Méthodes mathématiques de l'informatique

Série 12

à remettre jusqu'au Lundi 28.05.2012 12^{15}

Exercice *N*º 1

Convert the formulas

(a)
$$F = (\forall x \exists y P(x, g(y, f(x))) \vee \neg Q(z)) \vee \neg \forall x R(x, y)$$

(b)
$$F = \exists z (\exists x Q(x, z) \vee \exists x P(x)) \rightarrow \neg(\neg \exists x P(x) \wedge \forall x \exists z Q(z, x)).$$

into rectified prenex form.

Exercice *N*º 2

Prove that $\forall x \exists y P(x, y)$ is a consequence of $\exists u \forall v P(v, u)$, but not *vice versa*.

Not to hand in!

Exercice *N*º 3

(REVIEW for EXAM) Let A and B be finite sets with $|A| = n$ and $|B| = m$.

(a) How many maps $f : A \rightarrow B$ are there?

(b) How many injective maps $f : A \rightarrow B$ are there?

Exercice *N*º 4

(REVIEW for EXAM) In how many ways can five men and five women stand in a straight line (to buy movie tickets) if no two men are allowed to stand next to each other and no two women are allowed to stand next to each other?

Exercice *N*º 5

(REVIEW for EXAM) In how many ways can five men and five women sit at a round table with ten identical chairs if no two men are allowed to sit next to each other?

Exercice *N*º 6

(REVIEW for EXAM) Given an integer k, let a_k be the number of lists (x_1, x_2, x_3) of integers such that $x_1 > 10$, $x_2 > 5$, $x_3 > 20$ and $x_1 + x_2 + x_3 = k$. What is a_k?

Exercice *N*º 7

(REVIEW for EXAM) Use generating functions to find the number of ways to collect $15 from 20 distinct people if each of the first 19 people can give a dollar and the twentieth person can give either $1 or $5 or nothing.

Exercice *N*º 8

(REVIEW for EXAM) Let a_k be the number of ways one can distribute k identical balls into seven distinct boxes if the first box can have no more than 2 balls.

(a) Find the generating function for a_k.

(b) Find a_k.

Exercice *N*º 9

(REVIEW for EXAM) Convert the following formula into DNF:

$$(A \vee B) \rightarrow (C \wedge D).$$

Exercice *N*º 10

(REVIEW for EXAM) A clause is called *positive* if it contains only positive literals. Show that a clause set is satisfiable if it does not contain a positive clause.