

Coursework 1: Automated Reasoning and SE

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This piece of coursework is due by December, 8th, 2006

1. A. Give proofs (if provable) for the following formulae both in sequent calculi and natural deduction.

1. $\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \rightarrow \phi_2)$ (0.5 points)
2. $\neg\phi_1 \wedge \neg\phi_2 \vdash \phi_1 \rightarrow \phi_2$ (0.5 points)
3. $\neg\phi_1 \wedge \neg\phi_2 \vdash \neg(\phi_1 \wedge \phi_2)$ (1 point)
4. $\forall x(P(x) \rightarrow Q(x)), \forall xP(x) \vdash \forall xQ(x)$ (1 point)
5. $\forall x(P(x) \rightarrow Q(x)), \exists xP(x) \vdash \exists xQ(x)$ (1 point)
6. $\exists xP(x), \forall x\forall y(P(x) \rightarrow Q(y)) \vdash \forall yQ(y)$ (1 point)
7. $\neg\forall x\phi \vdash \exists x\neg\phi$ (1 point)

Total for part A: 7 Points.

B. Use PVS to prove the formulae 1–7. Use the basic commands *case*, *flatten*, *split*, *propax*, *instantiate* and *skolem*. These are the commands that implement the sequent calculus in PVS (See Chapter 3 of the PVS prover guide). For each formula provide the commands that you have used to prove it.

Total for part B: 7 points (one point for each proof).

2. Puzzle problem: Three friends Alex, Brendon, and Calum together with their girlfriends Ann, Brigitta and Claudia met to look at photos from their last holidays. There were photos from Australia, Bahamas, and Canada. Each couple brought food or drink; there were ale, bread and chocolate. The following is known. (i) Alex and Claudia are a couple and brought the bottles of ale with them. (ii) Brendon and his girlfriend, whose name is not Ann showed photos from Australia and brought the bread with them. The couple who brought the chocolate did not show photos from the Bahamas. They were not Calum and his girlfriend. Express the information using typed predicate logic, and later PVS specification language and try to answer the following questions. (i) Who is Brigitta's partner? (ii) Who showed the photos from Canada? (iii) Who brought the bread with them? Verify your answers using PVS.

Total for question 2: 6 points.