Méthodes mathématiques de l'informatique

Série 3

à remettre jusqu'au Lundi 19.03.2012 12^{15}

Exercice $N^{\underline{o}}$ 1

In how many ways can one distribute 24 toys to 6 children

a) without any restriction

b) such that each child receives 4 toys

c) such that 3 of the children receive 5 toys each, and the other 3 children receive 3 toys each?

Exercice $N^{\underline{o}}$ 2

Show that $p_2(k) = \lfloor \frac{k}{2} \rfloor + 1$ and $p_3(k) = \left[\frac{(k+3)^2}{12} \right]$ by using the generating function of $p_n(k)$ we have seen in class.

Exercice $N^{\underline{o}}$ 3

We consider

$$f(z) = \frac{1}{(1+3z)^2}.$$

a) Expand $f(z)$ into a formal serie $f(z) = \sum a_k z^k$.

b) Give a recurrence relation with initial conditions that define the sequence $(a_k)_{k\geq 0}$.

The Pigeonhole Principle

The so-called *Pigeonhole Principle* is the following obvious fact : if n objects are put in m boxes and $n > m$, then at least one box contains two or more of the objects.

Exercice $N^{\underline{o}}$ 4

Suppose $A \subset \{1, 2, \ldots, 2n\}$ with $|A| = n+1$. Show that there are two numbers in A such that one divides the other.

Hint : think of any number $a \in A$ as a product $a = 2^k m$ where $k \in \mathbb{N}$ and $m \in \{1, \ldots, 2n-1\}$ is odd.

Exercice $N^{\underline{o}}$ 5

Given five points inside a square whose sides have length 2, prove that two are within $\sqrt{2}$ of each other.