \mid \mathrm{x} = \mathrm{M},\ \mathrm{y} = \mathrm{N} \rangle \longrightarrow \langle \mathrm{M} \mid \mathrm{x} = \mathrm{M},\ \mathrm{y} = \mathrm{N} \rangle$$

  Garbage collection rules later remove unreferenced bindings.

- Cyclic bindings must sometimes be kept. Keeping bindings in the same place *and* substituting for all occurrences is just less flexible. Bindings can be removed from their current location by including copies with replacements [Ancona and Zucca, 1999], but this loses sharing and explodes size, e.g.:

$$\begin{aligned} &\langle (\mathrm{x},\mathrm{x}) \mid \mathrm{x} = (\mathrm{x},\mathrm{y}),\ \mathrm{y} = (\mathrm{y},\mathrm{x}) \rangle \\ &\quad \longrightarrow (\langle \mathrm{x} \mid \mathrm{x} = (\mathrm{x},\mathrm{y}),\ \mathrm{y} = (\mathrm{y},\mathrm{x}) \rangle, \langle \mathrm{x} \mid \mathrm{x} = (\mathrm{x},\mathrm{y}),\ \mathrm{y} = (\mathrm{y},\mathrm{x}) \rangle) \end{aligned}$$

Acyclic Substitution Restriction

- The m-calculus does not perform substitution along cycles:

$$\{\mathrm{f} \triangleright \mathrm{x} = (1, \mathrm{y}),\ \mathrm{g} \triangleright \mathrm{y} = (\underline{\mathrm{x}}, 2)\} \not\longrightarrow \{\mathrm{f} \triangleright \mathrm{x} = (1, \mathrm{y}),\ \mathrm{g} \triangleright \mathrm{y} = ((1, \mathrm{y}), 2)\}$$

- The restriction is needed for the m-calculus to be confluent. Ariola and Klop [1997] give an example showing that confluence is impossible without the restriction.

- Because only substitution into letrec bodies is needed for evaluation, this restriction is merely a loss in equational reasoning power.

- If this restriction were removed, we believe the m-calculus would still be consistent, but different proof methods would be needed to show this.

Dynamic Errors in the m-Calculus

- Linking modules with common output components:

$$\{\mathrm{f} \triangleright \mathrm{w} = \bullet,\ \mathrm{g} \triangleright \mathrm{x} = \mathrm{M}\} \oplus \{\mathrm{f} \triangleright \mathrm{y} = \mathrm{N},\ \mathrm{g} \triangleright \mathrm{z} = \mathrm{N}'\}$$

- Selecting from an incomplete module:

$$\{\mathrm{f} \triangleright \mathrm{w} = \bullet,\ \mathrm{g} \triangleright \mathrm{x} = \mathrm{M}\}.\mathrm{g}$$

- Selecting a nonexistent component:

$$\{\mathrm{f} \triangleright \mathrm{w} = \mathrm{M},\ \mathrm{g} \triangleright \mathrm{x} = \mathrm{N}\}.\mathrm{h}$$

- Hiding an input component:

$$\{\mathrm{f} \triangleright \mathrm{w} = \bullet,\ \mathrm{g} \triangleright \mathrm{x} = \mathrm{N}\}\backslash\mathrm{f}$$
$$\{\mathrm{f} \triangleright \mathrm{w} = \bullet,\ \mathrm{g} \triangleright \mathrm{x} = \mathrm{N}\}\backslash{-}\{\mathrm{g}\}$$

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λ-Calculus in m-Calculus

- The λ-calculus can be encoded by choosing two distinguished component names "arg" and "res":

$$(\lambda x.M) = \{\mathrm{arg} \triangleright x = \bullet,\ \mathrm{res} \triangleright \_ = M\}$$

$$(M M') = (M \oplus \{\mathrm{arg} \triangleright \_ = M'\}).\mathrm{res}$$

  (Example of rewriting simulation of β rule on next page.)

- The encoding is like that independently developed by Ancona and Zucca [1999]. It is only superficially like the encoding of λ-calculus in ς-calculus by Abadi and Cardelli [1996].

How the λ-Calculus Encoding Works

$$
\begin{array}{lrl}
 & & (\lambda x.x\,x)(y\,z) \\
 & = & (\{\mathrm{arg} \triangleright x = \bullet,\ \ \mathrm{res} \triangleright w = (x\,x)\} \oplus \{\mathrm{arg} \triangleright x = (y\,z)\}).\mathrm{res} \\
(\textbf{link}) & \longrightarrow & \{\mathrm{arg} \triangleright x = (y\,z),\ \ \mathrm{res} \triangleright w = (x\,x)\}.\mathrm{res} \\
(\textbf{select}) & \longrightarrow & \langle w \mid x = (y\,z),\ \ w = (x\,x)\rangle \\
(\textbf{subst-letrec}) & \longrightarrow & \langle (x\,x) \mid x = (y\,z),\ \ w = (x\,x)\rangle \\
(\textbf{gc-letrec}) & \longrightarrow & \langle (x\,x) \mid x = (y\,z)\rangle \\
(\textbf{subst-letrec}) & \longrightarrow & \langle (y\,z)\,x \mid x = (y\,z)\rangle \\
(\textbf{subst-letrec}) & \longrightarrow & \langle (y\,z)\,(y\,z) \mid x = (y\,z)\rangle \\
(\textbf{gc-letrec}) & \longrightarrow & \langle (y\,z)\,(y\,z) \mid \ \rangle \\
(\textbf{empty-letrec}) & \longrightarrow & (y\,z)\,(y\,z)
\end{array}
$$

Records in m-Calculus

- Records are encoded as modules with components whose internal names are not used:

$$\{f_1 = M_1,\ \ldots,\ f_n = M_n\} \quad = \quad \{f_1 \triangleright \_ = M_1,\ \ldots,\ f_n \triangleright \_ = M_n\}$$

- The expected behavior is obtained:

$$\begin{array}{lll} & & \{\mathrm{f} = 1,\ \mathrm{g} = \mathrm{x}\}.\mathrm{g} \\ & = & \{\mathrm{f} \triangleright \mathrm{y} = 1,\ \mathrm{g} \triangleright \mathrm{z} = \mathrm{x}\}.\mathrm{g} \\ (\textbf{select}) & \longrightarrow & \langle \mathrm{z} \mid \mathrm{y} = 1,\ \mathrm{z} = \mathrm{x}\rangle \\ (\textbf{subst-letrec}) & \longrightarrow & \langle \mathrm{x} \mid \mathrm{y} = 1,\ \mathrm{z} = \mathrm{x}\rangle \\ (\textbf{gc-letrec}) & \longrightarrow & \langle \mathrm{x} \mid \ \rangle \\ (\textbf{empty-letrec}) & \longrightarrow & \mathrm{x} \end{array}$$

- Henceforth we let "$f = M$" abbreviate "$f \triangleright \_ = M$".

Other Encodings

The long version [Wells and Vestergaard, 1999] shows how to encode the following in the m-calculus:

- A component renaming operator that can rename both input and output components.
- The ς-calculus of objects of Abadi and Cardelli [1996].
- The module calculus of Ancona and Zucca [1999].
- The calculus of first-class contexts of Hashimoto and Ohori [1997].
- The $\lambda\varepsilon$-calculus of first-class environments of Sato et al. [1999].

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C-style Modules

- Each object file O exporting components (functions, variables) $f_1, \ldots, f_n$, depending on imported components $g_1, \ldots, g_m$, and with private components $h_1, \ldots, h_o$ can be represented as a module M:

$$M = \{f_1 \triangleright x_1 = M_1,\ \ldots,\ g_1 \triangleright y_1 = \bullet,\ \ldots,\ \_ \triangleright h_1 = N_1,\ \ldots\}$$

- The linking of modules $M_1, \ldots, M_n$ to form a program and invoke its start routine is represented as:

$$P = (M_1 \oplus \ldots \oplus M_n).\mathrm{main}$$

- We are ignoring the call-by-value and imperative features of C-like languages here.

Package-style Modules

This example Haskell program:

```
module A (f) where
 f = N

module B (f, g) where
 import A
 g = P(A.f)
```

```
module Main (main) where
 import qualified B
 f = 5
 main = Q(B.f,B.g,f)
```

can be encoded as these m-calculus modules:

$$
\begin{aligned}
M_{A} &= \{A = \{f = N\}\} \\
M_{B} &= \{A \triangleright x = \bullet,\ B = \{g = P(x.f),\ f = x.f\}\} \\
M_{Main} &= \{B \triangleright x = \bullet,\ Main = \{\_ \triangleright y = 5,\ main = Q(x.f, x.g, y)\}\}
\end{aligned}
$$

Package-style Modules (more)

The previous example's modules were encoded as:

$$
\begin{array}{ll}
\mathrm{M}_{\mathrm{A}} & = \{\mathrm{A} = \{\mathrm{f} = \mathrm{N}\}\} \\
\mathrm{M}_{\mathrm{B}} & = \{\mathrm{A} \triangleright \mathrm{x} = \bullet,\ \mathrm{B} = \{\mathrm{g} = \mathrm{P}(\mathrm{x.f}),\ \mathrm{f} = \mathrm{x.f}\}\} \\
\mathrm{M}_{\mathrm{Main}} & = \{\mathrm{B} \triangleright \mathrm{x} = \bullet,\ \mathrm{Main} = \{\_ \triangleright \mathrm{y} = 5,\ \mathrm{main} = \mathrm{Q}(\mathrm{x.f}, \mathrm{x.g}, \mathrm{y})\}\}
\end{array}
$$

The evaluation of the program proceeds like this:

$$
\begin{array}{ll}
 & (\mathrm{M}_{\mathrm{A}} \oplus \mathrm{M}_{\mathrm{B}} \oplus \mathrm{M}_{\mathrm{Main}}).\mathrm{Main.main} \\
\longrightarrow\!\!\!\!\rightarrow & \left\{\begin{array}{l} \mathrm{A} \triangleright \mathrm{x} = \{\mathrm{f} = \mathrm{N}\},\ \mathrm{B} \triangleright \mathrm{z} = \{\mathrm{g} = \mathrm{P}(\mathrm{x.f}),\ \mathrm{f} = \mathrm{x.f}\}, \\ \mathrm{Main} = \{\_ \triangleright \mathrm{y} = 5,\ \mathrm{main} = \mathrm{Q}(\mathrm{z.f}, \mathrm{z.g}, \mathrm{y})\} \end{array}\right\}.\mathrm{Main.main} \\
\longrightarrow\!\!\!\!\rightarrow & \left\{\begin{array}{l} \mathrm{A} = \{\mathrm{f} = \mathrm{N}\},\ \mathrm{B} = \{\mathrm{g} = \mathrm{P}(\mathrm{N}),\ \mathrm{f} = \mathrm{N}\}, \\ \mathrm{Main} = \{\mathrm{main} = \mathrm{Q}(\mathrm{N}, \mathrm{P}(\mathrm{N}), 5)\} \end{array}\right\}.\mathrm{Main.main} \\
\longrightarrow\!\!\!\!\rightarrow & \mathrm{Q}(\mathrm{N}, \mathrm{P}(\mathrm{N}), 5)
\end{array}
$$

Handling Other Haskell Features

- Although the example used qualified names (e.g., "`B.f`"), nearly all of Haskell's behavior with unqualified names can be encoded.

- There is one strange quirk of Haskell's qualified names that would need an m-calculus extension to be handled: the "diamond import" behavior.

- Other Haskell module namespace management features can be handled, e.g., listing entities *not* to import with unqualified names, local aliases for imported modules, etc.

- Haskell's type classes should not be hard if one insists on working only with the result of the dictionary-passing translation. And no one knows how to do rewriting directly with type classes, so we don't feel bad about it.

SML-style Modules

- We are ignoring the type-level aspects of SML modules as well as SML's imperative features and call-by-value behavior.

- SML provides modules called *structures* as well as a λ-calculus (*functors* and *functor applications*) for manipulating them.

- SML structures can be encoded directly as m-calculus modules.

- SML functors and functor applications can be encoded using the λ-calculus encoding given earlier.

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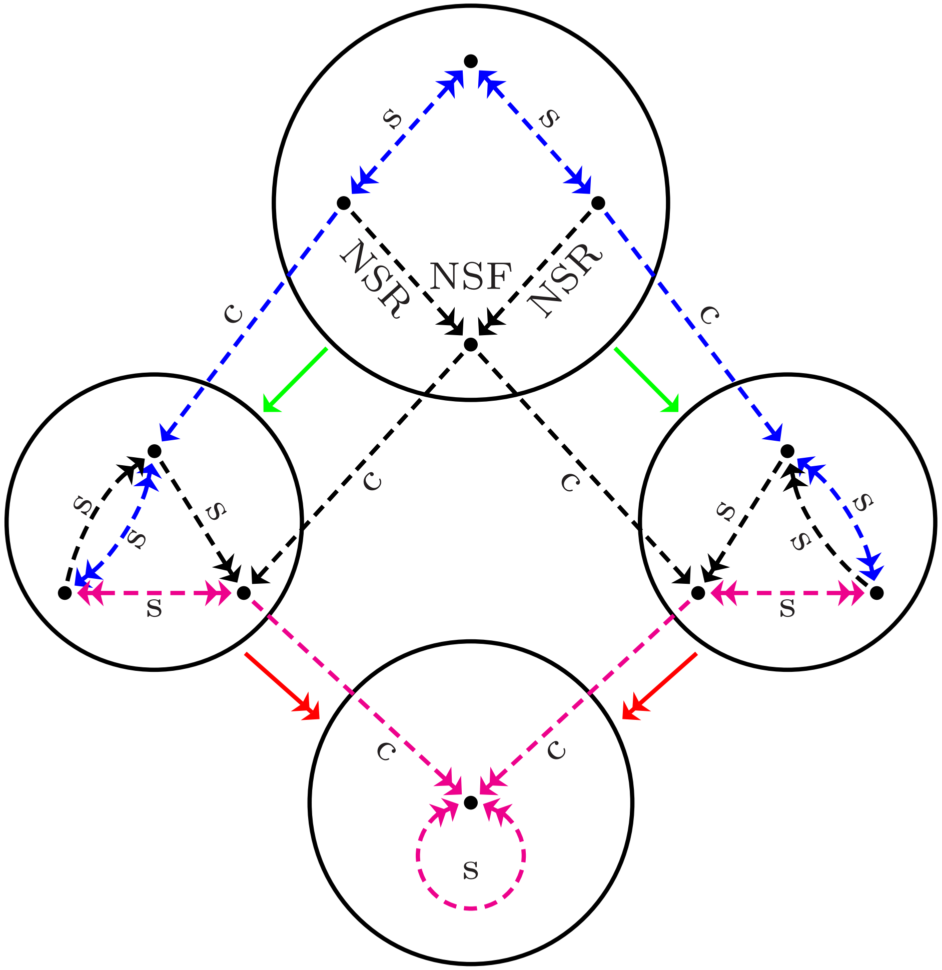
Confluence via Finite Developments

- Confluence of the m-calculus is shown via the well known method of *strong finite developments*, which means that any two coinitial developments can be joined by two further cofinal developments.

- Strong finite developments follows from *termination* and *weak confluence* of *marked rewriting*:
  - Termination of marked rewriting (a.k.a. finite developments) is proven using well known methods of weighted labels on terms.
  - Weak confluence of marked rewriting is proven as described on the following slide.

- The full proof can be found in [Wells and Vestergaard, 1999].

Weak Confluence of Marked Rewriting



○	(real) marked term
•	raw marked term
$\longrightarrow$	one step
$\twoheadrightarrow$	zero or more steps
$\twoheadleftarrow\!\!\twoheadrightarrow$	zero/more steps either way
——	on (real) marked terms
- - -	on raw marked terms
c	via computational rule(s)
s	via structural rule(s)
NSR	Nice Structural Rewriting
NSF	Nice Structural Form
——	premises
——	desired conclusion
- - -	equivalent to premises
- - -	equivalent to conclusion

Interesting Points about Proof Method

- Fully rigorous treatment of *structural* rewriting, including α-*conversion*.
- Nice structural term properties (e.g., "distinctly named bound variables") as side conditions on rules, *without* "Abracadabra!" style proofs.
- Convenient, like the convention in Barendregt [1984], but *rigorous*.
- The substitution on raw terms actually used is defined purely by *structural recursion* (as are similar term operations), making rigorous proofs much easier.

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Related Calculi with Modules and Linking

	Rewrite Semantics	1st-Class Modules	1-Target Subst.	Confluence	Typed	General Comp.
Cardelli [1997]	yes	no	no	yes	yes	no
Ancona and Zucca [1998]	yes?	no	no	yes	no	no
Ancona and Zucca [1999]	yes	yes	no	yes?	yes	yes
Machkasova and Turbak [2000]	yes	no	yes	no	no	yes
Wells and Vestergaard [2000]	yes	yes	yes	yes	no	yes
Flatt and Felleisen [1998]	no	yes	n/a	n/a	no	yes
Drossopoulou et al. [1999]	no	?	n/a	n/a	yes	?

Future Work

- Type inference. The best candidate type system seems to be one by Harper and Pierce [1991] for polymorphic record concatenation.
- Extending calculus with features such as call-by-value evaluation, assignment statements, etc.
- Formalizing proof method in Coq.
- Using the design ideas of the m-calculus to make a more powerful module system for Haskell.

Conclusion

The m-calculus:

- Supports smooth encodings of the λ-calculus, renaming, records, first-class environments, etc.
- Supports first-class modules, mutual intermodule dependencies, hierarchical namespaces, and black-box reuse.
- Can represent several existing styles of module system.
- Has an extremely well behaved rewrite relation.

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