Equations and formal languages in algebra

Schedule

Monday, March 7:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Murray Elder**: *Minicourse 1.*
9:50 - 10:15	coffee
10:15 - 11:05	**Casals/Kazachkov**: *Minicourse 2.*
11:15 - 12:05	**Dirk Nowotka** *On word equations in one variable with constants*
12:30	lunch
13:00 - 16:30	ski / free time
16:30 - 17:00	coffee at Hôtel les Sources
17:00 - 17:50	**Markus Lohrey** *Knapsack and subsetsum for nilpotent, polycyclic and co-context-free groups*
18:00 - 18:30	**Georg Zetzsche** *Knapsack in graph groups, HNN-extensions and amalgamated products*
18:40 - 19:10	**Maranda Franke** *Geodesic Language Complexity and Group Structure*
19:30	dinner

Tuesday, March 8:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Murray Elder**: *Minicourse 1.*
9:50 - 10:15	coffee
10:15 - 11:05	**Casals/Kazachkov**: *Minicourse 2*
11:15 - 12:05	**Geraud Senizergues** *First-order logic with idempotent variables over free inverse monoids*
12:30	lunch
13:00 - 16:30	ski / free time
16:30 - 17:00	coffee at Hôtel les Sources
17:00 - 17:50	**Artur Jeż** *Finding all Solutions in Equations in Free Groups*
18:00 - 18:30	**Albert Garetta-Fontselles**: *Systems of Equations in Random Nilpotent Groups*
18:40 - 19:10	**Armin Weiss**: *Word and conjugacy problem in Baumslag-Solitar groups*
19:30	dinner

Wednesday, March 9:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Olga Kharlampovich** *Equations and elementary classification questions in a group algebra of a free group*
9:50 - 10:15	coffee
10:15 - 11:05	**Bob Gilman** *The word problem of $\mathbb{Z} \times \mathbb{Z}$*
11:15 - 12:05	**Alina Vdovina** *Quadratic equations in hyperbolic groups are NP-complete*
12:30	lunch
13:00 - 16:30	ski / free time
16:30 - 17:00	coffee at Hôtel les Sources
17:00 - 17:50	**Yago Antolín**: *Formal conjugacy growth and hyperbolicity*
18:00 - 18:30	**Bryan Jacobson**: *Algebraic subgroups of acylindrically hyperbolic groups*
19:30	dinner

Thursday, March 10:

8:00 - 9:00	Breakfast
9:00 - 9:50	**Stepan Holub**: *Algebraic Properties of Word Equations*
9:50 - 10:15	coffee
10:15 - 11:05	**Alexei Miasnikov**: *Equations in groups and rings*
11:15 - 12:00	**Kazachkov**: *On the Lyndon's group of the free pro-p group*
12:30	lunch

Abstracts of minicourses

Murray Elder (Newcastle): *Languages and groups*

I will introduce a menagerie of formal language classes that sit somewhere between regular and context-sensitive, and somehow tie them in with problems concerning infinite finitely generated groups. Lecture 1 will focus on the word problem for a group, which is either: a decision problem to decide on input a word in the generators, whether or not the word evaluates to the identity; or a language, the set of all words in the generators that evaluate to the identity. I will discuss a range of examples and results classifying (or attempting to) groups having word problem in some language class. Lecture 2 will consider different problems for groups, including geodesics, growth, amenability, primitives in free groups, and solutions to equations over groups. By necessity the survey will be pretty fast and short on precise details, relevant references will be given instead.

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Montse Casals/Ilya Kazachkov (Bilbao) *Makanin-Razborov Diagrams*

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Abstracts of talks

Yago Antolín (Vanderbilt)
Formal conjugacy growth and hyperbolicity

Rivin conjectured that the formal conjugacy growth series of an hyperbolic group is rational if and only if the group is virtually cyclic. In this talk, I will present a proof of Rivin's conjecture and supporting evidence for the analogous statement for acylindrically hyperbolic groups. This is joint work with Laura Ciobanu.

Kazachkov (Bilbao)
Finite index subgroups of partially commutative groups

A famous result of Baumslag-Roseblade states that a finitely presented subgroup of the direct product of two free groups is virtually the direct product of two free groups. In this talk we will discuss the structure of subgroups of coherent pc groups from this perspective. On the one hand, we will show that an analogue of the Baumslag-Roseblade's result holds for finite index subgroups, namely, any finite index subgroup of a coherent pc group is virtually a pc group; and on the other hand, we will prove that this result cannot be extended to finitely presented subgroups. More precisely, we give an example of a subgroup of a coherent pc group of type FP which is not virtually partially commutative.

Albert Garetta-Fontelles (Stevens/Barcelona)
Systems of Equations in Random Nilpotent Groups

We introduce and study a model for choosing random (torsion-free, class 2) nilpotent groups G. In this model, G is chosen by randomly specifying a polycyclic presentation of G. This differs significantly from the *Gromov model* presented recently by Cordes, Duchin, Duong, Ho, and Snchez. We then provide several results concerning equations in nilpotent groups, and we address the following question: when are systems of equations decidable in a generic G? This is joint work with Alexei Miasnikov and Denis Ovchinnikov.

Bob Gilman (Stevens)
The word problem of $\mathbb{Z} \times \mathbb{Z}$

An introduction to and discussion of Sylvain Salvati's recently published result that the word problem of $\mathbb{Z} \times \mathbb{Z}$ is a multiple context free language.

Maranda Franke (Nebraska)
Geodesic Language Complexity and Group Structure

A finitely generated group has solvable word problem if its language of geodesics is computable; the complexity of this language has connections to algebraic and geometric properties of the group. Gilman, Hermiller, Holt and Rees showed that a group is virtually free if and only if there is a finite generating set which produces a locally excluding geodesic language. In this talk, I will discuss existence results that were motivated by the search for a group theoretic characterization of the related language restriction piecewise excluding.

Bryan Jeffrey Jacobson (Vanderbilt)
Algebraic subgroups of acylindrically hyperbolic groups

The Zariski topology on a group G is generated by taking the collection of solution sets to group equations to be a subbasis for the closed sets. A subset of G is called algebraic if it is Zariski-closed. We will discuss a structure theorem for non-elementary algebraic subgroups of acylindrically hyperbolic groups and provide a couple of basic applications.

Artur Jeż (Wroclaw)
Finding all Solutions in Equations in Free Groups

In this talk I will present an algorithm for solving equations in free groups, in particular, the algorithm gives a graph-like representation of all solutions.

The main idea is to reduce the case of groups to semigroups (with regular constraints and involution) and then employ a simple technique of local recompression. The technique is based on local modification of variables (replacing X by aX or Xa) and iterative replacement of pairs of letters occurring in the equation by a 'fresh' letter, which can be seen as a bottom-up compression of the solution of the given equation.

Ilya Kazachkov (Bilbao)
On the Lyndon's group of the free pro-p group

Lyndon introduced the free exponential group $F^{Z[t]}$ in the 60's in connection with Tarski's problem and proved that $F^{Z[t]}$ is discriminated by the free group. By a theorem of Kharlampovich-Miasnikov, the Lyndon's free group $F^{Z[t]}$ is the universe for limit groups over free group. In turn, Miasnikov and Remeslennikov have shown that $F^{Z[t]}$ can be constructed from the free group by a sequence of centraliser extensions. Hence, any limit group is a subgroup of an iterated centraliser extension.

Zalesskii and Kochloukova initiated the study of limit groups over the free pro-p group as subgroups of iterated extensions of centralisers in the category of pro-p groups. In this talk we will present a construction of a potential universe for limit groups over a free pro-p group.

O. Kharlampovich (Hunter College CUNY)
Equations and elementary classification questions in a group algebra of a free group

We will show that equations are undecidable in a group algebra of a free group over a field of characteristic zero. This is a surprising result that shows that answers to first-order questions in a free non-abelian group and in its group algebra are completely different. (Joint results with A. Miasnikov)

Stepan Holub(Prague)
Algebraic Properties of Word Equations

We survey possibilities of using polynomials to deal with word equations. We discuss in some detail the recently introduced method which allows to obtain nontrivial information about independence of word equations from determinants of polynomial vectors.

Markus Lohrey (Siegen)
Knapsack and subsetsum for nilpotent, polycyclic and co-context-free groups (joint work with Daniel Knig and Georg Zetzsche)

Recently, Myasnikov, Nikolaev and Ushakov considered classical knapsack related decision problems for arbitrary finitely generated (f.g.) groups. Among others, they studied the following problems for a f.g. group G (where elements of G are represented by finite words over the generators):

-Subset sum problem for G: Given elements $g_1, \ldots, g_k, g$ of G, decide whether there exist numbers $e_1 \ldots, e_k \in \{0, 1\}$ such that $g = g_1^{e_1} \ldots g_k^{e_k}$.

-Knapsack problem for G: Given elements $g_1, \ldots, g_k, g$ of G, decide whether there exist natural numbers $e_1, \ldots, e_k$ such that $g = g_1^{e_1} \ldots g_k^{e_k}$.

Among other results, Myasnikov et al. proved that for a f.g. virtually nilpotent group the subset sum problem can be solved in polynomial time. In the talk, I will speak about some recent results for knapsack and subset sum in nilpotent, polycyclic, and co-context-free groups that were jointly obtained with Daniel Knig and Georg Zetzsche. More precisely, we will show (if time permits):

- There exists a f.g. 2-step nilpotent group G with an undecidable knapsack problem.

- There exists a polycyclic group with an NP-complete subset sum problem.

- For every co-context-free group, knapsack is decidable.

Alexei Miasnikov (Stevens)
Equations in groups and rings

In this talk I will discuss decidability of equations in groups and rings, relations between them, outline some interesting open problems and describe recent approaches to their solutions.

Dirk Nowotka (Kiel)
On word equations in one variable with constants

Word equations with one variable and constants appear to be the most simple, non-trivial class of word equations. However, the structure of its solutions is not clear. It is known that such equations either have a solution set $(pq)^*p$, with pq primitive, or a solution set of size $O(\log n)$, where n is the number of occurrences of the variable. It has been conjectured that a solution set is either of infinite size or not larger than a constant c. Where we believe that $c = 2$. In this talk, the current state of affairs is presented and the connection to another longstanding open problem is drawn.

G. Sénizergues (LaBri)
First-order logic with idempotent variables over free inverse monoids

For every set X of cardinality ≥ 2, equations in the free inverse monoid generated by X are not recursively solvable ([Rozenblat, Sib. J. of Maths, 1985]). On the other hand, for equations where the solutions are prescribed to be idempotent, the existence of solutions becomes decidable [D-M-S, IJAC 2007] and even EXPTIME [D-D-M-S, CSR 2014].

We show here that this positive result cannot be extended to the full FO of such equations (with idempotent variables): we construct a FO interpretation of the free monoid into the free inverse monoid, using idempotent variables only. Since the idempotents of FIM(X) are essentially trees (the so-called Munn-trees) and the FO theory of terms is decidable [C,ICALP 91], this negative result was rather unexpected.

Alina Vdovina (Newcastle)
Quadratic equations in hyperbolic groups are NP-complete

Using geometry of closed surfaces we solve equations in hyperbolic and toral relatively hyperbolic groups. We also show that the tasks of solving quadratic equations in hyperbolic and toral relatively hyperbolic groups are NP-complete. The main results are based on joints works with O.Kharlampovich,A.Mohajeri, A.Taam.

Georg Zetzsche (Cachan)
Knapsack in graph groups, HNN-extensions and amalgamated products

This talk continues the one of Markus Lohrey on the knapsack problem in groups. It presents joint work with Markus Lohrey, in which it was shown that (i) the knapsack problem can be decided in NP for each graph group, and (ii) the property of having an NP algorithm for knapsack is passed on to finite extensions, HNN-extensions, and amalgamated products with finite identified subgroups. In particular, the important class of virtually special groups has this property. Moreover, the proof yields that the set of solutions of each knapsack instance in such groups is semilinear.

Armin Weiss (Stevens)
Word and conjugacy problem in Baumslag-Solitar groups

Baumslag-Solitar groups were introduced in 1962 by Baumslag and Solitar as examples for finitely presented non-Hopfian two-generator groups. Since then, they served as examples for a wide range of purposes. Concerning algorithmic aspects of generalized Baumslag-Solitar groups, several decidability results are known. Indeed, a straightforward application of standard algorithms leads to a polynomial time solution of the word problem. The conjugacy problem is more complicated; still decidability has been established by Anshel and Stebe in 1974. However, up to now no precise complexity estimates have been given.

I will outline how to design a LOGSPACE algorithm for both problems. More precisely, there is a uniform TC^0 many-one reduction of the word problem to the word problem of the free group. The known techniques for the conjugacy problem can be refined to show that it can be solved in LOGSPACE. Finally, there are uniform versions (where also the group is part of the input) of both word and conjugacy problem: while the word problem still is solvable in LOGSPACE, the conjugacy problem becomes EXPSPACE-complete for generalized Baumslag-Solitar groups.